## Intent of Question

The primary goals of this question are: (1) to assess a student's ability to use simple graphical displays (dotplots in this case) to compare and contrast two distributions; and (2) to evaluate a student's ability to recognize what statistical information is most useful in making different practical decisions.

## Solution

**Part (a):**

Both distributions of distances are roughly symmetric and somewhat mound-shaped. The center of the distances for catapult A (median A = 136 cm) is slightly lower than the center of the distances for catapult B (median B = 138 cm). There is more variability in the distances traveled by the Ping-Pong balls launched with catapult A. There are distances that are extreme enough to be called (potential) outliers in the catapult A distribution, but there are no outliers among the catapult B distances.

**Part (b):**

Catapult B would be best because the distances vary less about the center of the distribution for catapult B. If catapult B is properly placed, the balls launched will have a higher probability of landing in the narrow (only 5 cm wide) target band.

**Part (c):**

The catapult should be placed 138 cm from the target line. Since the distribution of distances for catapult B seems to be fairly symmetric and somewhat mound-shaped, the median (138 cm) is a good representation of the center of the distribution. Placing catapult B at this location would have resulted in a high proportion (30/40 = 0.75) of Ping-Pong balls from this sample of launches landing in the target band.

## Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is essentially correct (E) if the student correctly identifies similarities and differences in center, spread, and shape for the two distributions.

Part (a) is partially correct (P) if the student correctly identifies similarities and differences in two of the three characteristics (center, shape, and spread) for the two distributions.

Part (a) is incorrect (I) if the student correctly identifies no more than one similarity or difference of the three characteristics (center, shape, and spread) for the two distributions.

Notes:
- Correct comments regarding outliers should be viewed as a positive. However, comments about outliers do not count as one of the three required characteristics.

- Describing catapult A's distribution as "normal" or "skewed left" or "uniform" is not acceptable for the shape characteristic. Describing either distribution as "approximately normal" is acceptable.

- Giving separate lists of measures of center and/or spread for the two distributions with no linkage between them is not an acceptable discussion of similarities and differences for these characteristics.

**Part (b)** is essentially correct (E) if catapult B is chosen using a rationale based on the variability in the distances.

Part (b) is partially correct (P) if catapult B is chosen, but the explanation does not refer to the variability in the distances.

Part (b) is incorrect (I) if catapult B is chosen and no explanation is provided OR catapult A is chosen.

**Part (c)** is essentially correct (E) if:
  the catapult is placed at the median (or mean) of the distances traveled by the Ping-Pong balls, and the explanation addresses why the median (or mean) was selected based on a property of the chosen statistic that relates to the context of the problem;
    OR
  the catapult is placed at a distance of 137.5-139.5 cm from the target line, and the explanation indicates that the chosen distance resulted in a high proportion of the balls in the sample landing in the target band.

Part (c) is partially correct (P) if the catapult is placed at an acceptable distance from the target line, but the explanation is incomplete or incorrect.

Part (c) is incorrect (I) if the catapult is placed less than 137.5 centimeters or more than 139.5 centimeters from the target line.

Notes:
- Simply saying "because it's the median (or mean)" is an incomplete explanation.
- Some students may confuse the 5 cm band as meaning 5 cm on either side of the target line. If the student chooses the median (or mean) and satisfactorily addresses why the median (or mean) was selected OR chooses a value of 137-140 cm and the explanation indicates that the chosen distance resulted in a high proportion of the balls in the sample landing in the target band, score the response as partially correct.
- If a student gives the distance from the catapult to the front or back of the shaded band rather than the distance to the target line, but gives an otherwise correct response, score part (c) as partially correct.
- If a student picks catapult A in part (b) and follows through correctly in part (c), then part (c) should be scored as essentially correct.

**4**      **Complete Response**

>   All three parts essentially correct

**3**      **Substantial Response**

>   Two parts essentially correct and one part partially correct

**2**      **Developing Response**

>   Two parts essentially correct and no parts partially correct

OR

>   One part essentially correct and two parts partially correct

OR

>   Three parts partially correct

**1**      **Minimal Response**

>   One part essentially correct and either zero or one part partially correct

OR

>   No parts essentially correct and two parts partially correct

(a) Comment on any similarities and any differences in the two distributions of distances traveled by balls launched from catapult A and catapult B.

The distribution of catapult A and B appears to be roughly symmetric w/ no extreme skewness. However, the distribution of Cat A has a larger range (~33cm) than that of catapult B. (~11cm). Catapult A distribution's center is (around 135cm) lower than the center for the distribution of catapult B. (around 137cm or so). Also, for catapult A, there are more extreme distance values that might be outliers, while catapult B's distribution has a smaller spread (range).

(b) If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Justify your choice.

Catapult B would be better to use than catapult A. This is because catapult B's distribution has a smaller range and less variability. Most of the recorded data measurements landed between ~130cm - 145cm. This would make it easier for the parents to determine the distance at which their catapult should be placed. On the other hand, catapult A's distribution is more varied and has a larger range. The data's are not as concentrated around a center. This will be harder to maximize the prob. of having the ping pong balls land within the band. b/c the distances spread over a larger range than that of catapult B's distribution.

(c) Using the catapult that you chose in part (b), how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.

The target line, using catapult B, should be placed approx 138cm from the target line. This is b/c the distribution of of catapult B is roughly symmetric, so if we look at the center of the distribution, in this case we can find the approx median, it is ~138 cm. The band is 5cm so any ping pong ball that launches a distance of approx 138 ± 2.5cm would land within the band. In catapult B's distribution (data), ~80% of the balls launched landed between 135.5cm and 140.5cm. This is why the catapult should be placed ~138cm from the target line.

(a) Comment on any similarities and any differences in the two distributions of distances traveled by balls launched from catapult A and catapult B.

The mean ~~data~~ and median distance of Catapult B is slightly higher than Catapult A. Catapult A has much more spread than catapult B; its range is almost three times the range of C. Both distributions are approximately ~~&~~ symmetrical and bell-shaped.

(b) If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Justify your choice.

Catapult B would be a better choice. In order to have the Ping-Pong balls land within the band, a longer launch distance would ~~be~~ be more desirable in order to maximize the probability. Since the center of distribution B is higher than that of distribution A, ping pong balls launched from Catapult B will in general ~~be~~ slightly longer than balls launched from A, which would therefore increase the probability of the balls from B landing in the strip.

(c) Using the catapult that you chose in part (b), how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.

The catapult should be placed about 138 cm from the target line. Since the median of distribution B is 138 and the distribution is approximately bell-shaped and symmetrical, it is reasonable to assume that the expected distance of the ping pong ball is 138cm, so the catapult should be placed that far from the target line to make sure that the balls land in the strip.

**GO ON TO THE NEXT PAGE.**

(a) Comment on any similarities and any differences in the two distributions of distances traveled by balls launched from catapult A and catapult B.

Catapult A has a median of about 136, is relatively normal, and has a range of about 35.

Catapult B has a median of about 138, is also relatively normal, and has a much smaller range of about 11.

(b) If the parents want to maximize the probability of having the Ping-Pong balls land within the band, which one of the two catapults, A or B, would be better to use than the other? Justify your choice.

Catapult B would be the better choice because a smaller range indicates less variation among distances. This will result in a more accurate catapult that is more likely to hit the target.

(c) Using the catapult that you chose in part (b), how many centimeters from the target line should this catapult be placed? Explain why you chose this distance.

Catapult B should be placed about 138 cm away from the target because that is the median distance reached during the trials.

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## Question 1

**Overview**

The primary goals of this question were to: (1) assess a student's ability to use simple graphical displays (dotplots in this case) to compare and contrast two distributions; and (2) evaluate a student's ability to recognize what statistical information is most useful in making different practical decisions.

**Sample: 1A**
**Score: 4**

The essay not only addresses similarities and differences in shape, center, and spread correctly but also mentions the potential outliers in catapult A's distribution in part (a). The numerical estimates of center and spread lend support to the comments. In the first two sentences of part (b), catapult B is identified as the better catapult for the stated goal of maximizing the probability of launching Ping-Pong balls into the shaded band. The next few sentences elaborate further on the choice of catapult B. In part (c) a strong argument is given—based on the symmetry of the distance distribution and the percent of sample shots landing in the shaded band—for placing catapult B at the median of the distances traveled by the 40 balls launched in the sample. This essay earned a score of 4.

**Sample: 1B**
**Score: 3**

A clear description of the shape of the two distributions is given in part (a): "approximately symmetrical and bell-shaped." The differences in center and spread are also addressed. In part (b) the correct catapult is chosen, but for the wrong reason. The catapult is chosen based on which catapult shoots balls the farthest. In part (c) a compelling argument is given for using the median distance traveled by balls launched using catapult B in this sample—namely, the symmetric, roughly bell-shaped nature of the distribution—to position the catapult. This essay earned a score of 3.

**Sample: 1C**
**Score: 2**

At first glance, it appears that only a listing of characteristics is given—shape, center, and spread—for the two distributions. Upon further inspection, however, linkage is provided for spread in the description of catapult B's distribution—"a much smaller range of about 11." There is no linkage for center. In part (b) catapult B is chosen due to the smaller variability in its distribution of distances. The median distance that the 40 balls traveled when launched with catapult B is used to position the catapult. This is an acceptable location for the catapult relative to the target line. However, a justification is not provided for using the median distance other than "because that is the median distance." This essay earned a score of 2.