2019

AP® CollegeBoard

AP® Physics C: Mechanics

Scoring Guidelines
Set 2

General Notes About 2019 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. The requirements that have been established for the paragraph-length response in Physics 1 and Physics 2 can be found on AP Central at https://secure-media.collegeboard.org/digitalServices/pdf/ap/paragraph-length-response.pdf.

3. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

4. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point, and a student's solution embeds the application of that equation to the problem in other work, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics; Physics C: Mechanics, Physics C: Electricity and Magnetism Course Description* or "Terms Defined" in the *AP Physics 1: Algebra-Based Course and Exam Description* and the *AP Physics 2: Algebra-Based Course and Exam Description*.

5. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but the use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

6. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points

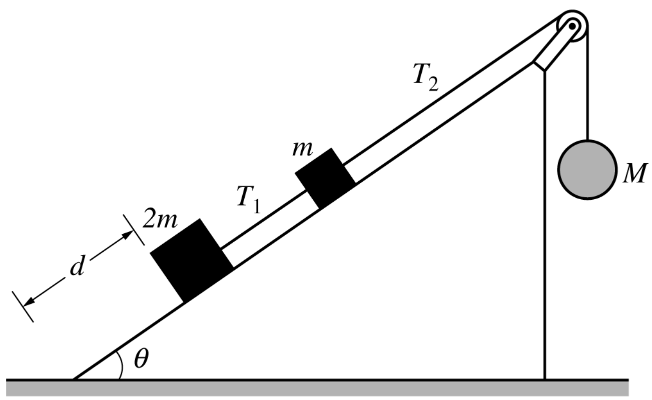
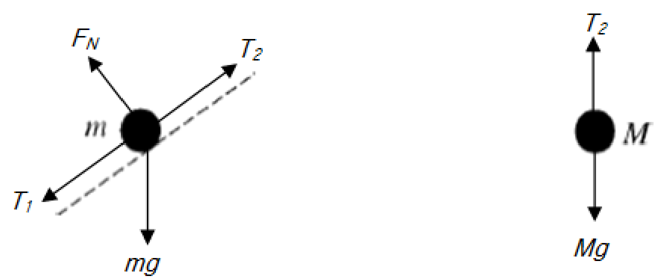


Figure 1

Blocks of mass m and $2m$ are connected by a light string and placed on a frictionless inclined plane that makes an angle θ with the horizontal, as shown in Figure 1 above. Another light string connecting the block of mass m to a hanging sphere of mass M passes over a pulley of negligible mass and negligible friction. The entire system is initially at rest and in equilibrium.

(a) LO INT-1.A, SP 3.D
3 points

On the dots below that represent the block of mass m and the sphere of mass M, draw and label the forces (not components) that act on each of the objects shown. Each force must be represented by a distinct arrow starting on and pointing away from the dot.



For correctly drawing and labeling vectors representing the normal force and the gravitational force on the block of mass m		1 point
For correctly drawing and labeling vectors representing the forces of tension on the block of mass m		1 point
For correctly drawing and labeling vectors representing the tension force and the gravitational force on the sphere of mass M		1 point
Note: A maximum of two points can be earned if there are any extraneous vectors.		

Question 1 (continued)

(b)

Derive expressions for the magnitude of each of the following. If you need to draw anything other than what you have shown in part (a) to assist in your solution, use the space below. Do NOT add anything to the figures in part (a).

i. LO INT-1.C.e, SP 1.D, 5.E
2 points

The force T_2 exerted on the block of mass m by the string. Express your answers in terms of m, θ , and physical constants, as appropriate.

For using an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{net} = (m + 2m)a = 0 \therefore T_2 - (m + 2m)g\sin\theta = 0$		
For a correct answer with supporting work		1 point
$T_2 = 3mg\sin\theta$		

ii. LO INT-1.D, SP 5.E
1 point

The mass M for which the system can remain in equilibrium. Express your answers in terms of m, θ , and physical constants, as appropriate.

For using a correct statement of Newton's second law for the whole system to derive an expression for M		1 point
$F_{net} = m_{tot}a \therefore Mg - 2mg\sin\theta - mg\sin\theta = 0$		
$M = 3m\sin\theta$		
Alternate Solution		*Alternate Points*
For a correct statement of Newton's second law for the sphere and an answer consistent with part (b)(i)		1 point
$T_2 - Mg = 0 \therefore Mg = T_2 \therefore M = T_2/g$		
$M = 3m\sin\theta$		

Question 1 (continued)

(c)

Now suppose that mass M is large enough to descend and that the sphere reaches the floor before the blocks reach the pulley. Answer the following for the moment immediately after the sphere reaches the floor.

i. LO INT-1.D, SP 7.A
1 point

Does the tension T_1 increase, decrease to a nonzero value, decrease to zero, or stay the same?

____ Increase ____ Decrease to a nonzero value

____ Decrease to zero ____ Stay the same

For correctly stating that the magnitude of T_1 drops to zero		1 point

ii. LO INT-1.D, SP 7.A
1 point

Is the velocity of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Up the ramp"		1 point

iii. LO INT-1.D, SP 7.A
1 point

Is the acceleration of the block of mass m up the ramp, down the ramp, or zero?

_____ Up the ramp _____ Down the ramp _____ Zero

For selecting "Down the ramp"		1 point

(d) LO INT-1.D, SP 5.A, 5.E
3 points

Consider the initial setup in Figure 1. Now suppose the surface of the incline is rough and the coefficient of static friction between the blocks and the inclined plane is μ_s. Derive an expression for the minimum possible value of M that will keep the blocks from moving down the incline. Express your answer in terms of m, μ_s, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the system		1 point
$F_{net} = m_{tot}a \therefore 2mg\sin\theta - f_{s2} + mg\sin\theta - f_{s1} - Mg = 0$		
For attempting to substitute in for the force of friction		1 point
$3mg\sin\theta - \mu_s F_{N2} - \mu_s F_{N1} = Mg$		
$3mg\sin\theta - \mu_s(2mg\cos\theta) - \mu_s(mg\cos\theta) = Mg$		
For a correct answer with supporting work		1 point
$M = 3m(\sin\theta - \mu_s\cos\theta)$		

Question 1 (continued)

(e) LO INT-1.D, CHA-1.A.b, SP 5.A, 5.E
3 points

The string connecting block m and the sphere of mass M then breaks, and the blocks begin to move from rest down the incline. The lower block starts a distance d from the bottom of the incline, as shown in Figure 1. The coefficient of kinetic friction between the blocks and the inclined plane is μ_k. Derive an expression for the speed of the blocks when the lower block reaches the bottom of the incline. Express your answer in terms of m, d, μ_k, θ, and fundamental constants, as appropriate.

For an attempt at a correct statement of Newton's second law for the two blocks		1 point
$F_{\text{net}} = m_{tot}a \therefore 2mg\sin\theta - f_{k2} + mg\sin\theta - f_{k1} = 3ma$		
Solve for the acceleration		
$3mg\sin\theta - \mu_k(2mg\cos\theta) - \mu_k mg\cos\theta = 3ma$		
$a = g(\sin\theta - \mu_k\cos\theta)$		
For using a correct kinematics equation to solve for the final velocity		1 point
$v_2^2 = v_1^2 + 2ad$		
$v_2^2 = 0^2 + 2g(\sin\theta - \mu_k\cos\theta)d$		
For a correct answer with supporting work		1 point
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		
Alternate Solution		*Alternate Points*
For an attempt at a correct statement of conservation of energy for the two blocks		*1 point*
$U_1 + K_1 = U_2 + K_2 + E_{\text{lost}}$		
$2mgh + mgh + 0 = 0 + \frac{1}{2}(2m)v^2 + \frac{1}{2}mv^2 + fd$		
For correct substitutions of height and friction		*1 point*
$3mgd\sin\theta = \frac{1}{2}(3m)v^2 + \mu_k(2mg\cos\theta)d + \mu_k(mg\cos\theta)d$		
$3gd\sin\theta - 3\mu_k gd\cos\theta = \frac{3}{2}v^2$		
For a correct answer with supporting work		*1 point*
$v = \sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$		

Question 1 (continued)

Learning Objectives

CHA-1.A.b: Calculate unknown variables of motion such as acceleration, velocity, or positions for an object undergoing uniformly accelerated motion in one dimension.
INT-1.A: Describe an object (either in a state of equilibrium or acceleration) in different types of physical situations such as inclines, falling through air resistance, Atwood machines, or circular tracks).
INT-1.C.e: Derive a complete Newton's second law statement (in the appropriate direction) for an object in various physical dynamic situations (e.g., mass on incline, mass in elevator, strings/pulleys, or Atwood machines).
INT-1.D: Calculate a value for an unknown force acting on an object accelerating in a dynamic situation (e.g., inclines, Atwood Machines, falling with air resistance, pulley systems, mass in elevator, etc.)

Science Practices

1.D: Select relevant features of a representation to answer a question or solve a problem.
3.C: Sketch a graph that shows a functional relationship between two quantities.
3.D: Create appropriate diagrams to represent physical situations.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
7.A: Make a scientific claim.

Question 2

15 points

A toy rocket of mass 0.50 kg starts from rest on the ground and is launched upward, experiencing a vertical net force. The rocket's upward acceleration a for the first 6 seconds is given by the equation $a = K - Lt^2$, where $K = 9.0\ \text{m/s}^2$, $L = 0.25\ \text{m/s}^4$, and t is the time in seconds. At $t = 6.0$ s, the fuel is exhausted and the rocket is under the influence of gravity alone. Assume air resistance and the rocket's change in mass are negligible.

(a) LO INT-5.E, SP 6.B, 6.C
2 points

Calculate the magnitude of the net impulse exerted on the rocket from $t = 0$ to $t = 6.0$ s.

For an expression for calculating impulse and correct substitution of $a(t)$ and m into the correct expression.		1 point
$J = \int F(t)\,dt$		
$J = \int ma(t)\,dt = (0.50)\int\left(9.0 - 0.25t^2\right)dt$		
For integrating with correct limits or including a constant of integration		1 point
$J = (0.50)\int_{t=0}^{t=6}\left(9.0 - 0.25t^2\right)dt = (0.50)\left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6}$		
$J = (0.50)\left[\left((9.0)(6) - \left(\frac{1}{12}\right)(6)^3\right)\right] - 0 = 18\ \text{N•s}$		
Alternate Solution (using an alternate solution from part (b))		*Alternate Points*
For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (b) into equation above		1 point
$J = (0.50\ \text{kg})(36 - 0)\ \therefore J = 18 N•s$		

Question 2 (continued)

(b) LO INT-5.A.a, SP 6.A, 6.C
2 points

Calculate the speed of the rocket at $t = 6.0$ s.

For correctly relating impulse to the speed of the rocket		1 point
$J = \Delta p = m(v_2 - v_1)$ OR $J = m\Delta v = mv_f$		
For correctly substituting answer from part (a) into equation above		1 point
$(18\text{ N}\cdot\text{s}) = (0.50\text{ kg})(v_2 - 0)\ \therefore\ v_2 = 36\text{ m/s}$		
Alternate Solution		*Alternate Points*
Integrate expression for a(t) (This may have already been done in solving part (a).)		1 point
$v = \int a dt = \int (9.0 - 0.25t^2)dt$		
For integrating with correct limits or including a constant of integration		1 point
$v = \int_{t=0}^{t=6} (9.0 - 0.25t^2)dt = \left[9.0t - \frac{1}{12}t^3\right]_{t=0}^{t=6} = 36\text{ m/s}$		

(c)
i. LO INT-4.C.c, SP 6.B, 6.C
2 points

Calculate the kinetic energy of the rocket at $t = 6.0$ s.

For substituting the mass of the rocket into the equation for kinetic energy		1 point
For substituting the answer from part (b) into the equation for kinetic energy		1 point
$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}\right)(0.50\text{ kg})(36\text{ m/s})^2 = 324\text{ J}$		

ii. LO CHA-1.B, CON-1.E, SP 6.A, 6.C
3 points

Calculate the change in gravitational potential energy of the rocket-Earth system from $t = 0$ to $t = 6.0$ s.

For integrating the acceleration twice to derive an expression for position		1 point
For integrating with correct limits or including a constant of integration		1 point
$v(t) = \int a(t)dt = \int (9.0 - 0.25t^2)dt = 9.0t - \frac{1}{12}t^3 + v_0 = 9.0t - \frac{1}{12}t^3$		
$\Delta y = \int v(t)dt = \int_{t=0}^{t=6} \left(9.0t - \frac{1}{12}t^3\right)dt = \left[\frac{9}{2}t^2 - \frac{1}{48}t^4\right]_{t=0}^{t=6}$		
$\Delta y = \left[\left(\frac{9}{2}\right)(6)^2 - \left(\frac{1}{48}\right)(6)^4\right] - 0 = 135\text{ m}$		
For substituting into equation for potential energy		1 point
$\Delta U_g = mg\Delta y = (0.50\text{ kg})(9.8\text{ m/s}^2)(135\text{ m}) = 660\text{ J}$		

Question 2 (continued)

(d) LO CHA-1.B, CON-2.B, SP 6.B, 6.C
3 points

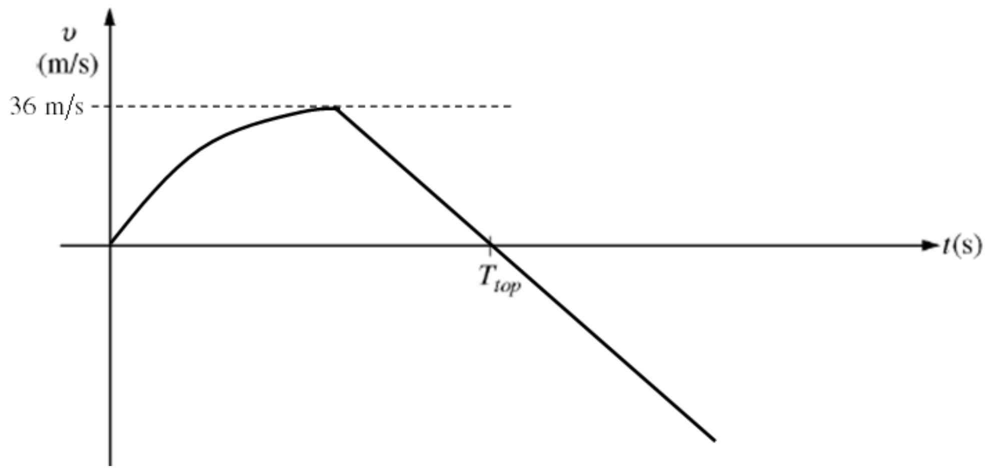
Calculate the maximum height reached by the rocket relative to its launching point.

For using $a = g$ in a correct kinematics equation to solve for height		1 point
$v_2^2 = v_1^2 + 2a(y_2 - y_1) \therefore 0 = v_1^2 + 2a(y_2 - y_1)$		
$y_2 - y_1 = -\dfrac{v_1^2}{2a} \therefore y_2 = -\dfrac{v_1^2}{2a} + y_1$		
For substituting the speed from part (b)		1 point
$y_2 = -\dfrac{(36\ \text{m/s})^2}{2(-9.8\ \text{m/s}^2)} + y_1$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 1		*Alternate Points*
For using energy conservation to find maximum height, consistent with the speed found in part (b).		1 point
$mg\Delta y = \frac{1}{2}mv^2 \therefore \Delta y = \dfrac{v^2}{2g}$		
For substituting the speed from part (b)		1 point
$\Delta y = \dfrac{(36\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 66\ \text{m}$		
For substituting the height from part (c)		1 point
$y_2 = 66\ \text{m} + 135\ \text{m} = 201\ \text{m}$ (199.8 m if $g = 10\ \text{m/s}^2$)		
Alternate Solution 2		*Alternate Points*
For using energy conservation to find maximum height, from kinetic and potential energies found in part (c)		1 point
$K_1 + U_1 = 0 + U_{top}$		
For substituting the kinetic energy from part (c)(i) into the equation above		1 point
For substituting the potential energy from part (c)(ii) into the equation above		1 point
$324 + 660 = (0.5)(9.8)\Delta y$		
$\Delta y = 201\ \text{m}$ (199.8 m if g = 10m/s^2)		

Question 2 (continued)

(e) LO CHA-1.C, SP 3.C
3 points

On the axes below, assuming the upward direction to be positive, sketch a graph of the velocity v of the rocket as a function of time t from the time the rocket is launched to the time it returns to the ground. T_{top} represents the time the rocket reaches its maximum height. Explicitly label the maxima with numerical values or algebraic expressions, as appropriate.



For an initial concave down curve that starts at the origin.		1 point
For a transition that occurs before T_{top} into a straight line with a negative slope.		1 point
For labeling the maximum value of the velocity, consistent with part (b), and a line that crosses the x-axis at T_{top}		1 point

Question 2 (continued)

Learning Objectives

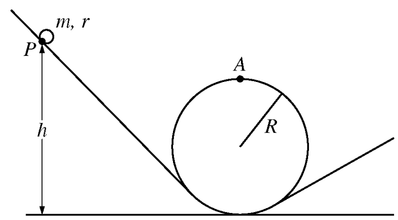
CHA-1.B: Determine functions of position, velocity, and acceleration that are consistent with each other, for the motion of an object with a nonuniform acceleration.
CHA-1.C: Describe the motion of an object in terms of the consistency that exists between position and time, velocity and time, and acceleration and time.
CON-1.E: Calculate the potential energy of a system consisting of an object in a uniform gravitational field.
CON-2.B: Describe kinetic energy, potential energy, and total energy in relation to time (or position) for a "conservative" mechanical system.
INT-4.C.c: Calculate changes in an object's kinetic energy or changes in speed that result from the application of specified forces.
INT-5.A.a: Calculate the total momentum of an object or system of objects.
INT-5.E: Calculate the change in momentum of an object given a nonlinear function, $F(t)$, for a net force acting on the object.

Science Practices

3.C: Sketch a graph that shows a functional relationship between two quantities.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.B: Apply an appropriate law, definition, or mathematical relationship to solve a problem.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.

Question 3

15 points



Note: Figure not drawn to scale.

The rotational inertia of a rolling object may be written in terms of its mass m and radius r as $I = bmr^2$, where b is a numerical value based on the distribution of mass within the rolling object. Students wish to conduct an experiment to determine the value of b for a partially hollowed sphere. The students use a looped track of radius $R >> r$, as shown in the figure above. The sphere is released from rest a height h above the floor and rolls around the loop.

(a) LO INT-2.D, SP 5.A, 5.E
2 points

Derive an expression for the minimum speed of the sphere's center of mass that will allow the sphere to just pass point A without losing contact with the track. Express your answer in terms of b, m, R, and fundamental constants, as appropriate.

For an expression relating the gravitational force to the centripetal force		1 point
$F_C = mg + F_N = \frac{mv^2}{R}$		
$mg + 0 = \frac{mv^2}{R}$		
For a correct substitution into a correct expression.		1 point
$v = \sqrt{Rg}$		

Question 3 (continued)

(b) LO INT-7.E, SP 5.A, 5.E
3 points

Suppose the sphere is released from rest at some point *P* and rolls without slipping. Derive an equation for the minimum release height *h* that will allow the sphere to pass point *A* without losing contact with the track. Express your answer in terms of *b, m, R,* and fundamental constants, as appropriate.

For using a correct expression of the conservation of energy for the object		1 point
$K_1 + U_1 = K_2 + U_2$		
For correct substitutions, including rotational kinetic energy		1 point
$0 + mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh_2$		
$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(bmr^2\right)\left(\frac{v}{r}\right)^2 + mgh_2$		
For correctly substituting for the final height and the answer from part (a) for the velocity		1 point
$gh = \frac{1}{2}\left(\sqrt{Rg}\right)^2 + \frac{1}{2}(b)\left(\sqrt{Rg}\right)^2 + g(2R)$		
$h = \frac{1}{2}R + \frac{1}{2}bR + 2R = \frac{1}{2}(b+5)R$		

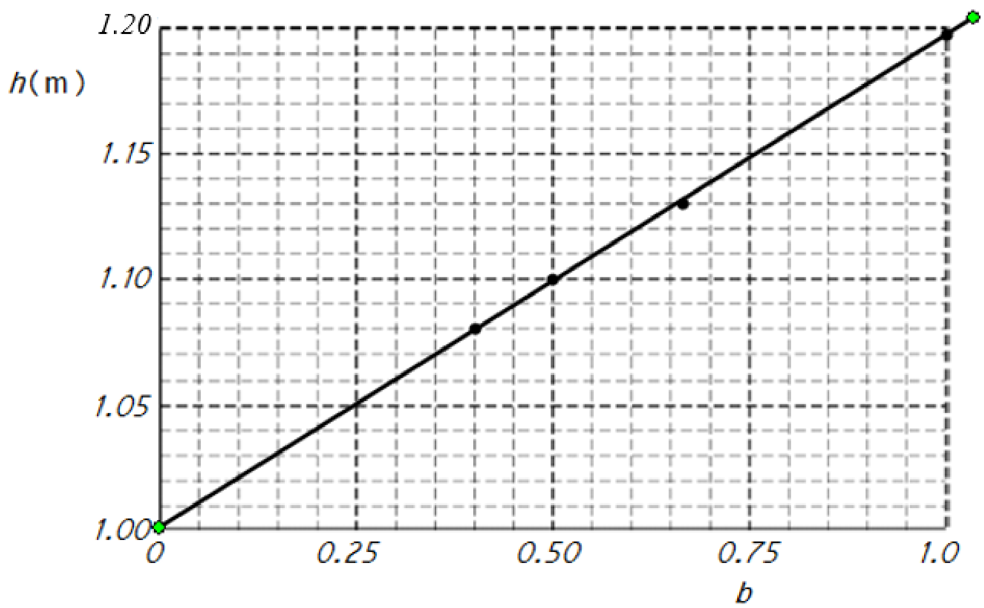
Question 3 (continued)

The students perform an experiment by determining the minimum release height h for various other objects of radius r and known values of b. They collect the following data.

Object	b	h (m)
Solid sphere	0.40	1.08
Hollow sphere	0.67	1.13
Solid cylinder	0.50	1.10
Hollow cylinder	1.0	1.20

(c) LO INT-7.E, SP 3.A, 4.C
4 points

On the grid below, plot the release height h as a function of b. Clearly scale and label all axes, including units, if appropriate. Draw a straight line that best represents the data.



For correctly labeling both axes with variables, units for h, no units for b		1 point
For correctly using and labeling the scale of the axes so that the data uses at least half the grid		1 point
For correctly plotting the data		1 point
For drawing a best-fit straight line that represents the data		1 point

Question 3 (continued)

(d) LO INT-7.E, SP 4.D, 6.C
2 points

The students repeat the experiment with the partially hollowed sphere and determine the minimum release height to be 1.16 m. Using the straight line from part (c), determine the value of *b* for the partially hollowed sphere.

For correctly calculating the slope from the best-fit straight line and substituting height into the line equation		1 point
$\text{slope} = \dfrac{1.20 - 1.00}{1.0 - 0.0} = 0.20\text{ m}$		
$y = mx + b \therefore h = 0.20b + 1.00$		
$1.16 = 0.20b + 1.00$		
For a correct answer for *b*		1 point
$b = 0.80$		
Note: Full credit can be earned for the correct answer if there is an indication that this was read from the graph.		

(e) LO INT-7.E, SP 6.A, 6.C
2 points

Calculate *R*, the radius of the loop.

For substituting into the expression from part (b)		1 point
$h = \dfrac{1}{2}(b+5)R \therefore R = \dfrac{2(1.16\text{ m})}{(0.80+5)}$		
For an answer consistent with previous parts		1 point
$R = 0.40\text{ m}$		
Alternate Solution		*Alternate Points*
For using an expression for the y-intercept (set b = 0)		1 point
$h = \dfrac{1}{2}(b+5)R = \dfrac{1}{2}(0+5)R = \dfrac{5}{2}R$		
For determining the value of the intercept (1.0 m) from the graph		1 point
$1.0\text{ m} = \dfrac{5}{2}R \therefore R = \dfrac{2}{5}\text{ m} = 0.40\text{ m}$		

Question 3 (continued)

(f) LO INT-7.E, SP 7.A, 7.C
2 points

In part (b), the radius r of the rolling sphere was assumed to be much smaller than the radius R of the loop. If the radius r of the rolling sphere was not negligible, would the value of the minimum release height h be greater, less, or the same?

____ Greater ____ Less ____ The same

Justify your answer.

For selecting "Less" and any justification		1 point
For a correct justification		1 point
Examples:		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the height needed to pass through the loop decreases according to the equation from part (b).		
If the radius of the object is not negligible, then the center of mass of the object travels in a radius less than R. If the radius of the circle decreases, the speed needed to pass through the loop decreases, and thus the height needed also decreases.		

Learning Objectives

INT-2.D: Derive expressions relating centripetal force to the minimum speed or maximum speed of an object moving in a vertical circular path.
INT-7.E: Derive expressions using energy conservation principles for physical systems such as rolling bodies on inclines, Atwood Machines, pendulums, physical pendulums, and systems with massive pulleys that relate linear or angular motion characteristics to initial conditions (such as height or position) or properties of rolling body (such as moment of inertia or mass).

Science Practices

3.A: Select and plot appropriate data.
4.C: Linearize data and/or determine a best-fit line or curve.
4.D: Select relevant features of a graph to describe a physical situation or solve problems.
5.A: Select an appropriate law, definition, or mathematical relationship or model to describe a physical situation.
5.E: Derive a symbolic expression from known quantities by selecting and following a logical algebraic pathway.
6.A: Extract quantities from narratives or mathematical relationships to solve problems.
6.C: Calculate an unknown quantity with units from known quantities, by selecting and following a logical computational pathway.
7.A: Make a scientific claim.
7.C: Support a claim with evidence from physical representations.