Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) interpret the meaning of a confidence level; (2) use a confidence interval to test the plausibility of a claim about the value of a population parameter; (3) perform a sample size calculation related to a confidence interval.

Solution

Part (a):

The 95 percent confidence level means that if one were to repeatedly take random samples of the same size from the population and construct a 95 percent confidence interval from each sample, then in the long run 95 percent of those intervals would succeed in capturing the actual value of the population proportion of households in the county that own at least one dog.

Part (b):

No. The 95 percent confidence interval 0.417 ± 0.119 is the interval $(0.298, 0.536)$. This interval includes the value 0.39 as a plausible value for the population proportion of households in the county that own at least one dog. Therefore, the confidence interval does not provide evidence that the proportion of dog owners in this county is different from the claimed national proportion.

Part (c):

The sample proportion is 0.417, and the margin of error is 0.119. Determining the sample size requires solving the equation $0.119 = 1.96 \times \sqrt{\dfrac{0.417 \times (1 - 0.417)}{n}}$ for n.

Thus, $n = \dfrac{1.96^2 \times 0.417 \times (1 - 0.417)}{0.119^2} \approx 65.95,$ so the humane society must have selected 66 households for its sample.

Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the student provides a correct interpretation of the confidence level in the context of the study. A correct interpretation can take one of two approaches:

1. Based on the concept of repeated sampling, the response must fulfill the following three requirements:
    *   Mentions repeated sampling or "in the long run" or "using this method"
    *   Mentions that 95 percent of the intervals will capture the population proportion
    *   Includes the context of the study

2. Based on probability, the response must state that there is a 0.95 probability that a random sample *selected in the future* will produce an interval that captures the actual value of the population proportion of households in the county that have at least one dog.

Partially correct (P) if the student provides an interpretation of the confidence level that includes two of the three components required for the repeated sampling interpretation *OR* provides a correct probability interpretation, but not in context.

Incorrect (I) if the student attempts to interpret a particular confidence *interval* rather than the confidence *level* (for example, by saying that we are 95 percent confident that an interval that has been obtained includes the population proportion of households in the county that have at least one dog) *OR* provides an interpretation of the confidence level that mentions at most one of the three components required for the repeated sampling interpretation.

Part (b) is scored as follows:

Essentially correct (E) if the student correctly states that because 0.39 (or "the claimed value") is in the computed interval, the interval does not provide evidence that the proportion of dog owners in the county is different from the claimed national proportion.

Partially correct (P) if the student indicates that the goal is to check whether the claimed value of 0.39 is in the computed interval but makes errors in implementation. Examples of errors include the following:
- The student notes that 0.39 is within the interval but does not draw a correct conclusion.
- The student makes an arithmetic error in computing the endpoints of the interval, but the conclusion is consistent with the computed interval.

  OR

The student correctly notes that 0.39 is in the interval and then concludes that 0.39 *is* the population proportion for the county.

Incorrect (I) if the student does not recognize how to check whether the claim is consistent with the confidence interval.

Part (c) is scored as follows:

Essentially correct (E) if the student provides a correct equation with correct numerical values substituted, as well as a correct integer solution.

Partially correct (P) if the student provides a correct equation (and substitutions) but makes one or more of the following errors:
- Does not complete the calculation or completes the calculation incorrectly
- Uses 0.5 or 0.39 rather than 0.417 as the sample proportion
- Uses an incorrect but plausible z^* value
- Reports the answer as a non-integer value
- Gives the calculated value of n as a lower bound for the required sample size

Incorrect (I) otherwise.

Notes
- It is acceptable to use $z^* = 2$ instead of 1.96.
- It is acceptable for the response to round up or down to get an integer answer.

4   **Complete Response**

All three parts essentially correct

3   **Substantial Response**

Two parts essentially correct and one part partially correct

2   **Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1   **Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

(a) Interpret the 95 percent confidence level in this context.

95% confidence means that if all possible confidence intervals of a given size were constructed, from this population 95% of them would capture the true population proportion of households in the county that own at least one dog.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was 0.417 ± 0.119.

(b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

$.417 \pm .119 = (.298, .536)$

The interval does not provide evidence that this county is different, because .39 is contained in its interval, indicating that that is a possible value of the true population proportion.

(c) How many households were selected in the humane society's sample? Show how you obtained your answer.

$M = 0.119 \qquad \hat{p} = .417$

$= z^* \sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

$.119 = 1.96 \sqrt{\dfrac{(.417)(.563)}{n}}$

$\dfrac{.119}{1.96} = \sqrt{\dfrac{.24311}{n}}$

$\dfrac{.119^2}{1.96^2} = \dfrac{.24311}{n}$

$.119^2 \cdot n = .24311 \cdot 1.96^2$

$n = \dfrac{(1.96^2)(.24311)}{(.119)^2}$

$= 65.9$

$\boxed{66 \text{ people}}$

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3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

(a) Interpret the 95 percent confidence level in this context.

If this test were repeated maaaaaany times, 95% of the resulting intervals would capture the true proportion of households in its county that own at least one dog.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was 0.417 ± 0.119.

(b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

No, because .39 is captured in the interval that the humane society achieved.

(c) How many households were selected in the humane society's sample? Show how you obtained your answer.

$$\text{Margin of Error} = Z^* \sqrt{\frac{\rho(1-\rho)}{n}}$$

$$.119 = 1.96 \sqrt{\frac{.5(1-.5)}{n}}$$

$$\left(\frac{.119}{1.96}\right)^2 = \sqrt{\frac{.5(.5)}{n}}^2$$

$$.0036886 = \frac{.5(.5)}{n}$$

$$n \cdot .0036886 = .25$$

$$n = 67.824$$

⟨68 households⟩

3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.

(a) Interpret the 95 percent confidence level in this context.

The real proportion of households in the county that own at least one dog will be in the confidence interval 95% of the time.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was 0.417 ± 0.119.

(b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.

$H_0 : p = .39$ $.417 + .119 = .536$ $(.298, .536)$

$H_a : p \neq .39$ $.417 - .119 = .298$

Because the interval contains .390, we fail to reject the null hypothesis. There is not sufficient evidence to suggest that the proportion of dog owners in its county is different from the claimed national proportion.

(c) How many households were selected in the humane society's sample? Show how you obtained your answer.

$.119 = $ margin of error

$.119 = $ critical value $\cdot$ SD.

$.119 = (1.959)\sqrt{\dfrac{p(1-p)}{n}}$

$.119 = 1.959\sqrt{\dfrac{.39(.41)}{n}}$

$.0746^2 = \sqrt{\dfrac{.2379}{n}}^2$

$.005577 = \dfrac{.2379}{n}$ $n = \dfrac{.2379}{.00557} = $

42.739 households

≈ 43 households

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Overview

The primary goals of this question were to assess students' ability to (1) interpret the meaning of a confidence level; (2) use a confidence interval to test the plausibility of a claim about the value of a population parameter; (3) perform a sample size calculation related to a confidence interval.

Sample: 3A
Score: 4

In part (a) the response provides a very nice interpretation of the confidence level, with the minor omission of the word *approximately* before "95%." Context is provided as well, so part (a) was scored as essentially correct. In part (b) the student answers the question and provides justification. This response provides a fine example of an essentially correct response to part (b). In part (c) the student provides the correct equation and substitutes the correct values. The calculations are shown in detail, and the final answer is rounded to an integer value, as required. There is a minor error in that the response says "66 people" instead of 66 households, but otherwise this is an excellent example of an essentially correct response to part (c). With all three parts essentially correct, the response earned a score of 4.

Sample: 3B
Score: 3

In part (a) the student provides a reasonable interpretation of the confidence level, including the concept of repeated sampling. The student's use of "test" in the response is interpreted to mean *procedure*. It would have been ideal if the student had added the word *approximately* before "95% of the resulting intervals," but the omission was considered to be minor in this situation. The response includes context as well, so part (a) was scored as essentially correct. In part (b) the response answers the question and provides the minimal justification required to be scored as essentially correct. In part (c) the student gives a correct equation but incorrectly substitutes 0.5 instead of 0.417 for $\hat{p}$. The value 0.5 would be correct if the question had asked for the minimum sample size required to obtain a margin of error of 0.119 in a future study. But in this case 0.119 is the obtained margin of error, and the point of the question is to work backward to find out what sample size must have been used to obtain it. Therefore, part (c) was scored as partially correct. With two parts essentially correct and one part partially correct, the response earned a score of 3.

Sample: 3C
Score: 2

In part (a) the response notes that "[t]he real proportion of households … will be in the confidence interval 95% of the time" but does not specify what is meant by "95% of the time." There is no mention that the intervals conceptually come from repeated sampling, so part (a) was scored as partially correct. In part (b) the student illustrates what is being tested by writing hypotheses and then uses the confidence interval to make the correct conclusion. This is a nice illustration of an essentially correct response for part (b). In part (c) the student provides the correct equation but uses the claimed value of 0.39 instead of the observed sample proportion of 0.417. In addition, there is an arithmetic error made in completing the calculation. Making either one of those errors would result in the same outcome as making both of them, which is why part (c) was scored as partially correct. With one part essentially correct and two parts partially correct, the response earned a score of 2.