

AP[®] Statistics 2005 Free-Response Questions Form B

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Formulas begin on page 3. Questions begin on page 6. Tables begin on page 12.

Formulas

(I) Descriptive Statistics

$$\overline{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum \left(x_i - \overline{x} \right)^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \overline{x})(y_i - \overline{y})}{\sum (x_i - \overline{x})^2}$$

 $b_0 = \overline{y} - b_1 \overline{x}$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \overline{x}}{s_x} \right) \left(\frac{y_i - \overline{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n - 2}}}{\sqrt{\sum (x_i - \overline{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_X = \sum x_i p_i$$

$$\operatorname{Var}(X) = \sigma_{x}^{2} = \sum \left(x_{i} - \mu_{x} \right)^{2} p_{i}$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

 $\mu_{\chi} = np$

$$\sigma_{\chi} = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If \bar{x} is the mean of a random sample of size *n* from an infinite population with mean μ and standard deviation σ , then:

 $\mu_{\overline{x}} = \mu$

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic \pm (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$rac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
	Special case when $\sigma_1 = \sigma_2$ $\sigma_1 \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$
	Special case when $p_1 = p_2$
	$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Chi-square test statistic =	$\sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$

STATISTICS SECTION II Part A Questions 1-5 Spend about 65 minutes on this part of the exam.

Percent of Section II grade—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

- 1. The graph below displays the scores of 32 students on a recent exam. Scores on this exam ranged from 64 to 95 points.
 - 6 ** 6 ** 7 *** 7 **** 8 **** 8 ***** 9 ***** 9 ****
 - (a) Describe the shape of this distribution.
 - (b) In order to motivate her students, the instructor of the class wants to report that, overall, the class's performance on the exam was high. Which summary statistic, the mean or the median, should the instructor use to report that overall exam performance was high? Explain.
 - (c) The midrange is defined as $\frac{\text{maximum} + \text{minimum}}{2}$. Compute this value using the data on the

preceding page.

Is the midrange considered a measure of center or a measure of spread? Explain.

2. For an upcoming concert, each customer may purchase up to 3 child tickets and 3 adult tickets. Let C be the number of child tickets purchased by a single customer. The probability distribution of the number of child tickets purchased by a single customer is given in the table below.

С	0	1	2	3	
p(c)	0.4	0.3	0.2	0.1	

- (a) Compute the mean and the standard deviation of C.
- (b) Suppose the mean and the standard deviation for the number of adult tickets purchased by a single customer are 2 and 1.2, respectively. Assume that the numbers of child tickets and adult tickets purchased are independent random variables. Compute the mean and the standard deviation of the total number of adult and child tickets purchased by a single customer.
- (c) Suppose each child ticket costs \$15 and each adult ticket costs \$25. Compute the mean and the standard deviation of the total amount spent per purchase.

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3. In search of a mosquito repellent that is safer than the ones that are currently on the market, scientists have developed a new compound that is rated as less toxic than the current compound, thus making a repellent that contains this new compound safer for human use. Scientists also believe that a repellent containing the new compound will be more effective than the ones that contain the current compound. To test the effectiveness of the new compound versus that of the current compound, scientists have randomly selected 100 people from a state.

Up to 100 bins, with an equal number of mosquitoes in each bin, are available for use in the study. After a compound is applied to a participant's forearm, the participant will insert his or her forearm into a bin for 1 minute, and the number of mosquito bites on the arm at the end of that time will be determined.

- (a) Suppose this study is to be conducted using a completely randomized design. Describe a randomization process and identify an inference procedure for the study.
- (b) Suppose this study is to be conducted using a matched-pairs design. Describe a randomization process and identify an inference procedure for the study.
- (c) Which of the designs, the one in part (a) or the one in part (b), is better for testing the effectiveness of the new compound versus that of the current compound? Justify your answer.
- 4. A researcher believes that treating seeds with certain additives before planting can enhance the growth of plants. An experiment to investigate this is conducted in a greenhouse. From a large number of Roma tomato seeds, 24 seeds are randomly chosen and 2 are assigned to each of 12 containers. One of the 2 seeds is randomly selected and treated with the additive. The other seed serves as a control. Both seeds are then planted in the same container. The growth, in centimeters, of each of the 24 plants is measured after 30 days. These data were used to generate the partial computer output shown below. Graphical displays indicate that the assumption of normality is not unreasonable.

	Ν	Mean	StDev	SE Mean
Control	12	15.989	1.098	0.317
Treatment	12	18.004	1.175	0.339
Difference	12	- 2.015	1.163	0.336

- (a) Construct a confidence interval for the mean difference in growth, in centimeters, of the plants from the untreated and treated seeds. Be sure to interpret this interval.
- (b) Based only on the confidence interval in part (a), is there sufficient evidence to conclude that there is a significant mean difference in growth of the plants from untreated seeds and the plants from treated seeds? Justify your conclusion.

5. John believes that as he increases his walking speed, his pulse rate will increase. He wants to model this relationship. John records his pulse rate, in beats per minute (bpm), while walking at each of seven different speeds, in miles per hour (mph). A scatterplot and regression output are shown below.



Regression Analysis:	Pulse Versus S	Speed				
Predictor Constant Speed	Coef 63.457 16.2809	SE Coef 2.387 0.8192	T 26.5 19.8	8 8	P 0.000 0.000	
S = 3.087	R-Sq = 98.7%	R-Sq (a	dj) = 98.5%			
Analysis of Variance						
Source	DF	SS	MS	F		Р
Regression	1	3763.2	3763.2	396.13		0.000
Residual	5	47.6	9.5			
Total	6	3810.9				

(a) Using the regression output, write the equation of the fitted regression line.

- (b) Do your estimates of the slope and intercept parameters have meaningful interpretations in the context of this question? If so, provide interpretations in this context. If not, explain why not.
- (c) John wants to provide a 98 percent confidence interval for the slope parameter in his final report. Compute the margin of error that John should use. Assume that conditions for inference are satisfied.

Part B

Question 6 Spend about 25 minutes on this part of the exam. Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

- 6. Regulations require that product labels on containers of food that are available for sale to the public accurately state the amount of food in those containers. Specifically, if milk containers are labeled to have 128 fluid ounces and the mean number of fluid ounces of milk in the containers is at least 128, the milk processor is considered to be in compliance with the regulations. The filling machines can be set to the labeled amount. Variability in the filling process causes the actual contents of milk containers to be normally distributed. A random sample of 12 containers of milk was drawn from the milk processing line in a plant, and the amount of milk in each container was recorded.
 - (a) The sample mean and standard deviation of this sample of 12 containers of milk were 127.2 ounces and 2.1 ounces, respectively. Is there sufficient evidence to conclude that the packaging plant is <u>not</u> in compliance with the regulations? Provide statistical justification for your answer.

Inspectors decide to study a particular filling machine within this plant further. For this machine, the amount of milk in the containers has a mean of 128.0 fluid ounces and a standard deviation of 2.0 fluid ounces.

- (b) What is the probability that a randomly selected container filled by this machine contains at least 125 fluid ounces?
- (c) An inspector will randomly select 12 containers filled by this machine and record the amount of milk in each. What is the probability that the minimum (smallest amount of milk) recorded in the 12 containers will be at least 125 fluid ounces? (Note: In order for the minimum to be at least 125 fluid ounces, each of the 12 containers must contain at least 125 fluid ounces.)

An analyst wants to use simulation to investigate the sampling distribution of the minimum. This analyst randomly generates 150 samples, each consisting of 12 observations, from a normal distribution with mean 128 and standard deviation 2 and finds the minimum for each sample. The 150 minimums (sorted from smallest to largest) are shown on the next page.

Sample	Minimum	Sample	Minimum	Sample	Minimum
1	121.45	51	124.28	101	125.25
2	122.51	52	124.29	102	125.31
3	122.53	53	124.30	103	125.36
4	122.72	54	124.31	104	125.38
5	122.75	55	124.34	105	125.40
6	122.89	56	124.36	106	125.42
7	122.93	57	124.37	107	125.48
8	122.99	58	124.37	108	125.49
9	123.04	59	124.39	109	125.50
10	123.08	60	124.39	110	125.52
11	123.09	61	124.41	111	125.54
12	123.10	62	124.44	112	125.56
13	123.31	63	124.53	113	125.61
14	123.34	64	124.53	114	125.67
15	123.39	65	124.54	115	125.72
16	123.40	66	124.55	116	125.76
17	123.41	67	124.55	117	125.77
18	123.41	68	124.55	118	125.78
19	123.46	69	124.55	119	125.79
20	123.49	70	124.58	120	125.84
21	123.51	71	124.67	121	125.87
22	123.57	72	124.69	122	125.87
23	123.58	73	124.73	123	125.90
24	123.59	74	124.77	124	125.90
25	123.60	75	124.78	125	125.93
26	123.66	76	124.78	126	125.93
27	123.67	77	124.80	127	125.93
28	123.72	78	124.80	128	125.94
29	123.75	79	124.81	129	125.98
30	123.77	80	124.85	130	126.00
31	123.78	81	124.91	131	126.03
32	123.84	82	124.92	132	126.05
33	123.91	83	124.92	133	126.05
34	123.93	84	124.96	134	126.06
35	123.95	85	125.00	135	126.09
36	123.95	86	125.01	136	126.15
37	123.98	87	125.02	137	126.15
38	123.99	88	125.02	138	126.16
39	124.05	89	125.03	139	126.19
40	124.05	90	125.04	140	126.19
41	124.06	91	125.05	141	126.25
42	124.12	92	125.07	142	126.26
43	124.14	93	125.08	143	126.33
44	124.15	94	125.09	144	126.35
45	124.16	95	125.14	145	126.45
46	124.19	96	125.18	146	126.50
47	124.23	97	125.21	147	126.57
48	124.27	98	125.21	148	126.62
49	124.28	99	125.22	149	126.64
50	124.28	100	125.25	150	126.95

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(d) Use the simulation results to estimate the probability that was requested in part (c) <u>and</u> compare this estimate with the theoretical value you calculated.

END OF EXAMINATION



Table entry for z is the probability lying below z.

Table A	Standard	normal	probabilities
I able A	Stanuaru	normai	probabilities

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z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.13/9
-0.9	.1841	.1814	.1/88	.1762	.1/36	.1/11	.1685	.1660	.1035	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1807
-0./	.2420	.2389	.2358	.2327	.2296	.2200	.2230	.2206	.21//	.2148
-0.6	.2745	.2709	.2070	.2045	.2011	.2378	.2340	.2314	.2483	.2431
-0.3	.3083	.3030	.3013	.2901	.2940	.2912	.2077	.2045	.2010	.2770
-0.4	.3440	.3409	.3312 3745	.5550	.5500	.5204	.3220	.3192	3520	3/83
-0.3	.3621	.5765	.3745	.5707	4052	.3032	3074	3036	3807	3850
-0.2	.4207	4562	4522	4090	.4052	44015	4364	4325	.3097	.3039 4947
-0.1	5000	4960	4920	4880	4840	4801	4761	4721	4681	4641
-0.0	.5000	.4900	.4920	.4000	.4840	.4801	.4701	.4721	.4001	.4041



Table entry for z is the probability lying below z.

Table A(Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

2005 AP® STATISTICS FREE-RESPONSE QUESTIONS (Form B)



 Table B
 t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
20	.084	.830	1.058	1.315	1.700	2.050	2.162	2.479	2.779	3.067	3.435	3.707
21	.084	.833	1.057	1.314	1.703	2.032	2.158	2.475	2.771	3.037	2.409	3.090
28	.083	.833	1.056	1.313	1.701	2.048	2.154	2.407	2.705	3.047	3.408	3.074
29	.005	.034	1.055	1.311	1.099	2.043	2.150	2.402	2.750	3.030	2 2 9 5	2.646
40	.005	.054	1.055	1.310	1.097	2.042	2.147	2.437	2.750	2 071	3.307	3.551
40 50	679	.851 849	1.030	1 200	1.676	2.021	2.125	2.423	2.704	2.971	3 261	3 496
50 60	679	848	1.047	1.295	1.671	2.002	2.109	2 390	2.670	2.937	3 232	3 460
80	678	846	1.043	1 292	1.671	1 990	2.099	2.370	2.000	2.913	3 195	3 416
100	677	845	1.043	1 290	1.660	1 984	2.000	2.364	2.626	2.807	3 174	3 390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3,300
0000	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
					(Confidence	level C					



Table C	χ^2	critical	values
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above it.

						Tail prol	bability p					
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2