Question 6

Intent of Question

The primary goals of this investigative task were to assess a student's ability to (1) identify clusters in a scatterplot and deduce what such clusters represent; (2) recognize a positive linear relationship and state what it means, in context, when all points lie below the line $y = x$; (3) understand that when data are paired, a paired t-test, rather than a two-sample t-test is appropriate when testing whether the means are equal; and (4) create an appropriate graphical display that makes the dividing point between two groups of measurements visually obvious.

Solution

Part (a):

The trait that distinguishes the two groups in the scatterplot is dominant foot (left or right). All of the points in the upper left cluster represent measurements from individuals whose dominant foot is the right foot, whereas all of the points in the lower right cluster represent measurements from individuals whose dominant foot is the left foot.

Part (b):

There is a positive (linear) relationship between swelling in the dominant foot and swelling in the nondominant foot. Swelling in the dominant foot tends to be greater than swelling in the nondominant foot.

Part (c):

Component 1: States a correct pair of hypotheses in words or symbols.

$$H_0 : \mu_d = 0 \qquad \text{OR} \qquad H_0 : \mu_D = \mu_N \qquad \text{OR} \qquad H_0 : \mu_D - \mu_N = 0$$
$$H_a : \mu_d \neq 0 \qquad \qquad \qquad H_a : \mu_D \neq \mu_N \qquad \qquad H_a : \mu_D - \mu_N \neq 0$$

where

μ_d = mean difference in swelling (*dominant – nondominant*)
μ_D = mean swelling for dominant foot
μ_N = mean swelling for nondominant foot

Question 6 (continued)

Component 2: Identifies a correct test (by name or by formula) and checks the conditions.

$$\text{Paired } t\text{-test} \quad OR \quad t = \frac{\overline{x}_d - 0}{s_d / \sqrt{n}}$$

Conditions

1. We are told that this is a random sample from the population of adult female patients with Morton's neuroma (MN).

2. A boxplot of the differences (*dominant – nondominant*) has no outliers or extreme skewness. Thus, it is reasonable to believe that the population of differences is approximately normal.
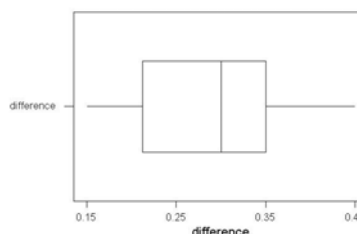
OR,

A normal probability plot of the differences has no obvious departures from a linear trend, so it is reasonable to believe that the differences come from a normal distribution.
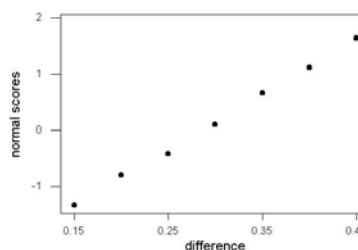
Component 3: Performs correct mechanics, including the value of the test statistic and *p*-value (or identifies a rejection region).

$$t = \frac{\overline{x}_d - 0}{\frac{s_d}{\sqrt{n}}} = \frac{0.2875 - 0}{\frac{0.0932}{\sqrt{12}}} = 10.68 \text{ with } df = 11, \text{ so the } p\text{-value is approximately } 0$$

OR,

From the calculator, $t = 10.68166837$, $df = 11$, p-value $= 0.00000038105102$

Component 4: Draws an appropriate conclusion in context, with linkage to the *p*-value (or rejection region).

Because the *p*-value is small (or is less than an α stated by the student), reject H_0. There is convincing evidence that the mean swelling is different for dominant and nondominant feet for women with MN.

Question 6 (continued)

Part (d):

Three acceptable suggestions are given below.

Separate the 24 swelling measurements into two groups—the 17 feet with MN and the 7 feet without MN. Construct a plot that displays the two groups, such as stacked dotplots or a back-to-back stemplot. The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for Morton's neuroma.
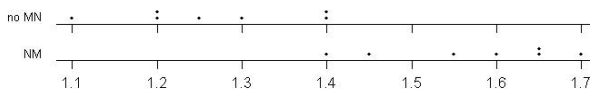
Dotplot of Swelling
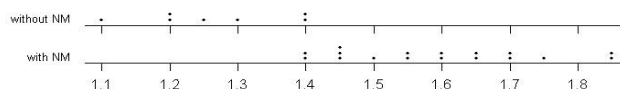
OR

Plot swelling measurements for only the seven individuals who do not have MN in both feet, plotting the measurements for their MN feet and their non-MN feet. The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for MN.

Dotplot of Swelling

OR

Plot the 12 measurements of MN in the dominant foot, the 5 measurements of MN in the nondominant foot, and the 7 measurements of no MN in the nondominant foot. (There are no individuals in the sample who do not have MN in the dominant foot.) The plot below suggests that a swelling measurement of about 1.4 or higher would be a reasonable criterion for MN.

Dotplot for NM Dominant-No NM Nondom
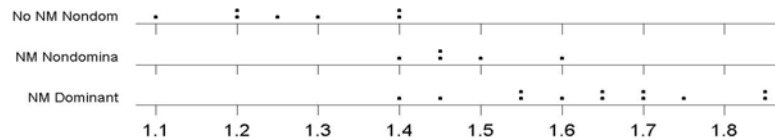
Scoring

Each of the four parts is scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response identifies the two groups as people with a left-dominant foot and people with a right-dominant foot. Students do not need to indicate which group is which.

Partially correct (P) if the response either identifies the two groups as people with greater swelling in the left foot and people with greater swelling in the right foot, missing the connection to foot dominance *OR* states only that "the dominant foot has more swelling than the nondominant foot" (making the connection back to the table) but without identifying the two groups.

Incorrect (I) if none of the three characterizations of the groups is present.

Part (b) is scored as follows:

Essentially correct (E) if the response contains both components below:

1. States that there is a positive (linear is optional) relationship *OR* states that as swelling in the dominant foot goes up, swelling in the nondominant foot tends to go up.
2. States that swelling in the dominant foot is greater than in the nondominant foot.

Note: One of these components may appear in part (a).

Partially correct (P) if only one of the two components is mentioned.

Incorrect (I) if neither component is mentioned.

Part (c) is scored as follows:

Essentially correct (E) if all four components of the significance test are correct.

Partially correct (P) if only two or three components of the significance test are correct.

Incorrect (I) if at most one component of the significance test is correct.

Part (d) is scored as follows:

Essentially correct (E) if the response recommends a reasonable criterion for deciding whether a foot has MN based on the given data and justifies the criterion graphically. There are other possibilities for graphs than the three outlined above, but the plot should make it easy to compare the measurements in feet with and without MN. Communication must be strong.

Partially correct (P) if a reasonable recommendation based on the data is made but the graphical display is missing, made incorrectly, or is difficult to interpret *OR* if a graphical display comparing measurements for feet with MN to those without MN is provided but a recommendation based on the display is weak or missing.

Incorrect (I) if neither a reasonable conclusion nor a graph is present.

Essentially correct responses count as 1 point, and partially correct responses count as ½ point.

4	**Complete Response**
3	**Substantial Response**
2	**Developing Response**
1	**Minimal Response**

If a response is between two scores (for example, 2½ points) use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication. Pay special attention to communication and to whether the correct test was performed in part (c), as well as to whether points were lost only because of omitted rather than incorrect elements, such as providing just one statement in part (b) and stating but not checking the condition of normality in part (c).

(a) A scatterplot of the ordered pairs (swelling in left foot, swelling in right foot), is shown below.
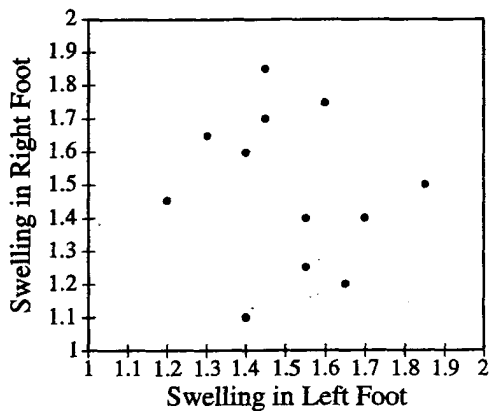
The scatterplot suggests there are two distinct groups of patients. Patients within each group share a common trait. Use the scatterplot above and the table on page 16 to determine the common trait and explain how this trait differs for the two groups.

The two groups are ~~those~~ those with larger swelling in the right foot then the left, and those with larger swelling in the left foot than the right. ~~In the first group, all subjects have a dominant right~~ The common trait in each group is their dominant foot. The first group described ~~has subjects with~~ a dominant right foot, while the second group has subjects with a dominant left ~~foot~~ foot.

(b) A scatterplot of the ordered pairs (swelling in dominant foot, swelling in nondominant foot), is shown below.
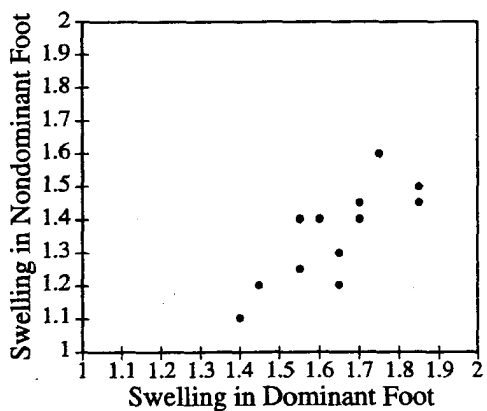
What conclusion can be drawn from this scatterplot that is not apparent from the scatterplot in part (a) ?

There is a~~n~~ (relatively strong) positive linear association between swelling in dominant foot and swelling in the nondominant foot. Generally, the larger the swelling in dominant foot, the larger the swelling in the non dominant foot. Also, swelling in the dominant foot is generally larger than the swelling in the non-dominant foot.

GO ON TO THE NEXT PAGE.

(c) Can you conclude that there is a difference between the mean swelling in the dominant foot and the mean swelling in the nondominant foot for adult females who have Morton's neuroma in at least one foot? Give a statistical justification to support your answer.

(For easy reference, the table of data from page 16 also appears at the bottom of page 19.)

$H_0: \mu_d = 0$

$H_a: \mu_d \neq 0$, where μ_d is the mean difference in swelling in the dominant foot and the swelling in the nondominant foot. (Dominant - nondominant)

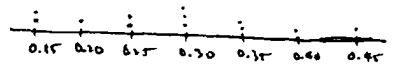We will perform a matched-pairs t-test on the mean difference.

$$t = \frac{\bar{x}_d - \mu_d}{s/\sqrt{n}}$$

Conditions

1. Data are SRS from population of interest. Yes. The question states that a random sample was taken.

2. Population distribution of differences is normal.
   Data of differences (swelling in dominant foot - swelling in nondominant foot)

$$\{0.3, 0.3, 0.45, 0.15, 0.3, 0.35, 0.25, 0.35, 0.2, 0.25, 0.4, 0.15\}$$

The dotplot shows a relatively symmetric distribution and appears normal. We can proceed.

$\bar{x}_d = 0.2875$

$s = 0.0932$

$n = 12$

$df = n - 1$

$\quad = 11$

$$t = \frac{\bar{x}_d - \mu_d}{s/\sqrt{n}}$$

$$= \frac{0.2875 - 0}{0.0932/\sqrt{11}}$$

$$= 10.682$$

P-value $= 2P(T \geq 10.682)$

$\quad = 3.811 \times 10^{-7}$

$\quad \approx 0 < 0.05$

Yes. Since the P-value $= 3.811 \times 10^{-7} < 0.05$, the results is significant at the $\alpha = 0.05$ significance level. We reject H_0, and conclude that there is significant evidence that there is a difference between the mean swelling in the dominant foot and the mean swelling in the nondominant foot.

GO ON TO THE NEXT PAGE.

-18-

(d) The nerve swelling measurement is used to indicate whether a foot has Morton's neuroma. Use the 24 measurements of nerve swelling to suggest a criterion for diagnosing Morton's neuroma. Justify your suggestion graphically.

(For easy reference, the table of data from page 16 also appears below.)

From the table, it appears that a swelling of ~~at least~~ of more than ~~1.30 or~~ 1.40.
~~suggest~~ the criterion for diagnosing Morton's

~~Stemplot of swellings~~
Dotplot of swellings

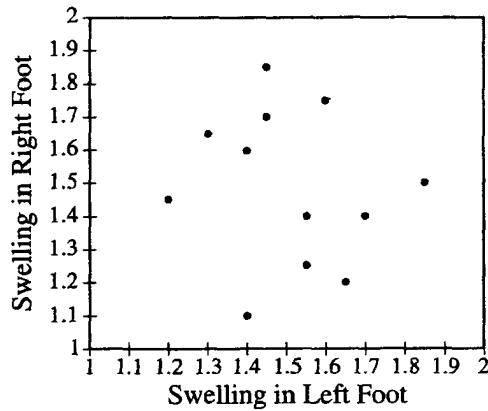range of swellings that were deemed as having Morton's neuroma.
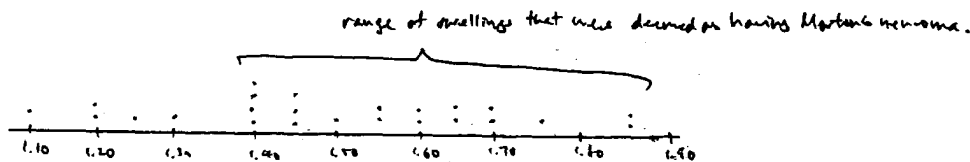
Swellings from 1.10-1.30 were on a foot were not classified as having neuroma, while those with swellings 1.40 and above were classified as having neuroma. Hence the criterion is somewhat larger than 1.30, for diagnosing Morton's neuroma possibly 1.40.

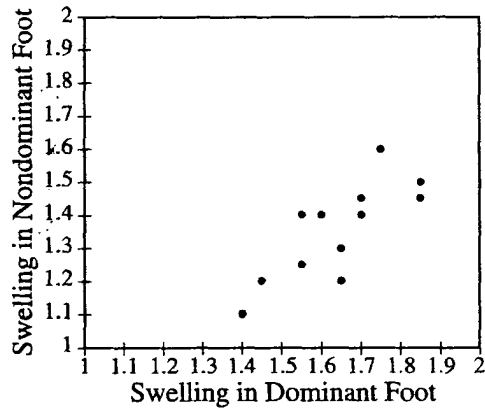| Dominant Foot	Swelling in Dominant Foot	Swelling in Nondominant Foot	Foot with Neuroma
Left	1.40	1.10	Left
Left	1.55	1.25	Left
Left	1.65	1.20	Left
Left	1.55	1.40	Both
Left	1.70	1.40	Left
Left	1.85	1.50	Both
Right	1.45	1.20	Right
Right	1.65	1.30	Right
Right	1.60	1.40	Right
Right	1.70	1.45	Both
Right	1.85	1.45	Both
Right	1.75	1.60	Both

GO ON TO THE NEXT PAGE.

(a) A scatterplot of the ordered pairs (swelling in left foot, swelling in right foot), is shown below.

The scatterplot suggests there are two distinct groups of patients. Patients within each group share a common trait. Use the scatterplot above and the table on page 16 to determine the common trait and explain how this trait differs for the two groups.

For the upper cluster, all individuals' dominant feet were their right foot while for the lower cluster, all their dominant feet were left foot. Hence their common trait is their dominant foot type.

(b) A scatterplot of the ordered pairs (swelling in dominant foot, swelling in nondominant foot), is shown below.

What conclusion can be drawn from this scatterplot that is not apparent from the scatterplot in part (a) ?

All patients There is a rather strong linear positive relationship between swelling in the dominant foot and swelling in the non-dominant foot.

There is more swelling in dominant foot than swelling in nondominant foot. The higher the swelling in the dominant foot, the higher the swelling in the nondominant foot, in general.

GO ON TO THE NEXT PAGE.

-17-

(c) Can you conclude that there is a difference between the mean swelling in the dominant foot and the mean swelling in the nondominant foot for adult females who have Morton's neuroma in at least one foot? Give a statistical justification to support your answer.

(For easy reference, the table of data from page 16 also appears at the bottom of page 19.)

We are interested in all adult females who have Morton's neuroma. Our parameter of interest is μ_1 = mean swelling in dominant foot for adult females with Morton's neuroma, and μ_2 = mean swelling in nondominant foot for adult females with Morton's neuroma. We are interested in the difference between the mean swellings.

$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$ $\alpha = 0.05$

We will use a two-sample t-test since we do not know s

We are interested in all adult females who have Morton's neuroma. Our parameter of interest is μ_{DIFF} = mean swelling in dominant foot − mean swelling in nondominant foot.

$H_0 : \mu_{DIFF} = 0$ $\alpha = 0.05$
$H_a : \mu_{DIFF} \neq 0$

We will use the one-sample t-test for this matched pairs design. We assume SRS, given that the sample was randomly selected. The plot shows the data distribution is approximately normal.

$\bar{X}_{DIFF} = 1.45$
$S_{DIFF} = 3.98457$
$df = 12-1 = 11$

$$t = \frac{\bar{X}-\mu}{S/\sqrt{n}}$$
$$= \frac{1.45-0}{3.98457/\sqrt{12}}$$
$$= 1.2606$$

$2 P(t > 1.2606) = 0.234 > 0.05 = \alpha$

Since the P-value of the test is more than $0.05 = \alpha$, we have insufficient evidence to reject that there is no difference in the mean swelling of the dominant & nondominant foot.

GO ON TO THE NEXT PAGE.

(d) The nerve swelling measurement is used to indicate whether a foot has Morton's neuroma. Use the 24 measurements of nerve swelling to suggest a criterion for diagnosing Morton's neuroma. Justify your suggestion graphically.
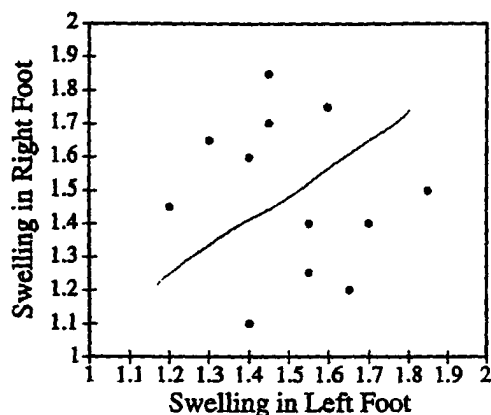
(For easy reference, the table of data from page 16 also appears below.)

Values of 1.45 and above for the swelling in foot should be a criterion for diagnosing Morton's neuroma.

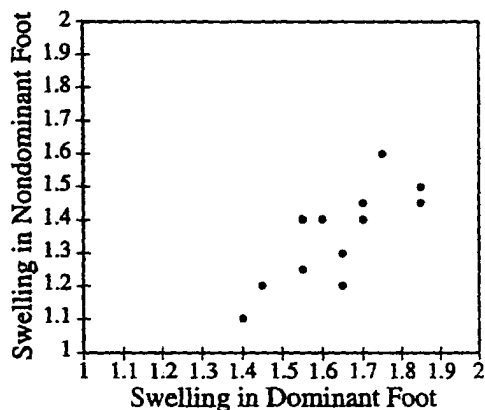| Dominant Foot	Swelling in Dominant Foot	Swelling in Nondominant Foot	Foot with Neuroma
Left	1.40	1.10	Left
Left	1.55	1.25	Left
Left	1.65	1.20	Left
Left	1.55	1.40	Both
Left	1.70	1.40	Left
Left	1.85	1.50	Both
Right	1.45	1.20	Right
Right	1.65	1.30	Right
Right	1.60	1.40	Right
Right	1.70	1.45	Both
Right	1.85	1.45	Both
Right	1.75	1.60	Both

GO ON TO THE NEXT PAGE.

-19-

(a) A scatterplot of the ordered pairs (swelling in left foot, swelling in right foot), is shown below.

The scatterplot suggests there are two distinct groups of patients. Patients within each group share a common trait. Use the scatterplot above and the table on page 16 to determine the common trait and explain how this trait differs for the two groups.

One group is the group that have neuroma in their left foot and who also have left foot as dominant. The other group is those who have neuroma in their right foot and have right foot as dominant.

(b) A scatterplot of the ordered pairs (swelling in dominant foot, swelling in nondominant foot), is shown below.

What conclusion can be drawn from this scatterplot that is not apparent from the scatterplot in part (a)?

Those who have more significant swelling in their dominant foot also have less swelling in their nondominant foot.

GO ON TO THE NEXT PAGE.

-17-

(c) Can you conclude that there is a difference between the mean swelling in the dominant foot and the mean swelling in the nondominant foot for adult females who have Morton's neuroma in at least one foot? Give a statistical justification to support your answer.

(For easy reference, the table of data from page 16 also appears at the bottom of page 19.)

Difference

$1.4 - 1.1 \rightarrow .3$

$1.55 - 1.25 \rightarrow .3$

$1.65 - \text{~~1.1~~} 1.2 \rightarrow .45$

$1.55 - 1.4 \rightarrow .15$

$1.7 - 1.4 \rightarrow .3$

$1.8 - 1.5 \rightarrow .3$

$1.45 - 1.2 \rightarrow .25$

$1.65 - 1.3 \rightarrow .35$

$1.6 - 1.4 \rightarrow .2$

$1.7 - 1.45 \rightarrow .25$

$1.85 - 1.45 \rightarrow .4$

$1.75 - 1.6 \rightarrow .15$

Paired T-Test (Matched Pairs)

$\mu > 0$

$H_0 = \mu_d = 0$ (no difference btw dom/nondom foot)

$H_A = \mu_d > 0$ (difference btw dom and nondom)

Assumptions

σ unknown ✓
large sample ✓
SRS ✓

$t = 10.7517$

$P = 1.78335 \times 10^{-7}$

$\bar{x} = .2833$

$S\bar{x} = .09$

$n = 12$

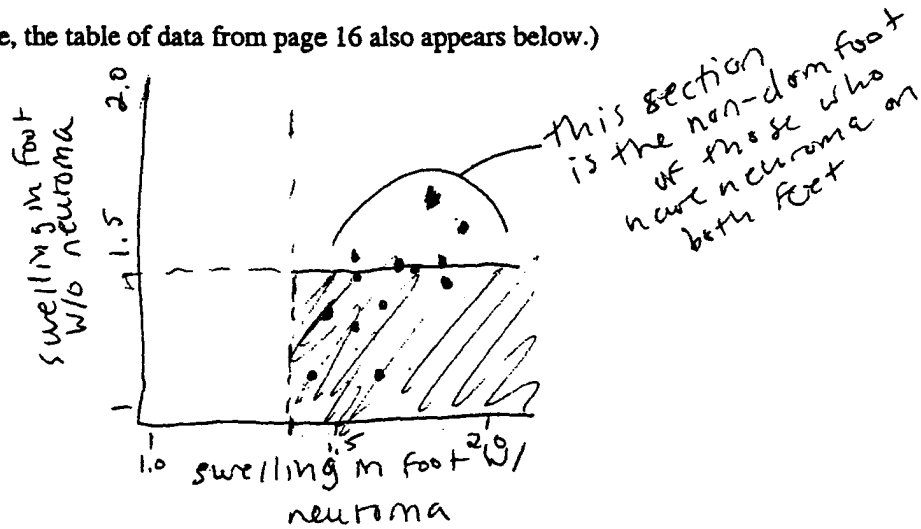P = prob. of getting such results when, in fact the null hypothesis is truly false

There is sufficient evidence to claim that there is a difference in mean swelling in dominant and nondominant feet at the .05 sig. level

GO ON TO THE NEXT PAGE.

-18-

(d) The nerve swelling measurement is used to indicate whether a foot has Morton's neuroma. Use the 24 measurements of nerve swelling to suggest a criterion for diagnosing Morton's neuroma. Justify your suggestion graphically.

(For easy reference, the table of data from page 16 also appears below.)

this section is the non-dom foot of those who have neuroma on both feet

swelling in foot w/o neuroma

1.0   swelling in foot w/ neuroma   2.0

In general, 1.4 is the marker that shows that feet with that much swelling have neuroma on that foot. This is shown in the graph above where all of the feet that have neuroma are at or greater than 1.4 (swelling)

Dominant Foot	Swelling in Dominant Foot	Swelling in Nondominant Foot	Foot with Neuroma
Left	1.40	1.10	Left
Left	1.55	1.25	Left
Left	1.65	1.20	Left
Left	1.55	1.40	Both
Left	1.70	1.40	Left
Left	1.85	1.50	Both
Right	1.45	1.20	Right
Right	1.65	1.30	Right
Right	1.60	1.40	Right
Right	1.70	1.45	Both
Right	1.85	1.45	Both
Right	1.75	1.60	Both

GO ON TO THE NEXT PAGE.

-19-

Question 6

Sample: 6A
Score: 4

In parts (a) and (b) the response includes the three conclusions that should be drawn: in the upper scatterplot, one group consists of people with a dominant right foot, and the other group consists of people with a dominant left foot; there is a positive (linear) relationship "between swelling in the dominant foot and swelling in the nondominant foot"; and swelling in the dominant foot is larger than swelling in the nondominant foot. Part (c) is a clear and correct matched pairs t-test, including a statement of conditions and evidence of whether they are met or not. (However, the value of n in the formula for t should be 12 rather than 11.) Part (d) is the weakest part of this response because the measurements of feet without Morton's neuroma are not clearly marked. There is no indication that two feet without Morton's neuroma have swelling measurements of 1.40. However, the entire answer was considered a complete response, based on all four parts.

Sample: 6B
Score: 3

In parts (a) and (b) the response includes the three conclusions that should be drawn. Both parts were scored as essentially correct. The test of significance in part (c) would be excellent except that there is no evidence that the plot of the differences used to check normality actually was constructed, and incorrect values were computed for the mean and standard deviation. Part (c) was scored as partially correct. However, part (d) is weak because the justification is not graphical as required. A reasonable criterion is given, based on circling the measurements in the table for the feet with Morton's neuroma. Part (d) was scored as partially correct. Overall, this answer was assessed as a substantial response.

Sample: 6C
Score: 2

In part (a) the two groups are correctly identified, but they should be defined in terms of having more swelling in the foot rather than that they "have neuroma." Part (a) was scored as essentially correct. The statement in part (b) does not include the idea that the relationship is positive, and the observation is missing that the dominant foot *always* has more swelling than the nondominant foot. Part (b) was scored as incorrect. The correct test is used in part (c), except that it should not be one-sided. There is an attempt to include all components of a t-test, but the conditions listed are not all correct, and the explanation of a p-value also is incorrect. The computations that take up most of the page in part (c) did not need to be shown. Part (c) was scored as partially correct. The criterion for diagnosing Morton's neuroma in part (d) is reasonable, but the graph is difficult to understand. The labels on the axes are incorrect because they indicate that, for all patients, one foot has neuroma and one foot does not. Part (d) was scored as partially correct. On the whole, this answer was judged a developing response.