2019

AP CollegeBoard

AP® Physics C: Mechanics

Sample Student Responses and Scoring Commentary Set 2

Inside:

Free Response Question 3

- ☑ Scoring Guideline
- ☑ Student Samples
- ☑ Scoring Commentary

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

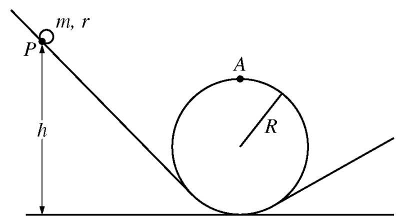
4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) LO INT-2.D, SP 5.A, 5.E
2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force		1 point
$F_C = mg + F_N = \frac{mv^2}{R}$		
$mg + 0 = \frac{mv^2}{R}$		
For a correct substitution into a correct expression.		1 point
$v = \sqrt{Rg}$		

Question 3 (continued)

(b) LO INT-7.E, SP 5.A, 5.E
3 points

Suppose the sphere is released from rest at some point *P* and rolls without slipping. Derive an equation for the minimum release height *h* that will allow the sphere to pass point *A* without losing contact with the track. Express your answer in terms of *b, m, R,* and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object		1 point
$K_1 + U_1 = K_2 + U_2$		
For correct substitutions, including rotational kinetic energy		1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$		
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(bmr^2\right)\left(\frac{v}{r}\right)^2 + mgh_2$		
For correctly substituting for the final height and the answer from part (a) for the velocity		1 point
$gh = \frac{1}{2}\left(\sqrt{Rg}\right)^2 + \frac{1}{2}(b)\left(\sqrt{Rg}\right)^2 + g(2R)$		
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$		

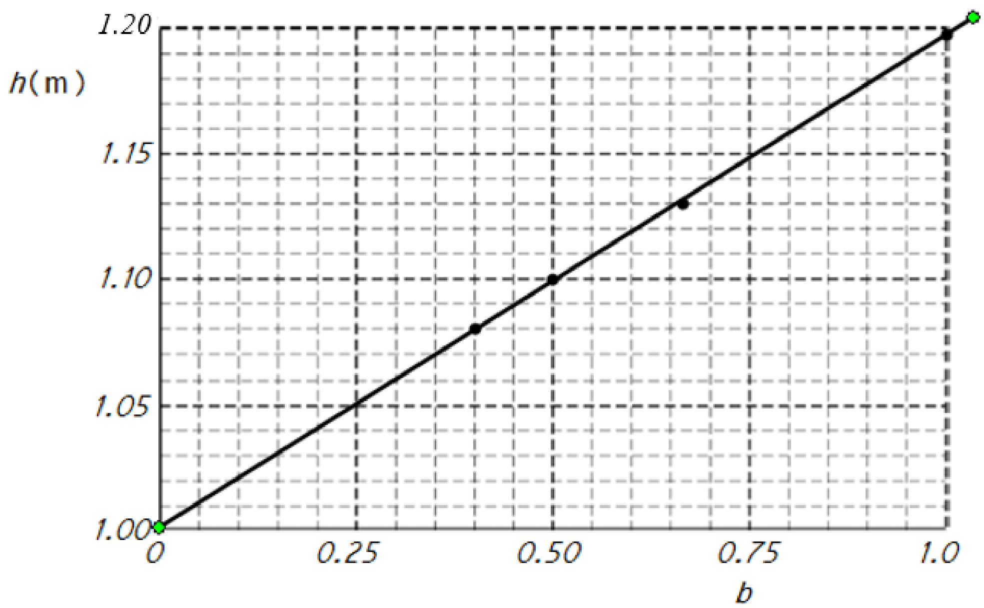
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h, no units for b		1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a best-fit straight line that represents the data		1 point

Question 3 (continued)

(d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation		1 point
$\text{slope} = \dfrac{1.20 - 1.00}{1.0 - 0.0} = 0.20 \text{ m}$		
$y = mx + b \therefore h = 0.20b + 1.00$		
$1.16 = 0.20b + 1.00$		
For a correct answer for *b*		1 point
$b = 0.80$		
Note: Full credit can be earned for the correct answer if there is an indication that this was read from the graph.		

(e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate *R*, the radius of the loop.

For substituting into the expression from part (b)		1 point
$h = \dfrac{1}{2}(b+5)R \therefore R = \dfrac{2(1.16 \text{ m})}{(0.80+5)}$		
For an answer consistent with previous parts		1 point
$R = 0.40 \text{ m}$		
Alternate Solution		*Alternate Points*
For using an expression for the y-intercept (set b = 0)		1 point
$h = \dfrac{1}{2}(b+5)R = \dfrac{1}{2}(0+5)R = \dfrac{5}{2}R$		
For determining the value of the intercept (1.0 m) from the graph		1 point
$1.0 \text{ m} = \dfrac{5}{2}R \therefore R = \dfrac{2}{5} \text{ m} = 0.40 \text{ m}$		

Question 3 (continued)

(f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

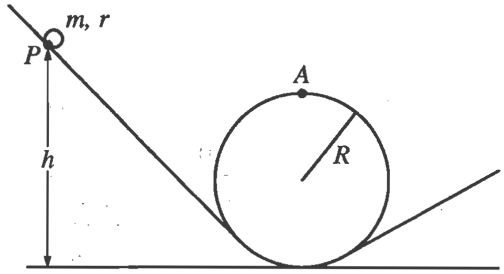
For selecting "Less" and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.
INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.



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3. The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$$\frac{mv^2}{R} = mg$$

$$v^2 = Rg \qquad v = \boxed{\sqrt{Rg}}$$

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$$mgh = \frac{1}{2} m Rg + 2mgR + \frac{1}{2} \cdot bmr^2 \cdot \left(\frac{\sqrt{Rg}}{r}\right)^2$$

$$gh = \frac{1}{2}Rg + 2Rg + \frac{1}{2} \cdot b \cdot Rg$$

$$h = \boxed{\frac{1}{2}R + 2R + \frac{1}{2}bR}$$

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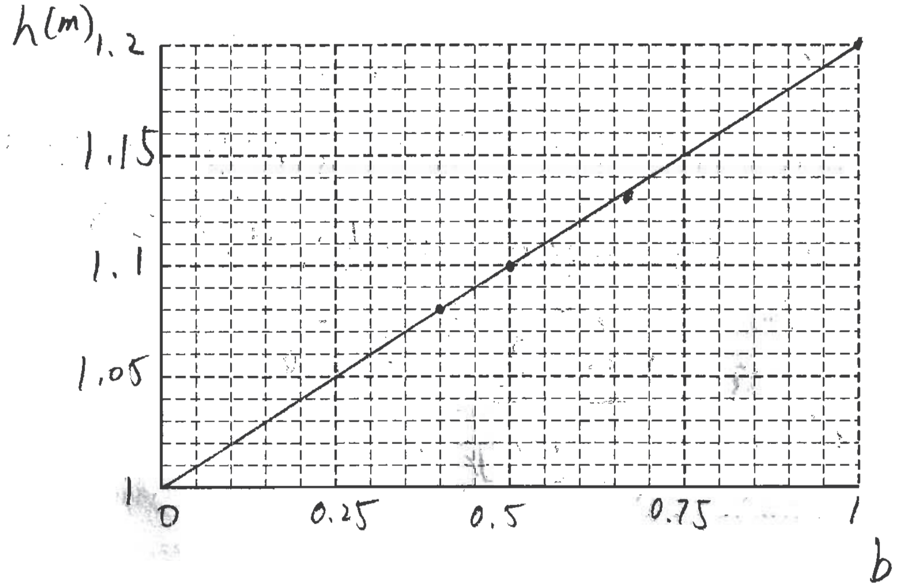
The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



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(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of b for the partially hollowed sphere.

used points (0.5, 1.1) and (1, 1.2) for slope

$\text{slope} = \frac{1.2 - 1.1}{1 - 0.5} = \frac{0.1}{0.5} = \frac{1}{5}$ y-intercept at (0, 1)

$1.16 = \frac{1}{5} \cdot b + 1$ $b = \boxed{0.8}$

(e) Calculate R, the radius of the loop.

$h = \frac{1}{2}R + 2R + \frac{1}{2}bR$ using point (1, 1.2)

$1.2 = 2\frac{1}{2}R + \frac{1}{2}R$

$1.2 = 3R$

$R = \boxed{0.4\,m}$

(f) In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater X Less ____ The same

Justify your answer.

The sphere will have a lower center of mass at the top of the loop, thus requiring a smaller centripetal acceleration. So the velocity at the top also does not need to be as high, meaning that the initial potential energy, which is determined partly by the release height, can be less.

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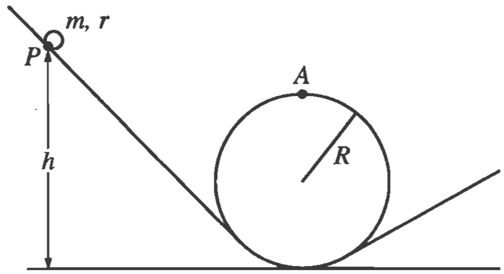

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(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$\Sigma F = ma$

$mg = \frac{mv^2}{R}$

$g = \frac{v^2}{R}$

$v = \sqrt{Rg}$

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$mgh = \frac{1}{2}mv^2 + mgh$

$mgh = \frac{1}{2}m(\sqrt{Rg})^2 + mg(2R)$

$h = \frac{\frac{1}{2}mRg + mg(2R)}{mg}$

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
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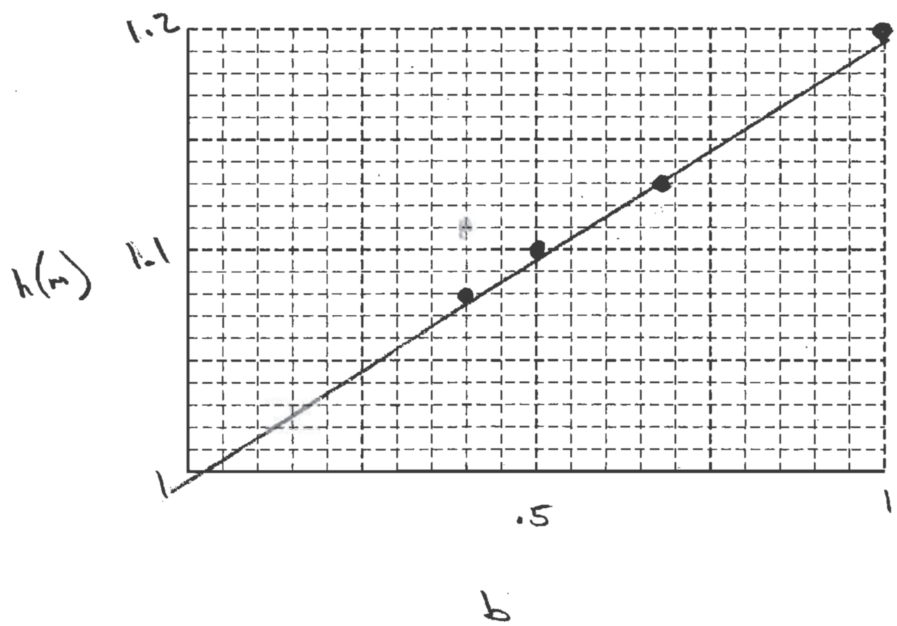

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(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

$b \approx .8$

(e) Calculate *R*, the radius of the loop.

$\text{Energy} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh$

(f) In part (b), the radius *r* of the rolling sphere was assumed to be much smaller than the radius *R* of the loop. If the radius *r* of the rolling sphere was not negligible, would the value of the minimum release height *h* be greater, less, or the same?

X Greater ____ Less ____ The same

Justify your answer.

A greater height would be necesary for the rolling sphere to have the required inertia to complete the loop.

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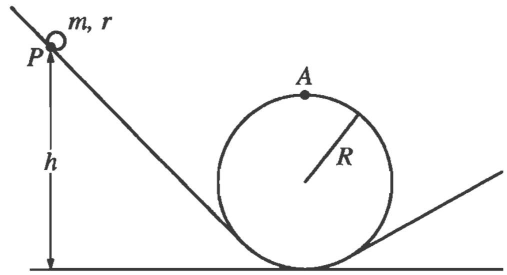

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(a) Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$a_c = \frac{v^2}{R}$ $F_c = F_N + F_g = ma = \frac{mv^2}{R}$

$F_g = mg = \frac{mv^2}{R}$ $v^2 = gR$ $v = \sqrt{gR}$

(b) Suppose the sphere is released from rest at some point P and rolls without slipping. Derive an equation for the minimum release height h that will allow the sphere to pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

$\Delta E = 0$

PE +

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
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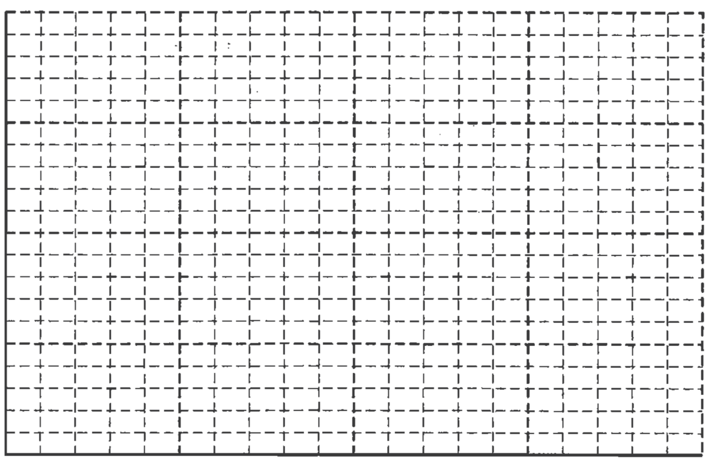

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(d) The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

(e) Calculate *R*, the radius of the loop.

(f) In part (b), the radius *r* of the rolling sphere was assumed to be much smaller than the radius *R* of the loop. If the radius *r* of the rolling sphere was not negligible, would the value of the minimum release height *h* be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The responses to this question were expected to demonstrate the following:

- Recognizing the forces in a system contributing to the centripetal force in a system with a vertical loop where the gravitational force needed to be considered properly
- Relating rotational velocity to linear velocity and the dependence on the radius of the circular motion involved in both spinning about a central axis
- Applying conservation of mechanical energy in a complex system with translational & rotational kinetic energy and with both initial and final gravitational potential energy
- Reading a data table and interpreting a graphical representation of that data, as well as constructing a graph and plotting given data in a meaningful way
- Being able to use this graph to deduce other related parameters of the system that were not directly measured

Sample: M Q3 A
Score: 15

This paper earned full credit. In part (a) 2 points were earned for relating centripetal force terms to gravitational force terms. In part (b) 3 points were earned for having a complete set of terms represented in the conservation of energy equation along with correct substitution for the final height and the answer from part (a). In part (c) 4 points were earned for correctly constructing, labeling, and best fitting data with a straight line along with showing use of more than half of the graph area available. In part (d) the points used off the graph for the slope are explicitly called out with calculation for b clearly shown, so 2 points were earned. In part (e) the calculation is started with an equation for h with a correct substitution from part (b), and a correct answer is shown, so 2 points were earned. In part (f) 2 points were earned for selecting the correct checkbox and providing a correct justification that calls out a lower release height based on a lower potential energy being needed as a result of the CM for the sphere being lower.

Sample: M Q3 B
Score: 8

Parts (a) and (c) earned full credit, 2 and 4 points, respectively. In part (b) the final height and the answer from part (a) for the velocity are substituted correctly, but a complete energy formulation is not shown, so 1 point was earned. In part (d) a correct answer for b is provided, but it is not clear reviewing the graph that the ".8" value is read explicitly from the graph, so 1 point was earned. In part (e) no clear attempt is made to solve for R using terms from the prior parts, so no points were earned. In part (f) no points were earned because the wrong checkbox is selected, and an incorrect explanation is provided.

Sample: M Q3 C
Score: 2

Part (a) earned full credit of 2 points. In part (b) an incomplete energy conservation relationship is shown, and substitution from part (a) is not shown, so no points were earned. Parts (c), (d), (e), and (f) are left blank, so no points were earned.