Question 6

Intent of Question

The primary goals of this question are to evaluate a student's ability to apply the concepts of significance testing to a new setting, in particular to: (1) state hypotheses for a parameter of interest, given a research question; (2) evaluate a new test statistic and use the probability distribution associated with that statistic to test the hypotheses of interest; (3) identify the values of the test statistic that would lead to rejection of the null hypothesis on a graph; and (4) interpret simulated sampling distributions for different populations.

Solution

Part (a):

Let σ^2 denote the variance in the temperatures measured by the thermostats recently produced by this manufacturer.

$H_0 : \sigma^2 = 1.52(^\circ F)^2$ OR Recently produced thermostats are not more variable than thermostats produced in the past.

$H_a : \sigma^2 > 1.52(^\circ F)^2$ OR Recently produced thermostats are more variable than thermostats produced in the past.

Part (b):

$$\frac{(n-1)s^2}{1.52} = \frac{9 \times (1.4277)^2}{1.52} = \frac{9 \times (2.0383)}{1.52} = 12.069$$

Part (c):

The test statistic has a χ^2 distribution with 9 degrees of freedom under H_0. The chance of exceeding the observed value of 12.069, under H_0, is

$$p\text{−value} = P(\chi_9^2 \geq 12.069) = 0.2094 .$$

(or, from the table, .20 < *p*-value < .25). Since the *p*-value is greater than 0.05, we cannot reject the null hypothesis. That is, we do not have statistically significant evidence that recent thermostats are less reliable (more variable) than in the past.

Part (d):

The smallest value that would have led to the rejection of the null hypothesis is the 95th percentile of the χ^2 distribution with 9 degrees of freedom, which is 16.92. The rejection region contains all values greater than or equal to 16.92. This region should be identified on the graph by indicating the approximate location of 16.92 on the axis and shading the region that is bounded by the vertical line through 16.92, the horizontal axis, and the χ^2 curve.

Question 6 (continued)

Part (e)

Indicate the region to the right of 16.92 on all three histograms.

Part (f)

The population with the largest variance will tend to produce the largest values of s^2 in the simulation and hence the largest test statistics. Histogram III has the largest probability of producing a sample that would lead to the rejection of H_0 so Histogram III corresponds to the population with the largest variance.

Similarly, the test statistics will tend to be smallest for the population with variance closest to 1.52. Histogram II has the smallest probability of producing a sample that would lead to the rejection of H_0 so Histogram II corresponds to the population with the smallest variance.

Scoring

Each of four components are scored as essentially correct (E), partially correct (P), or incorrect (I).

I. **Parts (a) and (b)** are combined into one component and scored as essentially correct (E) if both part (a) and part (b) are correct.

Parts (a) and (b) are partially correct (P) if one of the two parts is correct.

Notes*:*

1. If a two-sided alternative is used or the hypotheses involve a mean, then part (a) is not correct.
2. Nonstandard notation for the population variance must be defined.
3. If the value of s (or of s^2) is not shown in part (b), then part (b) is incorrect.

II. **Part (c)** is scored as essentially correct (E) if both:

- The p-value is given (or the test statistic compared to the critical value).
- The conclusion is written in context and linked to the p-value.

Part (c) is partially correct (P) if one of the two bulleted items is correct.

Notes*:*

1. Conditions (SRS, normal population) are given in the problem so it is not necessary to restate them. However, if incorrect conditions are given, the first bullet is incorrect.
2. If the null hypothesis is "accepted" or equivalent, the second bullet is incorrect.
3. If both an α and a p-value are given, the linkage is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.

III. **Parts (d) and (e)** are combined into one component and scored as essentially correct (E) if both:

- The critical value is identified as 16.92.
- The region to the right of a cut-off point of between 15 and 20 is identified in part (d). AND the same region is identified in each of the three histograms in part (e).

Question 6 (continued)

Parts (d) and (e) are partially correct (P) if one of the two bulleted items above is correct.

IV. **Part (f)** is essentially correct (E) if both:

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively.
- The justification refers to fact that Histogram III came from the population with the largest variance because the identified region is largest, and so it will be more likely to reject the null hypothesis. (Similarly for the smallest variance.)

Part (f) is partially correct (P) if both:

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively.
- The justification says only that Histogram III represents the population with the largest variance because the identified region is largest. OR The justification refers to the fact that the simulated sampling distribution for the population with the largest variance should result in sample variances—and hence test statistics—that are centered at the largest values. (Similarly for the smallest variance.) OR The justification refers to the fact that the simulated sampling distribution for the population with the largest variance should result in sample variances—and hence test statistics—that are more variable and Histogram III has the more variable values of the test statistic. (Similarly for the smallest variance.)

Part (f) is incorrect (I) if

- Histograms III and II are identified as the simulated sampling distributions for the populations with the largest and smallest variances, respectively, but the justification refers only to the fact that these histograms themselves have the largest and smallest variability.

Note*:*

1. If only one of Histogram III or Histogram II is identified and correctly justified, the response is scored partially correct.

For each of the four components,

Essentially Correct (E) = 1
Partially Correct (P) = 1/2
Incorrect (I) = 0

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, 2½ points), use a holistic approach to determine whether to score up or down depending on the strength of the response and communication.

6A1

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68°F. The readings for those 10 thermostats are given in the table below. $\sigma^2 = 1.52$

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

$H_0: \sigma^2 = 1.52$ where σ^2 is the true population variance

$H_a: \sigma^2 > 1.52$

It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\frac{(n-1)s^2}{1.52}$ for these data.

$$\chi^2 = \frac{(n-1)s^2}{1.52} = \frac{(10-1)(1.428)^2}{1.52} = 12.074$$

GO ON TO THE NEXT PAGE.

(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\frac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

$\chi^2 = 12.074$

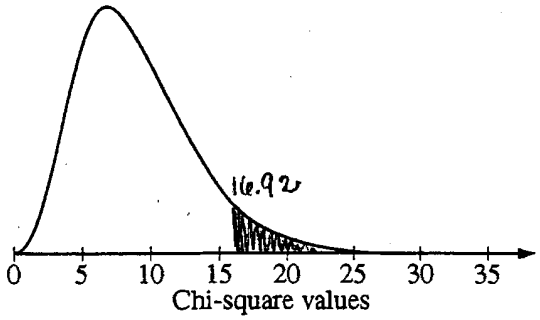
$Pr(\chi^2 \geq 12.074, df = 9) = .2092 > \alpha = .05$

We do not reject H_0. It is not unlikely to get results such as ours, and $s^2 = (1.428)^2 = 2.039$, given that $\sigma^2 = 1.52$, based on chance alone. We do not have strong evidence that w/ $\alpha = .05$ the reading of thermostats has become less reliable.

(d) For the test conducted in part (c), what ~~is the~~ smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

The smallest test statistic that would have led to rejection is 16.92

Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.



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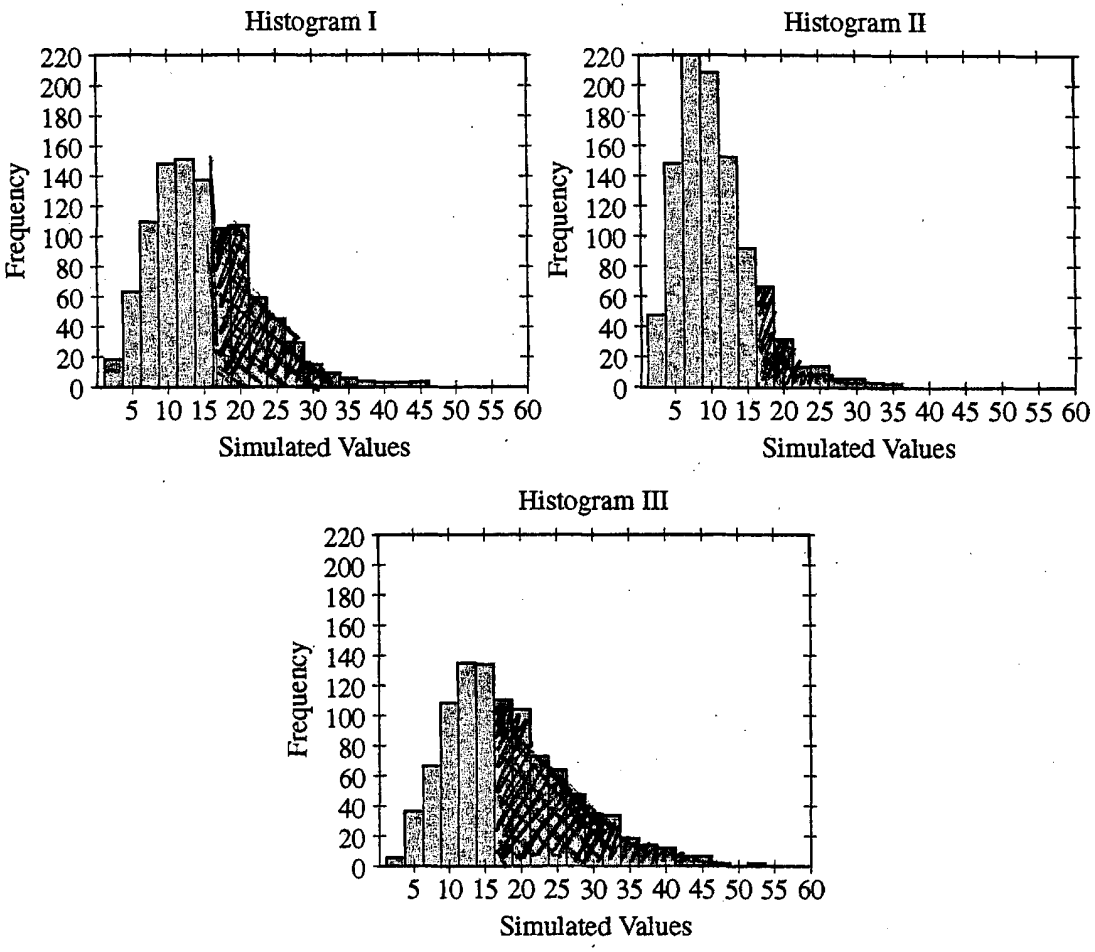
n=10

6A3

(e) Using simulation, 1,000 samples, each of size 10, were randomly generated from 3 populations with different variances. Each population was normally distributed with mean 68 and variance greater than 1.52.

The histograms below show the simulated sampling distribution of $\frac{(n-1)s^2}{1.52}$ for each population.

Mark the region identified in part (d) on each of the histograms below.



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6A4

(f) Based on the regions that you marked in part (e), identify the simulated sampling distribution that corresponds to the population with the largest variance. Then identify the simulated sampling distribution that corresponds to the population with the smallest variance. Justify your choices.

The population that corresponds to the largest variance is in Histogram III, because there is the greatest amt of area under the curve after 16.92, meaning that it would be more likely to choose a sample that would reject the null hypothesis that $\sigma^2 = 1.52$. correctly

The population that corresponds to the smallest variance is that of Histogram II. This distribution has the least amt of area under the curve after 16.92. Although we know that the variance is greater than 1.52 for this population, it would be the easiest true population with which to make Type II error because its variance is the smallest & the closest to 1.52.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

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6B1

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68°F. The readings for those 10 thermostats are given in the table below.

Thermostat	1	2	3	4	5	6	7	8	9	10
Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

$H_0: \sigma^2 = 1.52$

$H_a: \sigma^2 > 1.52$

It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\frac{(n-1)s^2}{1.52}$ for these data.

$s = 1.35144$

$s^2 = 1.8344$

$n = 10$

$\frac{(10-1)\,1.8344}{1.52} = 10.8616$

GO ON TO THE NEXT PAGE.

6B2

(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\frac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

$p > .05$

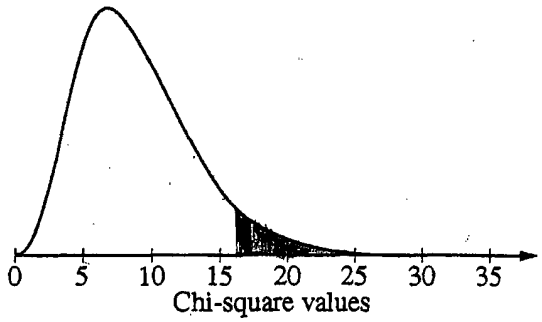
the probability is greater than .25

Do not Reject H_0. The difference is not statistically significant.

(d) For the test conducted in part (c), what is the smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

16.92

Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.



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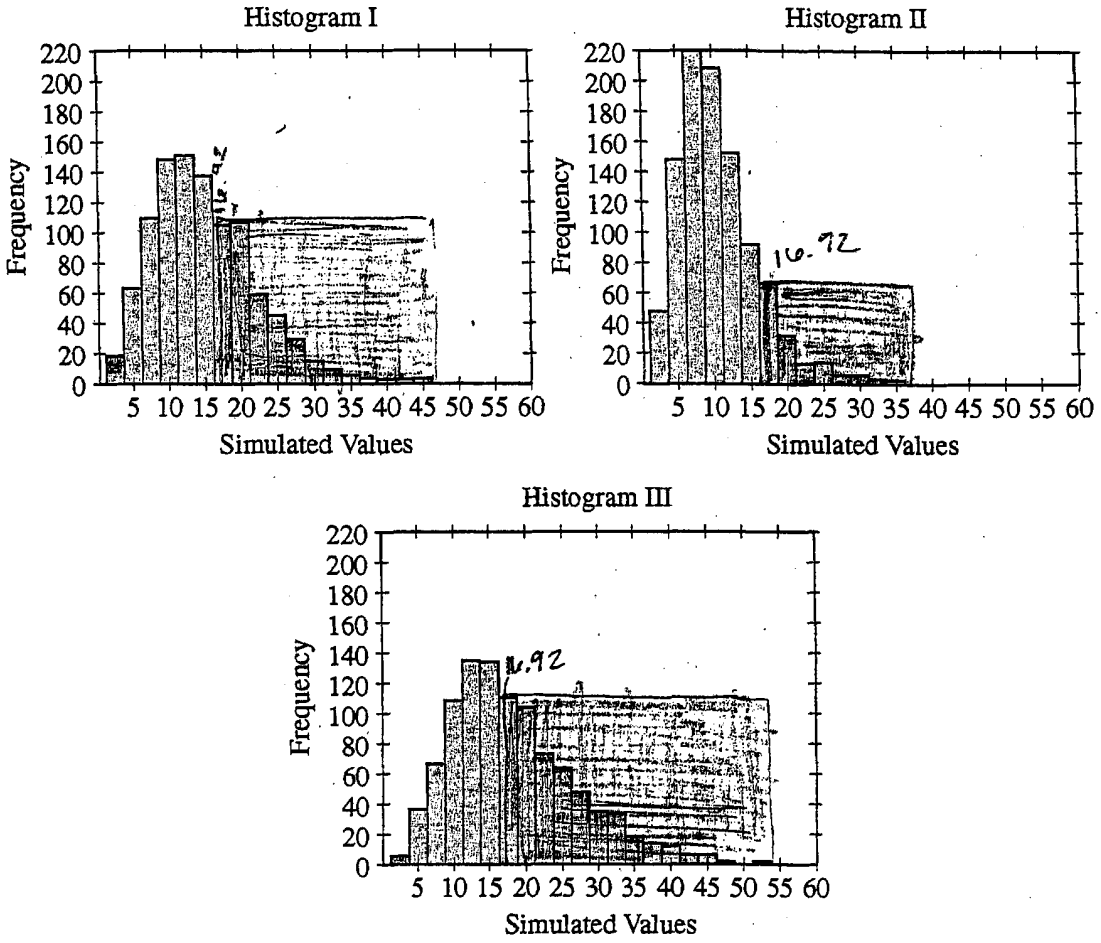
(e) Using simulation, 1,000 samples, each of size 10, were randomly generated from 3 populations with different variances. Each population was normally distributed with mean 68 and variance greater than 1.52.

The histograms below show the simulated sampling distribution of $\frac{(n-1)s^2}{1.52}$ for each population.

Mark the region identified in part (d) on each of the histograms below.



(f) Based on the regions that you marked in part (e), identify the simulated sampling distribution that corresponds to the population with the largest variance. Then identify the simulated sampling distribution that corresponds to the population with the smallest variance. Justify your choices.

Histogram III contains the largest variance because in this one there are many values that would be large enough for me to reject the null hypothesis that the variance is equal to 1.5 Z Histogram II, contains the smallest variance because in this one there are not many values that would be large enough for me to reject the null hypothesis.

STOP

END OF EXAM

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6C1

STATISTICS
SECTION II
Part B
Question 6
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Percent of Section II grade—25

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6. A manufacturer of thermostats is concerned that the readings of its thermostats have become less reliable (more variable). In the past, the variance has been 1.52 degrees Fahrenheit (F) squared. A random sample of 10 recently manufactured thermostats was selected and placed in a room that was maintained at 68°F. The readings for those 10 thermostats are given in the table below.

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Temperature (°F)	66.8	67.8	70.6	69.3	65.9	66.2	68.1	68.6	67.9	67.2

(a) State the null and alternative hypotheses that the manufacturer is interested in testing.

$H_0: \sigma^2 = 1.52$
$H_a: \sigma^2 > 1.52$

where σ^2 = the population variance for thermostats, ø

It can be shown that if the population of thermostat temperatures is normally distributed, the sampling distribution of $\frac{(n-1)s^2}{\sigma^2}$ follows a chi-square distribution with $n-1$ degrees of freedom.

(b) Calculate the value of $\frac{(n-1)s^2}{1.52}$ for these data.

$n = 10 \quad s = 1.42766$ For these sample values.

$$\frac{9 \cdot (1.42766)^2}{1.52} = 12.0684$$

GO ON TO THE NEXT PAGE.

6C2

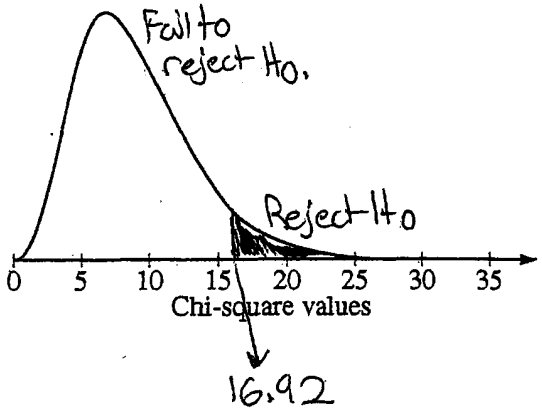
(c) Assume that the population of thermostat temperatures follows a normal distribution. Use the test statistic $\frac{(n-1)s^2}{1.52}$ from part (b) and the chi-square distribution to test the hypotheses in part (a).

Using 9 degrees of freedom, (n-1) and the chi-square distribution chart, we can determine that the tail probability is between .25 and .2 (closer to .2). Since the P-value of the test statistic is so high, we cannot reject the null hypothesis under any reasonable level of significance ($\alpha = .05$).

(d) For the test conducted in part (c), what is the smallest value of the test statistic that would have led to the rejection of the null hypothesis at the 5 percent significance level?

The smallest value of the test statistic that would have lead to the rejection of the null hypothesis would be 16.92.

Mark this value of the test statistic on the graph of the chi-square distribution below. Indicate the region that contains all of the values that would have led to the rejection of the null hypothesis.



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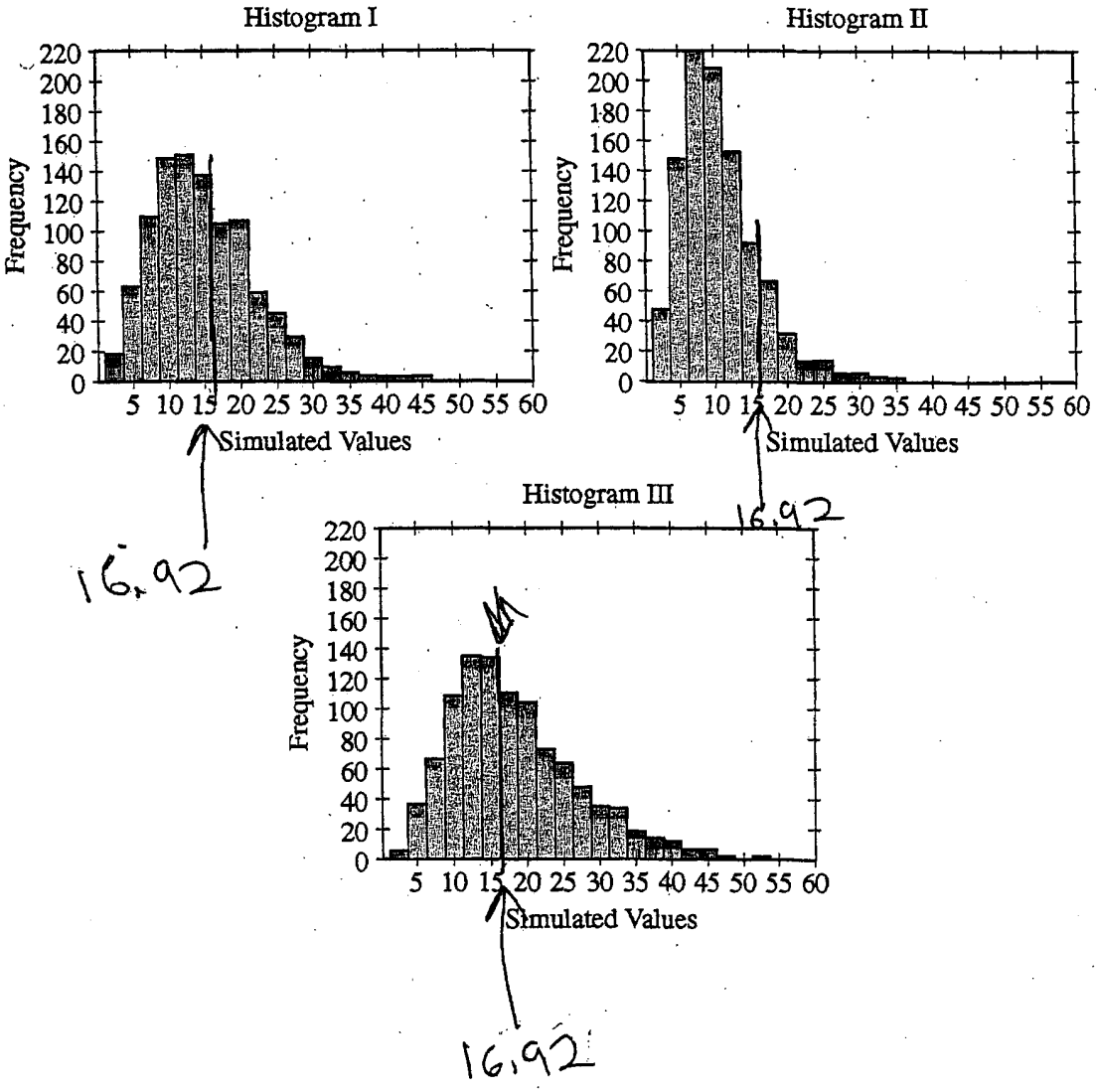
6C3

(e) Using simulation, 1,000 samples, each of size 10, were randomly generated from 3 populations with different variances. Each population was normally distributed with mean 68 and variance greater than 1.52.

The histograms below show the simulated sampling distribution of $\frac{(n-1)s^2}{1.52}$ for each population.

Mark the region identified in part (d) on each of the histograms below.



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6C4

(f) Based on the regions that you marked in part (e), identify the simulated sampling distribution that corresponds to the population with the largest variance. Then identify the simulated sampling distribution that corresponds to the population with the smallest variance. Justify your choices.

The population with the largest variance would be Histogram III. Its values deviate much farther towards each end over a wider range.

The simulated sampling distribution that corresponds to the population with the smallest variance is Histogram II. Its values do not deviate much and are distributed over a narrower range.

STOP

END OF EXAM

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AP® STATISTICS
2006 SCORING COMMENTARY

Question 6

Overview

The primary goals of this question were to evaluate a student's ability to apply the concepts of significance testing to a new setting; in particular to: (1) state hypotheses for a parameter of interest, given a research question; (2) evaluate a new test statistic and use the probability distribution associated with that statistic to test the hypotheses of interest; (3) identify the values of the test statistic that would lead to rejection of the null hypothesis on a graph; and (4) interpret simulated sampling distributions for different populations.

Sample: 6A
Score: 4

This is a complete essay that reflects insight on how to use the newly-presented test statistic to test a single variance. There are two population variances in this situation: the variance of the readings of the population of thermostats in the past (known to be 1.52 degrees Fahrenheit squared) and the variance of the readings of recently manufactured thermostats. The hypotheses in part (a) could be improved by stating which of these populations is meant by the "true population." The conclusion not to reject the null hypothesis in part (c) is linked to the *p*-value and is written in the context of the variability of the thermostat readings. The satisfactory explanation of the *p*-value in the conclusion would be better if it defined "results such as ours": It is not unlikely to get a sample variance as large as or even larger than in this sample, given that $\sigma^2 = 1.52$ based on chance alone. In part (f) the essay nicely describes how the shaded regions in part (e) relate to the concept of Type II error. This essay earned a score of 4.

Sample: 6B
Score: 3

The hypotheses in part (a) could be improved by stating which of the variances is represented by the symbol σ^2. In part (b) the sample variance $s^2 = 1.43^2$ should have been used in the computation. The value given, 1.354, is computed by dividing by the sample size of 10 rather than by $n - 1 = 9$. In part (c) the *p*-value follows consistently from the (incorrect) test statistic in part (b), but the *p*-value is not linked to the conclusion. Linkage could have been achieved by appealing either to a rejection region or to the strength of the evidence against the null hypothesis, as stated below.

> Select a significance level, say $\alpha = 0.05$ and state that the *p*-value is larger than α. Conclude that we do not reject H_0, followed by a statement to that effect in context.
>
> State that if the variance has remained 1.52, there is a 0.2092 chance of getting a sample variance as large as or even larger than the one from the sample. Thus, with a *p*-value this large, the evidence against the null hypothesis is not strong, and so there is no reason to conclude that the variance has increased.

Further, the conclusion lacks context and should refer to the "increase" rather than the "difference" in variance. The conclusion correctly states, "Do not reject H_0." If the conclusion had been written, "Accept H_0" (or the equivalent, such as stating that the variance of the recently manufactured thermostats is still 1.52), it would have been scored as incorrect. Part (f) correctly links the ability to reject the null hypothesis with the largest region but does not make the link to why the population with the largest variance will have the largest region. This essay earned a score of 3.

AP® STATISTICS
2006 SCORING COMMENTARY

Question 6 (continued)

Sample: 6C
Score: 2

The hypotheses in part (a) could be improved by stating which of the two populations (readings of thermostats in the past or readings of recently manufactured thermostats) has its variance represented by σ^2. The conclusion in part (c) nicely links the correct *p*-value to the conclusion but is not written in the context of the situation. In part (e) the critical value is marked, but no region (right tail) is indicated. In part (f) there is no evidence of understanding what the histograms represent. The justification for selecting Histograms III and II refers only to the spread of those histograms themselves. A connection to the populations has not been established, i.e., why the population with the variance farthest above 1.52 would result in a sampling distribution of this test statistic with the largest region and hence be most likely to reject the incorrect null hypothesis. This essay earned a score of 2.