AP® STATISTICS
2008 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) state the appropriate hypotheses; (2) identify and compute the appropriate test statistic; (3) make a conclusion in the context of the problem; and (4) compare two sets of proportions to identify the preferred habitat.

Solution

Part (a):

Step 1: States a correct pair of hypotheses.

H_0 : Moose have no preference for habitat type.
H_a : Moose have a preference for habitat type.

OR

H_0 : The number of moose in each habitat type is proportional to the amount of acreage of that habitat type.
H_a : The number of moose in at least one habitat type is not proportional to the amount of acreage of that habitat type.

OR

H_0 : $p_1 = 0.340, p_2 = 0.101, p_3 = 0.104, p_4 = 0.455$, where p_i = the proportion of moose in habitat type *i*.
H_a : At least one of these proportions is incorrect.

Step 2: Identifies a correct test (by name or formula) and checks appropriate conditions.

- Chi-square goodness-of-fit test (or test for more than two proportions)
$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$
- The stem of the problem stated that conditions for inference are met.

Step 3: Correct mechanics, including the value of the test statistic, df, and *p*-value (or rejection region).

- The test statistic, with df = 4 − 1 = 3, is
$$\chi^2 = \frac{(25-39.780)^2}{39.780} + \frac{(22-11.817)^2}{11.817} + \frac{(30-12.168)^2}{12.168} + \frac{(40-53.235)^2}{53.235} = 43.6893.$$
- The *p*-value is $P(\chi_3^2 \geq 43.6893) < 0.0005$ (a calculator gives the *p*-value as 1.7569×10^{-9}).

Question 5 (continued)

Step 4: States a correct conclusion in the context of the problem, using the result of the statistical test.

The data are not consistent with the researchers' expectation. Because the *p*-value is less than $\alpha = 0.05$, we reject H_0. There is strong evidence that moose have a preference for habitat type.

OR

The data are not consistent with the researchers' expectation. If the null hypothesis is true and the number of moose in each of the habitat types is proportional to the acreage in that habitat type, then we would observe a test statistic of 43.69 or one more extreme less than 0.05 percent of the time. There is strong evidence that moose have a preference for habitat type.

Part (b):

The moose seem to prefer habitat types 2 and 3. Relative to the proportion of total acreage, a higher proportion of moose were observed in each of these habitat types than expected. In habitat types 1 and 4, the observed proportion of moose was less than the expected proportion of moose, indicating that these two habitat types are less desirable.

OR

Habitat type 3 seems to be the most preferred—it has a positive difference between the observed (30) and expected (12.168) counts of moose and the largest contribution to the chi-square statistic (26.1325). Alternatively, habitat type 3 has the largest positive difference between the observed proportion of moose (0.256) and the expected proportion of moose (0.104).

Scoring

This problem is scored in four sections. Section 1 consists of part (a), step 1. Section 2 consists of part (a), steps 2 and 3. Section 3 consists of part (a), step 4. Section 4 consists of part (b). Sections 1, 2, and 3 are scored as essentially correct (E) or incorrect (I), and section 4 is scored as essentially correct (E), partially correct (P), or incorrect (I).

If an inappropriate inference procedure is used in part (a), then all three sections must be scored as incorrect (I).

Section 1 [part (a), step 1]: States a correct pair of hypotheses.

- Hypotheses must be given in context—which includes some reference to moose and the different habitat types—to earn an E. Hypotheses that clearly address sample data (like "observed number of moose") are incorrect.

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Question 5 (continued)

Section 2 [part (a), steps 2 and 3]: Identifies a correct test and checks appropriate conditions. Mechanics are correct.

- A discussion of conditions for inference should generally be treated as extraneous. However, if the response includes inappropriate conditions—like normality or independent samples—the response cannot receive a score of 4.
- An inappropriate method of calculating df will result in these combined steps being scored incorrect.

Section 3 [part (a), step 4]: States a correct conclusion in the context of the problem.

- If an incorrect *p*-value in steps 2 and 3 is obtained from a chi-square goodness-of-fit test, but the conclusion is consistent with this *p*-value, step 4 can be considered correct.
- If both an α and a *p*-value are given together, the linkage between the *p*-value and the conclusion is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the *p*-value *OR* explaining how the conclusion follows from the *p*-value.

Section 4 [part (b)] is scored as follows:

Essentially correct (E) if habitat types 2 and 3 are identified as the preferred habitat types with a justification that indicates there is a higher proportion (or a higher number) of moose than expected relative to the proportion of total acreage in those areas. One way to do this is to compare the observed density of moose across the four habitat types. Note that habitat types 2 and 3 also happen to make the largest contribution to the chi-square statistic.

OR

Habitat type 3 is identified as the most preferred because it has a higher proportion (or higher number) of moose than expected and the largest chi-square contribution *OR* the largest positive difference in observed and expected proportions *OR* the highest density of moose.

Partially correct (P) if habitat types 2 and 3 (or habitat type 3 alone) are identified with an incomplete justification. For example, a student might select habitat type 3 as most preferred based on the fact that it yields the largest contribution to the chi-square statistic but not indicate that there is a higher proportion (or higher number) of moose than expected in these areas.

Incorrect (I) if habitat types 2 and 3 (or just habitat type 3) are identified with no or incorrect justification *OR* habitat types 1 or 4 are identified.

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (b) counts as $\frac{1}{2}$ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

Question 5 (continued)

1 Minimal Response

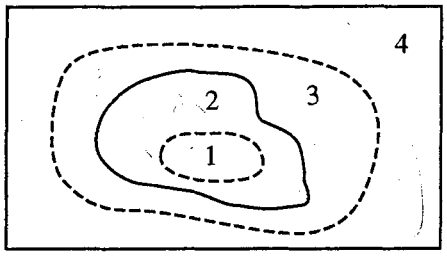
If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.

5A1

5. A study was conducted to determine where moose are found in a region containing a large burned area. A map of the study area was partitioned into the following four habitat types.

(1) Inside the burned area, not near the edge of the burned area,
(2) Inside the burned area, near the edge,
(3) Outside the burned area, near the edge, and
(4) Outside the burned area, not near the edge.

The figure below shows these four habitat types.



Note: Figure not drawn to scale.

The proportion of total acreage in each of the habitat types was determined for the study area. Using an aerial survey, moose locations were observed and classified into one of the four habitat types. The results are given in the table below.

Habitat Type	Proportion of Total Acreage	Number of Moose Observed
1	0.340	25
2	0.101	22
3	0.104	30
4	0.455	40
Total	1.000	117

(a) The researchers who are conducting the study expect the number of moose observed in a habitat type to be proportional to the amount of acreage of that type of habitat. Are the data consistent with this expectation? Conduct an appropriate statistical test to support your conclusion. Assume the conditions for inference are met.

H_0: number of moose proportional to acreage of habitat
H_A: number of moose not proportional to acreage of habit

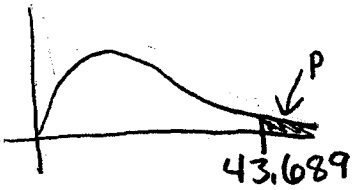
Habitat	1	2	3	4
observed	25	22	30	40
expected	39.78	11.817	12.168	53.235

GO ON TO THE NEXT PAGE.

If you need more room for your work in part (a), use the space below.

Assumptions for goodness of fit test: expected counts ≥ 5 ✓

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}} = 43.689$$



$df = \#\text{ of cells} - 1 = 4 - 1 = 3$

$p = P(\chi^2 \geq 43.689) = 1.757 \times 10^{-9}$

Since p is very small (1.757×10^{-9}), I reject the null and conclude the number of moose observed in a habitat type is not proportional to the acreage of that type of habitat.

(b) Relative to the proportion of total acreage, which habitat types did the moose seem to prefer? Explain.

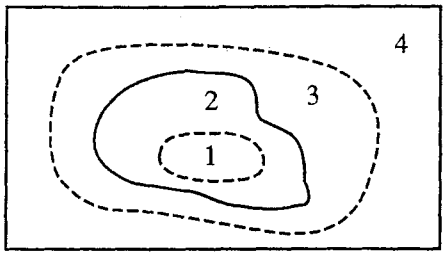
The moose seem to prefer habitats 2 and 3 — living inside the burned area, near the edge or outside the burned area, near the edge. This conclusion can be drawn because the observed count of moose in each of these habitats was greater than the expected count of moose based on the proportion of total acreage. 22 moose were observed in habitat 2, though only 11.817 were expected and 30 were observed in habitat 3 though only 12.168 were expected.

GO ON TO THE NEXT PAGE.

5. A study was conducted to determine where moose are found in a region containing a large burned area. A map of the study area was partitioned into the following four habitat types.

(1) Inside the burned area, not near the edge of the burned area,
(2) Inside the burned area, near the edge,
(3) Outside the burned area, near the edge, and
(4) Outside the burned area, not near the edge.

The figure below shows these four habitat types.



Note: Figure not drawn to scale.

The proportion of total acreage in each of the habitat types was determined for the study area. Using an aerial survey, moose locations were observed and classified into one of the four habitat types. The results are given in the table below.

Habitat Type	Proportion of Total Acreage	Number of Moose Observed	Expected #
1	0.340	25	(.34)(117) = 39.78
2	0.101	22	(.101)(117) = 11.817
3	0.104	30	(.104)(117) = 12.168
4	0.455	40	(.455)(117) = 53.235
Total	1.000	117	

(a) The researchers who are conducting the study expect the number of moose observed in a habitat type to be proportional to the amount of acreage of that type of habitat. Are the data consistent with this expectation? Conduct an appropriate statistical test to support your conclusion. Assume the conditions for inference are met.

$\chi^2 = \sum \frac{(obs-exp)^2}{exp}$ $\alpha = .05$

H_0: The # of moose are proportional to area
H_a: The # of moose are not proportional to area.

Since all conditions are assumed to be met, a Chi-square goodness of fit test can be used with 3 degrees of freedom.

Habitat	$\frac{(obs-exp)^2}{exp}$
1	5.491
2	8.775
3	26.132
4	3.290
tot =	$\chi^2 = 43.688$

$\chi^2 cdf(43.688, 99, 3) = 1.76 \times 10^{-9}$

The p-value is nearly zero, providing significant evidence to reject the null and support the alternative hypothesis that the number of moose in each habitat is not proportional to its area.

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If you need more room for your work in part (a), use the space below.

(b) Relative to the proportion of total acreage, which habitat types did the moose seem to prefer? Explain.

	Residuals (obs-exp)
1	-14.78
2	10.183
3	17.832
4	-13.235

The moose preferred the two habitats, relative to the proportion of area, near the edge of the burned area, especially on the outer edge. The residuals for these habitats are much greater than zero.

5C1

5. A study was conducted to determine where moose are found in a region containing a large burned area. A map of the study area was partitioned into the following four habitat types.

(1) Inside the burned area, not near the edge of the burned area,
(2) Inside the burned area, near the edge,
(3) Outside the burned area, near the edge, and
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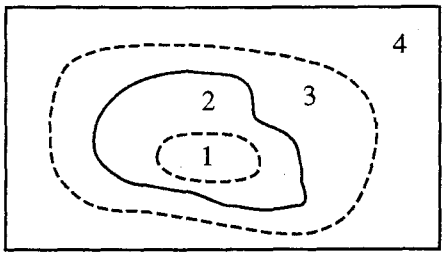

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The proportion of total acreage in each of the habitat types was determined for the study area. Using an aerial survey, moose locations were observed and classified into one of the four habitat types. The results are given in the table below.

Habitat Type	Proportion of Total Acreage	Number of Moose Observed
1	0.340	25
2	0.101	22
3	0.104	30
4	0.455	40
Total	1.000	117

(a) The researchers who are conducting the study expect the number of moose observed in a habitat type to be proportional to the amount of acreage of that type of habitat. Are the data consistent with this expectation? Conduct an appropriate statistical test to support your conclusion. Assume the conditions for inference are met.

expected # moose

habitat	# moose
1	$.34 \cdot 117 = 39.78$
2	$.101 \cdot 117 = 11.817$
3	$.104 \cdot 117 = 12.168$
4	$.455 \cdot 117 = 53.235$

$\alpha = .05$

$\chi^2 = \sum\left(\frac{(Obs - Exp)^2}{Exp}\right)$, $3 = d.f.$

$= 43.6893$

$p =$ basically $0 << .05$

Using a chi-squared goodness-of-fit test, the probability of fitting the expectation with this data is extremely small. The numbers of moose in each habitat are not proportional to the acreage of each habitat.

GO ON TO THE NEXT PAGE.

If you need more room for your work in part (a), use the space below.

(b) Relative to the proportion of total acreage, which habitat types did the moose seem to prefer? Explain.

standardized χ^2-value = $\frac{(obs - exp)}{\sqrt{exp}}$

habitat	stand. # moose
1	-2.956
2	2.171
3	3.2557
4	-2.093

The moose seem to prefer habitat types 2 and 3, those near the edge of the burned area. They have disproportionately large amounts of ~~data~~ moose in the area than acreage would account for, or than the other two habitats, which the moose seemed to greatly avoid, as there are so many fewer than expected moose in those areas

AP® STATISTICS
2008 SCORING COMMENTARY

Question 5

Overview

The primary goals of this question were to assess a student's ability to (1) state the appropriate hypotheses; (2) identify and compute the appropriate test statistic; (3) make a conclusion in the context of the problem; and (4) compare two sets of proportions to identify the preferred habitat.

Sample: 5A
Score: 4

The response in part (a) includes an acceptable pair of hypotheses with appropriate reference to context: "moose" and "acreage." This response shows solid mechanics—from naming the test; to calculating the test statistic, *p*-value, and degrees of freedom; to sketching and shading the appropriate chi-square distribution. Based on the small *p*-value, the response makes a correct decision about the null hypothesis and states an appropriate conclusion in context. Sections 1–3, corresponding to part (a), were scored as essentially correct. In part (b), corresponding to section 4, the response correctly identifies habitat types 2 and 3 as preferred "because the observed count of moose . . . was greater than the expected count" in these two areas. Section 4 was thus scored as essentially correct. The full answer, including all four sections, was judged a complete response and earned 4 points.

Sample: 5B
Score: 3

The response to part (a) includes a pair of hypotheses that are stated in context but that do not adequately describe how the moose are distributed across the four habitat types. A correct chi-square test statistic, degrees of freedom, and *p*-value are provided. With clear linkage to the computed *p*-value, the response makes an appropriate decision about the stated null hypothesis and gives a clear conclusion that is consistent with stated hypotheses. Section 1 of part (a) was scored as incorrect, and sections 2 and 3 were scored as essentially correct. Using calculated residuals in part (b), the response correctly identifies the habitat types "near the edge of the burned area" as the ones preferred by moose. Section 4, corresponding to part (b), was thus scored as essentially correct. Overall, this answer was considered a substantial response and was awarded 3 points.

Sample: 5C
Score: 2

This response does not include any hypotheses in part (a). However, the response does give the name of the appropriate statistical test, along with correct calculations of the chi-square test statistic, degrees of freedom, and *p*-value. Although there is clear linkage between the *p*-value and a chosen significance level of 0.05, the conclusion includes an inappropriate comment about what this *p*-value represents. In addition, no decision can be made about hypotheses, as none have been previously stated. Section 2 was thus scored as essentially correct, but sections 1 and 3 were scored as incorrect. In part (b) the student makes a convincing argument in favor of habitat types 2 and 3 that is based on "stand. [standardized] # of moose" per habitat. Section 4 was thus scored as essentially correct. The answer as a whole was deemed a developing response and received 2 points.