Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate the proper sample size for a given margin of error and (2) relate that value to practical restrictions given in the problem.

Solution

Part (a):

The required sample size for 95% confidence and a margin of error (E) of 2 is

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(12)^2}{2^2} = 138.3$$

So, use $n = 139$.

Note: It is also acceptable to use $z = 2$ in the formula for sample size, in which case $n = 144$.

The cost of carrying out the study would be \$13,900 (or \$14,400), which exceeds the amount budgeted for the study. Sufficient funds are not available.

OR

For \$12,000, the company can afford to use a sample size of 120. The margin of error (with 95% confidence) expected for this sample size is

$$1.96\frac{12}{\sqrt{120}} = 2.15 \quad \textit{OR}\text{, using 2 in place of 1.96,} \quad 2\frac{12}{\sqrt{120}} = 2.19$$

It is not possible to estimate the true mean stopping distance to within 2 feet with 95% confidence for \$12,000.

Part (b):

To be within 2 feet of the true mean with 95% confidence, 139 observations are required. The budget of \$12,000 only allows 120 observations to be taken. Therefore, the company will not be able to meet the regulatory agency's requirements with the allocated budget.

OR

Given the fact that \$12,000 will allow 120 observations to be taken, the margin of error will be 2.15. Therefore, the company will not be able to meet the regulatory agency's requirements with the allocated budget.

Question 3 (continued)

Scoring

There are two components to this question: component 1 consists of the calculations; component 2 consists of the conclusion/consequences. Components 1 and 2 are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Component 1 is scored as follows:

Essentially correct (E) if:

(1) A correct approach to calculating either the margin of error or the sample size is used.
AND
(2) The required sample size (139) is correctly computed OR the margin of error that could be achieved for $12,000 is correctly computed (2.15).

Partially correct if:

One or more minor errors are made in the formula and/or calculation. Examples of minor errors are the following:

- A square or square root sign is missing or is included erroneously
- A multiplier of 1.645 is used instead of 1.96
- The margin of error is rounded to 2, so the student thinks there are sufficient funds
- The sample size is rounded down instead of up, or left as 138.3
- The student thinks the $12,000 will only allow 12 cars to be tested

Incorrect if:

One or more major errors are made. Examples of major errors are the following:

- Not including any multiplier in the margin of error formula
- Thinking the full width rather than half the width of the interval is the margin of error
- Stating that n must be greater than 30 (to have a "large sample")

Component 2 is scored as follows:

Essentially correct (E) if:

(1) A correct conclusion that the budget is insufficient (based on the sample size or margin of error calculation) is stated and clearly explained. (If the sample size or margin of error calculation is incorrect, the conclusion may be that the budget is sufficient.)
AND
(2) The consequence that the car manufacturer will not be able to meet the requirement of the regulatory agency is stated. (If the sample size or margin of error calculation is incorrect, the conclusion may be that the manufacturer will be able to meet the requirement of the regulatory agency.)

Note: One would expect the conclusion (1) to be given in part (a) and the consequence (2) to be given in part (b); if both are given in part (a), then there must be an additional consequence in context specified in part (b) for component 2 to be essentially correct.

Question 3 (continued)

Partially correct (P) if only one of (1) and (2) is correctly stated.

Incorrect (I) if neither (1) nor (2) is correctly stated.

4 Complete Response

Both components essentially correct

3 Substantial Response

One component essentially correct and one component partially correct

2 Developing Response

One component essentially correct and the other component incorrect

OR

Both components partially correct

1 Minimal Response

One component partially correct and the other component incorrect

3. A car manufacturer is interested in conducting a study to estimate the mean stopping distance for a new type of brakes when used in a car that is traveling at 60 miles per hour. These new brakes will be installed on cars of the same model and the stopping distance will be observed. The cost of each observation is $100. A budget of $12,000 is available to conduct the study and the goal is to carry it out in the most economical way possible. Preliminary studies indicate that $\sigma = 12$ feet for stopping distances.

3A

(a) Are sufficient funds available to estimate the mean stopping distance to within 2 feet of the true mean stopping distance with 95% confidence?

No

Explain your answer.

To find within 2feet at 95% confident:

$2 = CV \times \sigma$; $CV = 1.96$

$2 = 1.96 \times \sigma$

$\sigma = \frac{2}{1.96} = 1.020$

$\sigma = \frac{\sigma_s}{\sqrt{n}}$; $\sigma_s = 12ft$

$\therefore \frac{12}{\sqrt{n}} = \frac{2}{1.96} \Rightarrow 2\sqrt{n} = 23.52 \Rightarrow \sqrt{n} = 11.76$

$n = 138.3$

therefore need at least 139 observations at $100 an observation, total need is 100×139 = $13,900

∴ No enough budgets.

(b) A regulatory agency requires a 95% level of confidence for an estimate of mean stopping distance that is within 2 feet of the true mean stopping distance. The car manufacturer cannot exceed the budget of $12,000 for the study. Discuss the consequences of these constraints.

As $12,000 gives 120 observations, the 95% confidence interval will be within $1.96 \times \frac{12}{\sqrt{120}} = 2.147$ feet of the true mean. This means that the manufacturer will not be able to fulfil the required accuracy of the regulatory agency which means the agency will not be able approve of the new brakes.

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3B

3. A car manufacturer is interested in conducting a study to estimate the mean stopping distance for a new type of brakes when used in a car that is traveling at 60 miles per hour. These new brakes will be installed on cars of the same model and the stopping distance will be observed. The cost of each observation is $100. A budget of $12,000 is available to conduct the study and the goal is to carry it out in the most economical way possible. Preliminary studies indicate that $\sigma = 12$ feet for stopping distances.

(a) Are sufficient funds available to estimate the mean stopping distance to within 2 feet of the true mean stopping distance with 95% confidence?

Explain your answer.

100 | 12000

n=120

$N = \frac{(1.64 \cdot 12)^2}{2}$

n=193

No, to estimate the true mean stopping dist. to within 2 feet, w/ 95% conf. would cost $19,300, over $7,000 more than you have.

(b) A regulatory agency requires a 95% level of confidence for an estimate of mean stopping distance that is within 2 feet of the true mean stopping distance. The car manufacturer cannot exceed the budget of $12,000 for the study. Discuss the consequences of these constraints.

As you can see above, it will cost $19,300 to be 95% confident. With $12000 the closest you can get is w/in 3.227 feet [redacted] of the true stopping distance

$120 = \left(\frac{1.64 \times 12}{\text{M.error}}\right)^2$

M.error = 3.227

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3C

3. A car manufacturer is interested in conducting a study to estimate the mean stopping distance for a new type of brakes when used in a car that is traveling at 60 miles per hour. These new brakes will be installed on cars of the same model and the stopping distance will be observed. The cost of each observation is $100. A budget of $12,000 is available to conduct the study and the goal is to carry it out in the most economical way possible. Preliminary studies indicate that $\sigma = 12$ feet for stopping distances.

(a) Are sufficient funds available to estimate the mean stopping distance to within 2 feet of the true mean stopping distance with 95% confidence?

yes

Explain your answer.

OK, say the mean stopping distance is 20 and you can conduct 120 tests with your budget and we have a $\sigma = 12$

$20 \pm 1.960\, {}^{12}/\sqrt{120} =$ 18ft to 22ft with a 95% confidence level

so there are sufficient funds available

(b) A regulatory agency requires a 95% level of confidence for an estimate of mean stopping distance that is within 2 feet of the true mean stopping distance. The car manufacturer cannot exceed the budget of $12,000 for the study. Discuss the consequences of these constraints.

With a budget of $12,000 that allows you to conduct 120 tests. Well, It seems as if it would be hard to get exactly what you want with a fixed number because if one thing is wrong it could take many tests to figure it out

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Question 3

Sample: 3A
Score: 4

This response successfully determines the number of observations needed by rounding the computed number up to 139. A calculation provides justification for the clear statement that there are not sufficient funds for 139 observations. The response to part (b) provides a second way to analyze this problem, computing the margin of error associated with the 120 observations that the budget allows. Either analysis would justify the final sentence, which identifies the consequences of the constraints. Each component of this question was scored as essentially correct, so the entire answer was considered a complete response.

Sample: 3B
Score: 3

In part (a) the formula is missing a power of 2 and uses a critical value of 1.64 instead of 1.96. The number is then rounded down to get the number of needed observations. These three minor errors resulted in a score of partially correct for the computations. The clear statement in part (a) and the second way of analyzing the problem in part (b) was considered a sufficient identification of the consequences, so that component was scored as essentially correct. The overall answer, based on the two components of this question, was deemed a substantial response.

Sample: 3C
Score: 2

The computations in part (a) use a formula for a confidence interval rather than for the margin of error, which would have been acceptable if a clear interpretation of the interval had been included. No margin of error is given, nor is the computed value compared to the required distance of 2 feet, however. (The interval is rounded to a whole number of feet, making it appear that the margin of error with 120 observations is 2 feet.) Thus, the computations were scored as partially correct. The response to part (b) does not add to the conclusion already given in part (a), so the consequences component was also scored as partially correct. On the whole this answer was judged a developing response.