

# AP<sup>®</sup> STATISTICS

## 2008 SCORING GUIDELINES

### Question 1

#### Intent of Question

The primary goals of this question were to assess a student's ability to (1) compare two distributions; (2) reevaluate shape, center, and spread for comparing the two distributions after one of the distributions is transformed by multiplying each of the data points by a constant; and (3) make a prediction about the means of the two distributions based on information derived about the behavior of the distributions from the boxplots.

#### Solution

##### **Part (a):**

The cereals that list a serving size of one cup have a median sugar amount larger than the median for the cereals that list a serving size of three-quarters of a cup. There is more variability (larger range and larger IQR) for the one-cup cereals. The shapes of the two distributions differ. The distribution of sugar content for three-quarter-cup cereals is reasonably symmetric: notice that the median is in the middle of the box. The distribution of sugar content for one-cup cereals is clearly skewed to the left (skewed toward the lower values): notice that the median is pulled to the right side of the central box closer to the third quartile.

##### **Part (b):**

The distribution of sugar content in the cereals that list one-cup serving sizes remains the same as in part (a) because no transformations were applied to this distribution. There is a noticeable shift toward higher sugar content for the cereals that list three-quarter-cup servings after the transformation was applied to this distribution. The two types of cereals (one-cup and three-quarter-cup) now have similar medians, and the two distributions now show similar maximum values. In addition, the variability in the sugar content for cereals with a three-quarter-cup serving size increased by a factor of  $\frac{4}{3}$  after the transformation was applied to the data in this distribution.

##### **Part (c):**

Judging from the boxplots in part (b), we would expect the mean amounts of sugar per serving to be different. By the symmetry of the boxplot for the three-quarter-cup cereals, we would expect the mean and median to be similar. Because the boxplot for the one-cup cereals is skewed to the left, we would expect the mean to be lower than the median. Thus, because both types of cereal have similar medians, we would expect the mean amount of sugar per cup for cereals with a one-cup serving size to be lower than the mean amount of sugar per cup for cereals with a three-quarter-cup serving size.

#### Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the student correctly compares center, shape, and spread of the two distributions. Specific numerical values are not required.

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### Question 1 (continued)

Partially correct (P) if the student correctly compares any two of the three characteristics (center, shape, and spread) of the two distributions.

Incorrect (I) if the student compares no more than one of the three characteristics.

*Note:* If the student uses “normal,” “mound-shaped,” or “uniform” for “symmetric,” then no credit is given for shape. If the student uses “mean” for “median,” then no credit is given for center.

**Part (b)** is scored as follows:

Essentially correct (E) if the student correctly indicates that the three-quarter-cup cereals shift to higher values *AND* that the three-quarter-cup distribution becomes more variable.

Partially correct (P) if the student recognizes one of the two changes (shift to higher values or increase in variability) for the distribution of the three-quarter-cup cereals.

Incorrect (I) if the student identifies neither the shift to higher values nor the increased variability for the three-quarter-cup distribution.

**Part (c)** is scored as follows:

Essentially correct (E) if the student predicts that the mean for cereals with a one-cup serving size would be lower than the mean for cereals with a three-quarter-cup serving size *AND* provides a reasonable justification based on the left skewness of the distribution for the one-cup serving size.

Partially correct (P) if the student predicts that the mean will be lower for the one-cup serving size but provides a weak justification *OR* if the student correctly compares mean and median for each serving size with a justification based on the shapes of the distributions but fails to compare the means of the two serving sizes.

Incorrect (I) if the student predicts that the means will be different with no justification *OR* predicts that the means will be similar *OR* makes no prediction.

#### **4 Complete Response**

All three parts essentially correct

#### **3 Substantial Response**

Two parts essentially correct and one part partially correct

#### **2 Developing Response**

Two parts essentially correct and no parts partially correct

*OR*

One part essentially correct and one or two parts partially correct

*OR*

Three parts partially correct

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**Question 1 (continued)**

**1 Minimal Response**

One part essentially correct and no parts partially correct

*OR*

No parts essentially correct and two parts partially correct

## STATISTICS

## SECTION II

## Part A

## Questions 1-5

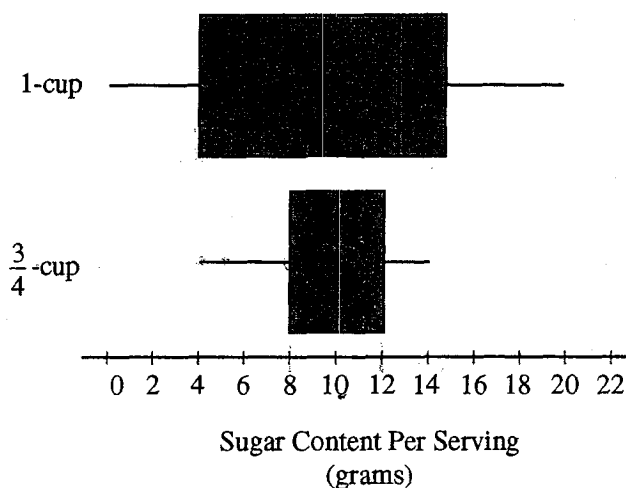
Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store.

The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels. To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.

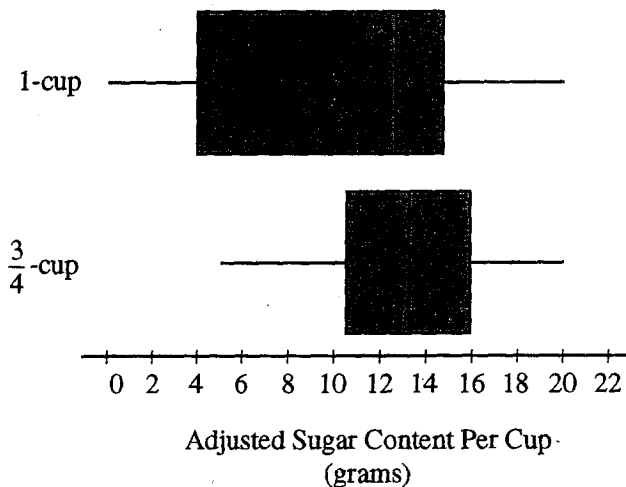


- (a) Write a few sentences to compare the distributions of sugar content per serving for the two serving sizes of cereals.

The boxplot for the sugar content in a 1 cup serving size has a higher median than <sup>that of</sup> the boxplot for the sugar content in a  $\frac{3}{4}$ -cup serving. The IQR for the 1 cup serving size boxplot is approximately 11 grams of sugar, which is much larger than the IQR of 4 grams of sugar for the  $\frac{3}{4}$ -cup serving size boxplot. The shape of the boxplot for the  $\frac{3}{4}$ -cup serving size is roughly symmetrical while the shape of the 1-cup serving size boxplot is skewed left. Both boxplots have no outliers.

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After analyzing the boxplots on the preceding page, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by  $\frac{4}{3}$ . The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.



(b) What new information about sugar content do the boxplots above provide?

The medians of the two distributions are approximately the same, the range of the adjusted sugar content for the boxplot of the data from the original  $\frac{3}{4}$ -cup serving size is now much wider and higher than that of the original boxplot. Overall, the adjusted boxplot for the  $\frac{3}{4}$ -cup serving size reveals that it contains a higher sugar count than the first boxplot for this data revealed. The IQR for the new  $\frac{3}{4}$ -cup serving size boxplot is also slightly greater than that of the original boxplot.

(c) Based on the boxplots shown above on this page, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain.

I would expect the mean amount of sugar per cup to be greater for the  $\frac{3}{4}$ -cup serving size because the  $\frac{3}{4}$ -cup distribution is centered at a higher number of sugar content per cup than the 1-cup distribution. The 1-cup serving size would also have a lower mean because the skewed distribution to the left means that these numbers will decrease the mean because it is non-resistant to skewness and outliers and will be affected by these low numbers.

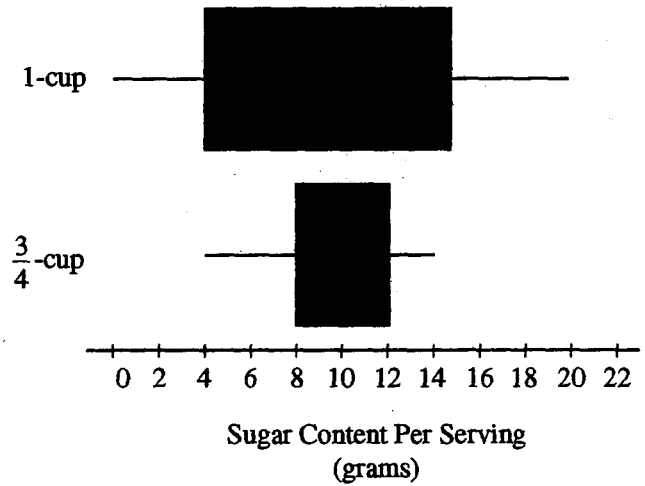
STATISTICS  
SECTION II  
Part A  
Questions 1-5

Spend about 65 minutes on this part of the exam.  
Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- 1. To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store.

The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels. To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.

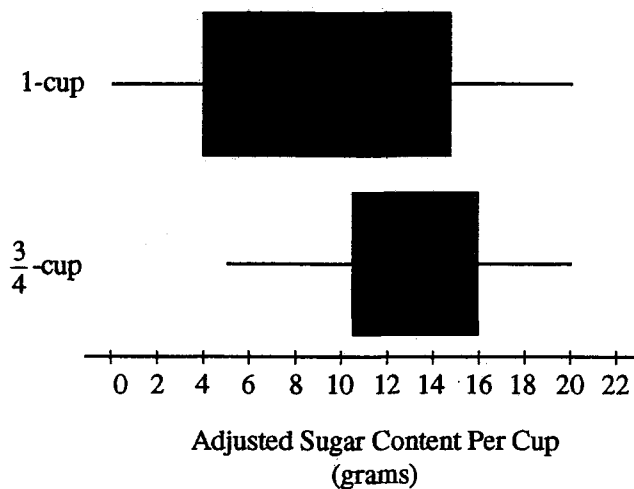


- (a) Write a few sentences to compare the distributions of sugar content per serving for the two serving sizes of cereals.

The 1-cup serving distribution has a much wider range than the 3/4-cup distribution. It is wider by about 10 grams. The distribution of 3/4 cup servings is more symmetrical than the 1-cup serving, which is more strongly skewed left. The median sugar content is higher in the 1-cup distribution.

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After analyzing the boxplots on the preceding page, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by  $\frac{4}{3}$ . The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.



(b) What new information about sugar content do the boxplots above provide?

The median sugar content is higher for the  $\frac{3}{4}$ -cup servings than it is for the one-cup serving distributions. There is actually a higher sugar content in the cereals that have  $\frac{3}{4}$ -cup serving labels.

(c) Based on the boxplots shown above on this page, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain.

The mean amount of sugar/cup for the  $\frac{3}{4}$ -cup serving size would be about 13, or close to the median since the distribution is roughly symmetrical. The mean amount of sugar/cup for the 1-cup serving size would be less than 13 since the distribution is skewed left, and the mean will be shifted toward lower values.

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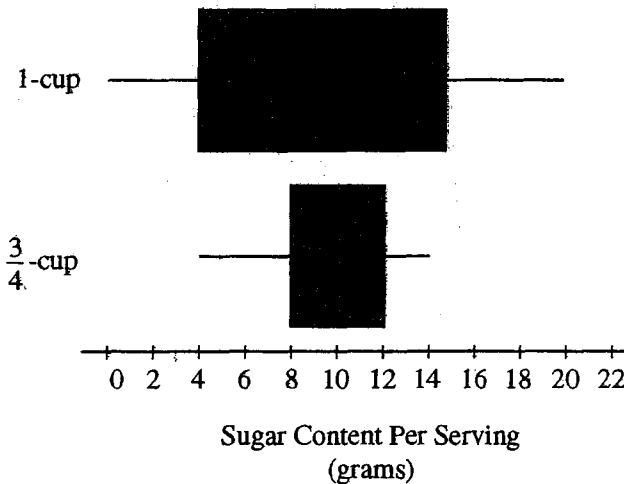
**STATISTICS  
SECTION II  
Part A  
Questions 1-5**

**Spend about 65 minutes on this part of the exam.  
Percent of Section II score—75**

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

- 1. To determine the amount of sugar in a typical serving of breakfast cereal, a student randomly selected 60 boxes of different types of cereal from the shelves of a large grocery store.

The student noticed that the side panels of some of the cereal boxes showed sugar content based on one-cup servings, while others showed sugar content based on three-quarter-cup servings. Many of the cereal boxes with side panels that showed three-quarter-cup servings were ones that appealed to young children, and the student wondered whether there might be some difference in the sugar content of the cereals that showed different-size servings on their side panels. To investigate the question, the data were separated into two groups. One group consisted of 29 cereals that showed one-cup serving sizes; the other group consisted of 31 cereals that showed three-quarter-cup serving sizes. The boxplots shown below display sugar content (in grams) per serving of the cereals for each of the two serving sizes.



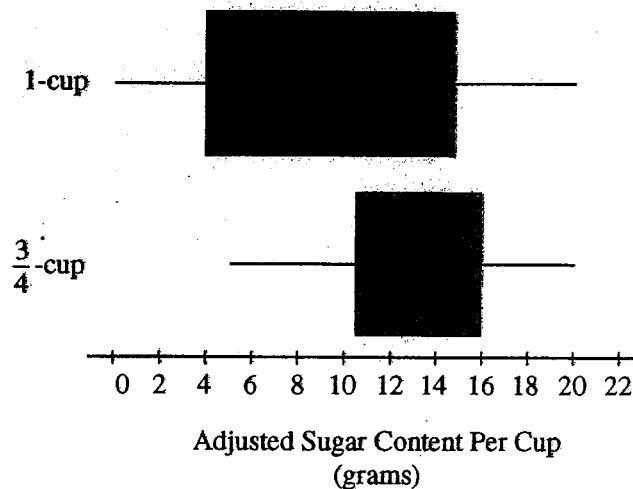
- (a) Write a few sentences to compare the distributions of sugar content per serving for the two serving sizes of cereals.

The IQR of the 1-cup boxes is 10 while for 3/4-cup boxes it is only 4. This shows a larger range of sugar content in 1-cup boxes. Both distributions are normal, but the 1-cup has a higher median. There are no outliers.

**GO ON TO THE NEXT PAGE.**



After analyzing the boxplots on the preceding page, the student decided that instead of a comparison of sugar content per recommended serving, it might be more appropriate to compare sugar content for equal-size servings. To compare the amount of sugar in serving sizes of one cup each, the amount of sugar in each of the cereals showing three-quarter-cup servings on their side panels was multiplied by  $\frac{4}{3}$ . The bottom boxplot shown below displays sugar content (in grams) per cup for those cereals that showed a serving size of three-quarter-cup on their side panels.



(b) What new information about sugar content do the boxplots above provide?

Now, it is clear that the  $\frac{3}{4}$ -cup boxes contain more sugar because their adjusted median is larger than the 1-cup boxes. The IQR of the  $\frac{3}{4}$ -cup boxes also increased from 4 to about 5.5. Because the sizes are now the same, you can see that the more child oriented cereals may contain more sugar.

(c) Based on the boxplots shown above on this page, how would you expect the mean amounts of sugar per cup to compare for the different recommended serving sizes? Explain.

I would expect the mean amount of sugar in the  $\frac{3}{4}$ -cup to be larger because its median is higher, and it has a far less amount of values on the lower end of the # line. The 1-cup has more low sugar contents and the  $\frac{3}{4}$ -cup has the highest sugar content.

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**AP<sup>®</sup> STATISTICS**  
**2008 SCORING COMMENTARY**

**Question 1**

**Overview**

The primary goals of this question were to assess a student's ability to (1) compare two distributions; (2) reevaluate shape, center, and spread for comparing the two distributions after one of the distributions is transformed by multiplying each of the data points by a constant; and (3) make a prediction about the means of the two distributions based on information derived about the behavior of the distributions from the boxplots.

**Sample: 1A**  
**Score: 4**

In part (a) the response clearly compares the medians, the IQRs, and the shapes of the distributions of sugar content per serving for the two serving sizes of cereals. Part (a) was scored as essentially correct. In part (b) the response states that the medians of the two distributions are “approximately the same” after the adjustment and that the range and the IQR for the three-quarter-cup serving size are both larger after the adjustment than they were in the original boxplot. Part (b) was scored as essentially correct. The response in part (c) correctly predicts that the mean amount of sugar per cup will be larger for the cereals with a three-quarter-cup serving size than for the cereals with a one-cup serving size and gives a justification based on the left skewness of the distribution for the one-cup serving size. Part (c) was scored as essentially correct. The entire answer, based on all three parts, was judged a complete response and earned 4 points.

**Sample: 1B**  
**Score: 3**

In part (a) the response clearly compares the medians, the ranges, and the shapes of the distributions of sugar content per serving for the two serving sizes of cereals. Part (a) was scored as essentially correct. In part (b) the response states that after the adjustment the median sugar content is higher for the cereals with a three-quarter-cup serving size than for the cereals with a one-cup serving size. The response fails to mention the increase in variability that results from the adjustment. Part (b) was scored as partially correct. The response in part (c) correctly predicts that the mean amount of sugar per cup for the cereals with a three-quarter-cup serving size will be “about 13 [grams]” and that the mean amount of sugar per cup for the cereals with a one-cup serving size will be “less than 13 [grams]” and gives a correct justification based on the shapes of both distributions. Part (c) was scored as essentially correct. The overall answer was considered a substantial response and was awarded 3 points.

**Sample: 1C**  
**Score: 2**

In part (a) the response compares variability (IQR and range) and center (median) of the distributions of sugar content per serving for the two serving sizes of cereals. The response incorrectly states that “Both distributions are normal”; it is not possible to establish the normality of a distribution from a boxplot. Part (a) was scored as partially correct. The response in part (b) clearly states that the median sugar content of cereals with a three-quarter-cup serving size is larger after the adjustment than the median sugar content of the cereals with a one-cup serving size and that the IQR for the three-quarter-cup serving size increases with the adjustment. Part (b) was scored as essentially correct. In part (c) the response correctly predicts that the mean amount of sugar per cup will be larger for the cereals with a three-quarter-cup serving size than for the cereals with a one-cup serving size. The weak justification is based on the fact that there are fewer “values on the lower end of the # line” for the cereals with a three-quarter-cup serving size, rather than on the shapes of the distributions. Part (c) was scored as partially correct. The whole answer was deemed a developing response and received 2 points.