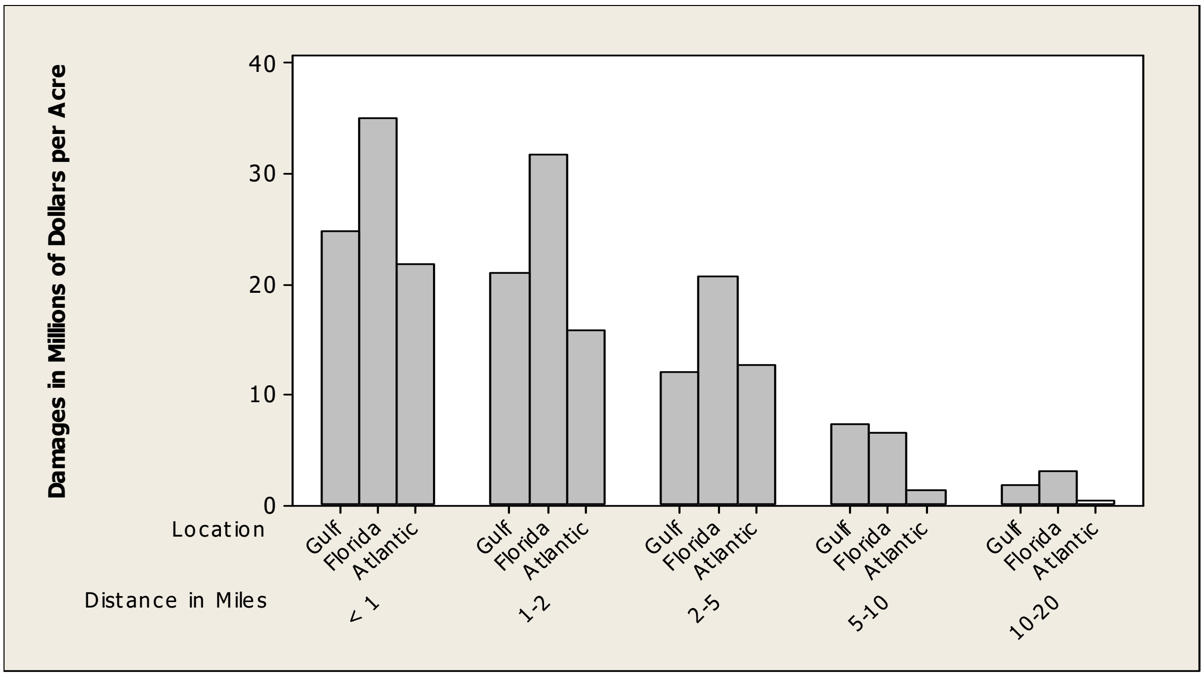
Question 6

Intent of Question

The primary goals of this investigative task were to assess students' ability to (1) produce and comment on a graphical display; (2) calculate a test statistic based on rank data; (3) use simulation results to draw an appropriate conclusion.

Solution

Part (a):



Part (b):

In all three regions (Gulf Coast, Florida, Lower Atlantic) the hurricane damage amounts tend to decrease as distance from the coast increases. For almost all given distances from the coast, the Florida region has the largest damage amounts. Also, for any given distance, the Gulf Coast and Lower Atlantic regions have similar damage amounts but with the Lower Atlantic damage amounts generally smaller.

Part (c):

For the "10 to 20 miles" distance category: The Florida region has the most damage (3.0 million dollars per acre) and so has rank 1. The region with the second-most damage is the Gulf Coast (1.7 million dollars), obtaining rank 2. The Lower Atlantic region has the least damage (0.3 million dollars) and so has rank 3. The last columns of the table should be filled in as follows:

Question 6 (continued)

	10 to 20 miles	Average Rank
Gulf Coast	2	2.0
Florida	1	1.2
Lower Atlantic	3	2.8

The average ranks are computed $\frac{2+2+3+1+2}{5}=2.0$ for the five Gulf Coast damage ranks, $\frac{1+1+1+2+1}{5}=1.2$ for the five Florida damage ranks and $\frac{3+3+2+3+3}{5}=2.8$ for the five Lower Atlantic damage ranks.

Part (d):

The calculated value of the test statistic *Q* is

$$Q=5\left[(2.0-2)^2+(1.2-2)^2+(2.8-2)^2\right]=5[0+0.64+0.64]=6.4.$$

Part (e):

A *Q* value of 6.4 or larger occurred in $\frac{39}{1,000}=0.039$ (or 3.9 percent) of the 1,000 repetitions. All 1,000 repetitions of the simulation assumed there was no difference in the distribution of damage amounts among the three regions. This is a fairly small (approximate) *p*-value (less than 0.05), indicating that a test statistic as large or larger than the observed test statistic of $Q=6.4$ would be fairly unlikely to occur by chance alone if there really was no difference among the regions for each distance category. The sample data therefore provide reasonably strong evidence that there is a difference in the distributions of hurricane damage amounts among these three regions.

<u>Scoring</u>

This question is scored in four sections. Section 1 consists of part (a); section 2 consists of part (b); section 3 consists of parts (c) and (d); section 4 consists of part (e). Each of the four sections is scored as essentially correct (E), partially correct (P) or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes a well-labeled, statistically valid graph that allows for comparing damage amounts both across regions *AND* at varying distances from the coast.

Partially correct (P) if the response includes a well-labeled, statistically valid graph that allows for comparing damage amounts across regions or at varying distances from the coast but not both *OR* if it includes a statistically valid graph that allows for both comparisons but lacks labels for either the regions or the distances (or both).

Incorrect (I) if the graph does not allow for either comparison *OR* if it is poorly labeled and allows for only one comparison.

Question 6 (continued)

Section 2 is scored as follows:

Essentially correct (E) if the response includes *BOTH* of the following:

- Mention of a difference — a contrast among the three regions across distance categories (e.g., "Florida has greater damage than the other two regions at most distances")
- Mention of a similarity — a statement that in all regions hurricane damage decreases with greater distance from the coast

Partially correct (P) if the response includes a valid statement about only a difference or about only a similarity.

Incorrect (I) if the response has neither a valid statement about the difference nor a valid statement about the similarity.

Note: A statement that "Florida has more damage" is not sufficient to describe a difference if "at most distances" is not specified. Phrases that imply comparisons across distances such as "generally" or "for the most part" are acceptable for the "at most distances" specification.

Section 3 is scored as follows:

Essentially correct (E) if the response includes the correct ranks in part (c) *AND* the correct test statistic calculation in part (d).

Partially correct (P) if the response includes either of the following:

- The correct ranks in part (c) but not the correct test statistic calculation in part (d)
- Incorrect ranks in part (c) but the test statistic calculated correctly in part (d), using those incorrect ranks

Incorrect (I) otherwise.

Notes

- Calculation of the average ranks need not be shown to receive credit for correctly calculating Q.
- A miscalculation of Q results in no credit for part (d), but the severity of the miscalculation may be considered later in holistic scoring.

Section 4 is scored as follows:

Essentially correct (E) if the response includes *BOTH* of the following components:

- A valid quantification of the tail probability of the test statistic Q based on the simulated Q values
- A conclusion, in context, consistent with the rarity (or likelihood) of the calculated test statistic Q relative to the simulated Q values

Partially correct (P) if the response includes only one of these two components.

Incorrect (I) if the response does not include either of these two components.

Question 6 (continued)

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

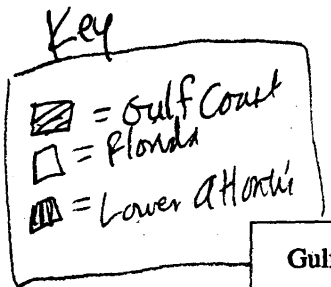
If a response is between two scores (for example, 2½ points), use a holistic approach to determine whether to score up or down, depending on the overall strength of the response and communication.

STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

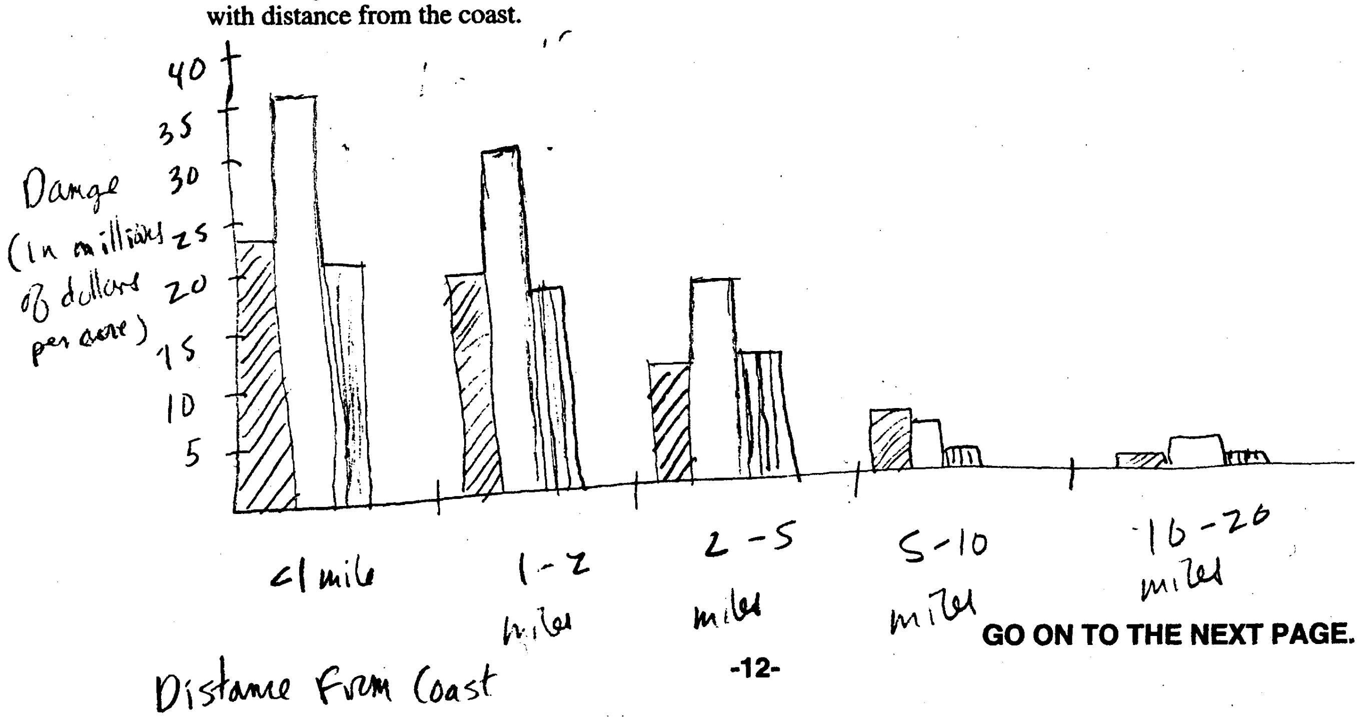
6. Hurricane damage amounts, in millions of dollars per acre, were estimated from insurance records for major hurricanes for the past three decades. A stratified random sample of five locations (based on categories of distance from the coast) was selected from each of three coastal regions in the southeastern United States. The three regions were Gulf Coast (Alabama, Louisiana, Mississippi), Florida, and Lower Atlantic (Georgia, South Carolina, North Carolina). Damage amounts in millions of dollars per acre, adjusted for inflation, are shown in the table below.



HURRICANE DAMAGE AMOUNTS IN MILLIONS OF DOLLARS PER ACRE

	Distance from Coast				
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles
Gulf Coast	24.7	21.0	12.0	7.3	1.7
Florida	35.1	31.7	20.7	6.4	3.0
Lower Atlantic	21.8	15.7	12.6	1.2	0.3

(a) Sketch a graphical display that compares the hurricane damage amounts per acre for the three different coastal regions (Gulf Coast, Florida, and Lower Atlantic) and that also shows how the damage amounts vary with distance from the coast.



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6A2

(b) Describe differences and similarities in the hurricane damage amounts among the three regions.

Florida had the most hurricane damage in 4 of the 5 strata. (Gulf coast had more damage 5-10 miles from the coast).

a similarity for all three regions, as expected, the damage decreased as we went farther from the coast, and it was greatest for all 3 regions when we went less than one mile from the coast.

Because the distributions of hurricane damage amounts are often skewed, statisticians frequently use rank values to analyze such data.

(c) In the table below, the hurricane damage amounts have been replaced by the ranks 1, 2, or 3. For each of the distance categories, the highest damage amount is assigned a rank of 1 and the lowest damage amount is assigned a rank of 3. Determine the missing ranks for the 10-to-20-miles distance category and calculate the average rank for each of the three regions. Place the values in the table below.

ASSIGNED RANKS WITHIN DISTANCE CATEGORIES

	Distance from Coast					
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles	Average Rank
Gulf Coast	2	2	3	1	2	2
Florida	1	1	1	2	1	1.2
Lower Atlantic	3	3	2	3	3	2.8

GO ON TO THE NEXT PAGE.

6 A3

(d) Consider testing the following hypotheses.

H_0: There is no difference in the distributions of hurricane damage amounts among the three regions.

H_a: There is a difference in the distributions of hurricane damage amounts among the three regions.

If there is no difference in the distribution of hurricane damage amounts among the three regions (Gulf Coast, Florida, and Lower Atlantic), the expected value of the average rank for each of the three regions is 2. Therefore, the following test statistic can be used to evaluate the hypotheses above:

$$Q = 5\left[\left(\bar{R}_G - 2\right)^2 + \left(\bar{R}_F - 2\right)^2 + \left(\bar{R}_A - 2\right)^2\right]$$

where $\bar{R}_G$ is the average rank over the five distance categories for the Gulf Coast (and $\bar{R}_F$ and $\bar{R}_A$ are similarly defined for the Florida and Lower Atlantic coastal regions).

Calculate the value of the test statistic Q using the average ranks you obtained in part (c).

$5[(2-2)^2 + (1.2-2)^2 + (2.8-2)^2]$

$5[0 + .64 + .64]$

$Q = 6.4$

(e) One thousand simulated values of this test statistic, Q, were calculated, assuming no difference in the distributions of hurricane damage amounts among the three coastal regions. The results are shown in the table below. These data are also shown in the frequency plot where the heights of the lines represent the frequency of occurrence of simulated values of Q.

Frequency Table for Simulated Values of Q

Q	Frequency	Cumulative Frequency	Percent	Cumulative Percent
0.0	48	48	4.80	4.80
0.4	262	310	26.20	31.00
1.2	168	478	16.80	47.80
1.6	153	631	15.30	63.10
2.8	186	817	18.60	81.70
3.6	59	876	5.90	87.60
4.8	33	909	3.30	90.90
5.2	52	961	5.20	96.10
6.4	16	977	1.60	97.70
7.6	15	992	1.50	99.20
8.4	6	998	0.60	99.80
10.0	2	1000	0.20	100.00

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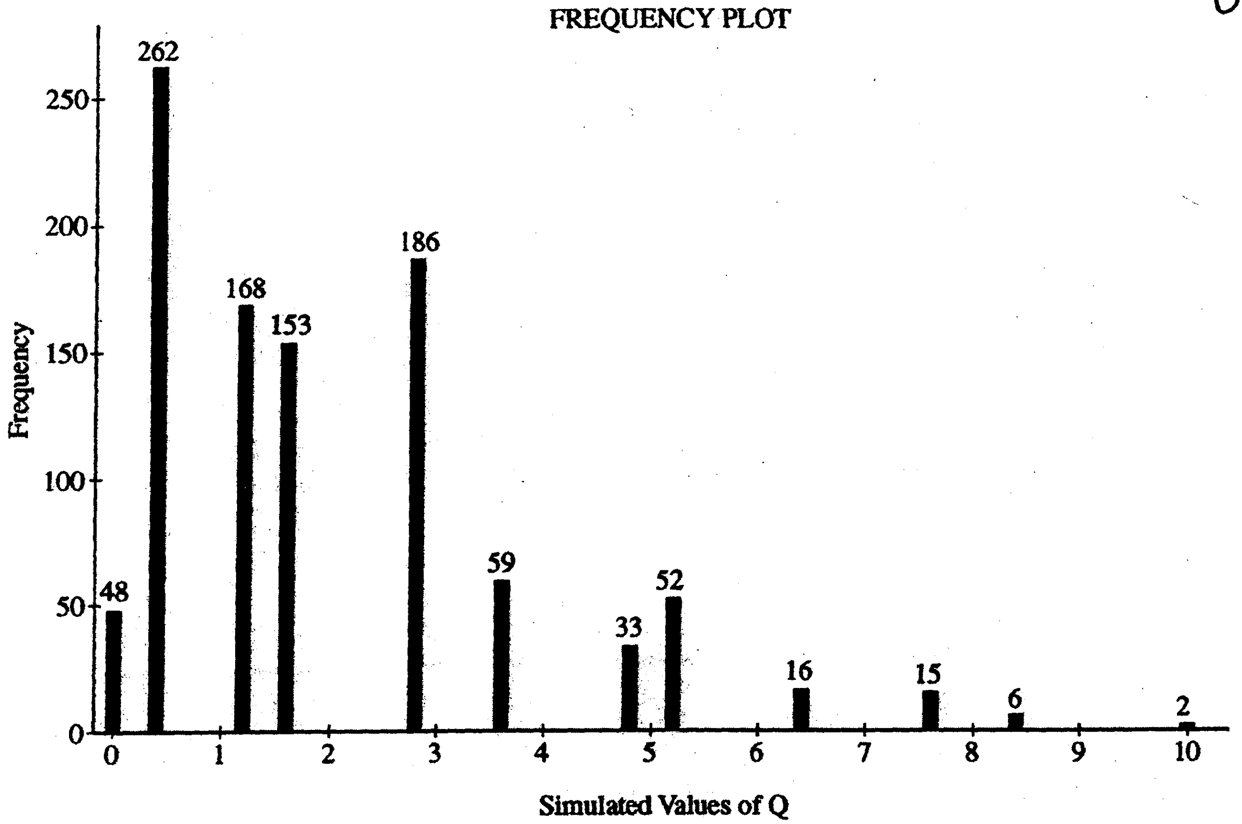
6A4



Use these simulated values and the test statistic you calculated in part (d) to determine if the observed data provide evidence of a significant difference in the distributions of hurricane damage amounts among the three coastal regions. Explain.

The Q statistic for the observed data was 6.4.
a Q-value of 6.4 or greater only occured 39 times out of the 1000 simulations.

Our P-value would be $\frac{39}{1000} = .039$.

at the significance level of $\alpha = .05$, we reject the null hypothesis with a P-value of .039. Based on this data, there is evidence to suggest that there is a difference in the distribution of hurricane damage in these 3 regions.

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STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Hurricane damage amounts, in millions of dollars per acre, were estimated from insurance records for major hurricanes for the past three decades. A stratified random sample of five locations (based on categories of distance from the coast) was selected from each of three coastal regions in the southeastern United States. The three regions were Gulf Coast (Alabama, Louisiana, Mississippi), Florida, and Lower Atlantic (Georgia, South Carolina, North Carolina). Damage amounts in millions of dollars per acre, adjusted for inflation, are shown in the table below.

HURRICANE DAMAGE AMOUNTS IN MILLIONS OF DOLLARS PER ACRE

	Distance from Coast				
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles
Gulf Coast	24.7	21.0	12.0	7.3	1.7
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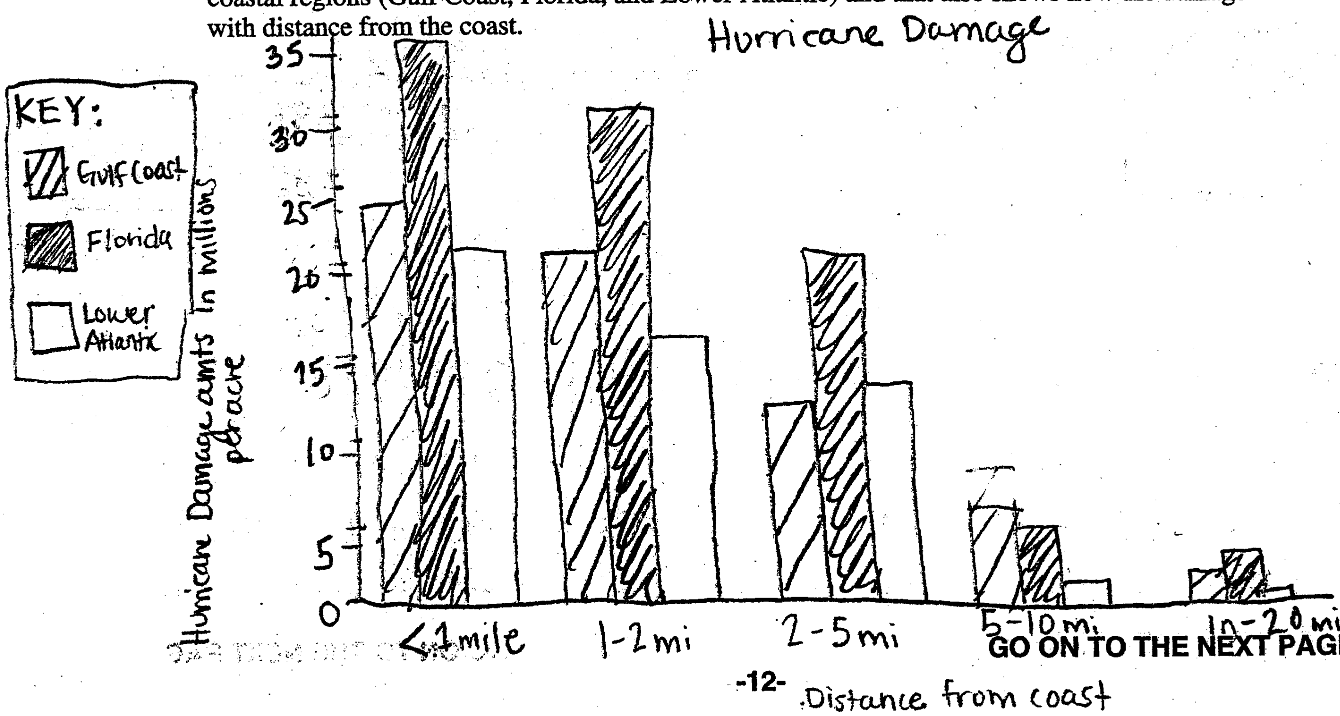
Lower Atlantic	21.8	15.7	12.6	1.2	0.3

(a) Sketch a graphical display that compares the hurricane damage amounts per acre for the three different coastal regions (Gulf Coast, Florida, and Lower Atlantic) and that also shows how the damage amounts vary with distance from the coast.



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6 B2

(b) Describe differences and similarities in the hurricane damage amounts among the three regions.

All three coastal regions' damage amounts are right skewed, which means that the closer any of these regions' areas were to the coast, the more damage in millions occurred. Also, the Florida had significantly more damage close to the coast line 5 miles or less. However, the Gulf Coast had the most damage amounts 5-10 miles from the coast. Overall, the lower atlantic had the least hurricane damage amounts among the 3 areas.

Because the distributions of hurricane damage amounts are often skewed, statisticians frequently use rank values to analyze such data.

(c) In the table below, the hurricane damage amounts have been replaced by the ranks 1, 2, or 3. For each of the distance categories, the highest damage amount is assigned a rank of 1 and the lowest damage amount is assigned a rank of 3. Determine the missing ranks for the 10-to-20-miles distance category and calculate the average rank for each of the three regions. Place the values in the table below.

ASSIGNED RANKS WITHIN DISTANCE CATEGORIES

	Distance from Coast					
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles	Average Rank
Gulf Coast	2	2	3	1	2	2
Florida	1	1	1	2	1	1.2
Lower Atlantic	3	3	2	3	3	2.8

Since my graph showed the lower atlantic had the least damage overall, I gave it a 3 for 10-20 miles. I gave Florida a 1 because it had significantly the most damage while I gave the Gulf Coast a 2 because it had a fair amount of damage.

$\frac{3+3+2+3+3}{5} = 2.8$ Lower Atlanti

$\frac{2+2+3+1+2}{5} = 2$ Gulf Coast

$\frac{1+1+1+2+1}{5} = 1.2$ Florida

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6 B3

(d) Consider testing the following hypotheses.

H_0: There is no difference in the distributions of hurricane damage amounts among the three regions.

H_a: There is a difference in the distributions of hurricane damage amounts among the three regions.

If there is no difference in the distribution of hurricane damage amounts among the three regions (Gulf Coast, Florida, and Lower Atlantic), the expected value of the average rank for each of the three regions is 2. Therefore, the following test statistic can be used to evaluate the hypotheses above:

$$Q = 5\left[\left(\bar{R}_G - 2\right)^2 + \left(\bar{R}_F - 2\right)^2 + \left(\bar{R}_A - 2\right)^2\right]$$

where $\bar{R}_G$ is the average rank over the five distance categories for the Gulf Coast (and $\bar{R}_F$ and $\bar{R}_A$ are similarly defined for the Florida and Lower Atlantic coastal regions).

Calculate the value of the test statistic Q using the average ranks you obtained in part (c).

$\bar{R}_G = 2$ $\quad Q = 5[(2-2)^2 + (1.2-2)^2 + (2.8-2)^2]$

$\bar{R}_F = 1.2$ $\quad Q = 5(0 + .64 + .64)$

$\bar{R}_A = 2.8$ $\quad \boxed{Q = 6.4}$

(e) One thousand simulated values of this test statistic, Q, were calculated, assuming no difference in the distributions of hurricane damage amounts among the three coastal regions. The results are shown in the table below. These data are also shown in the frequency plot where the heights of the lines represent the frequency of occurrence of simulated values of Q.

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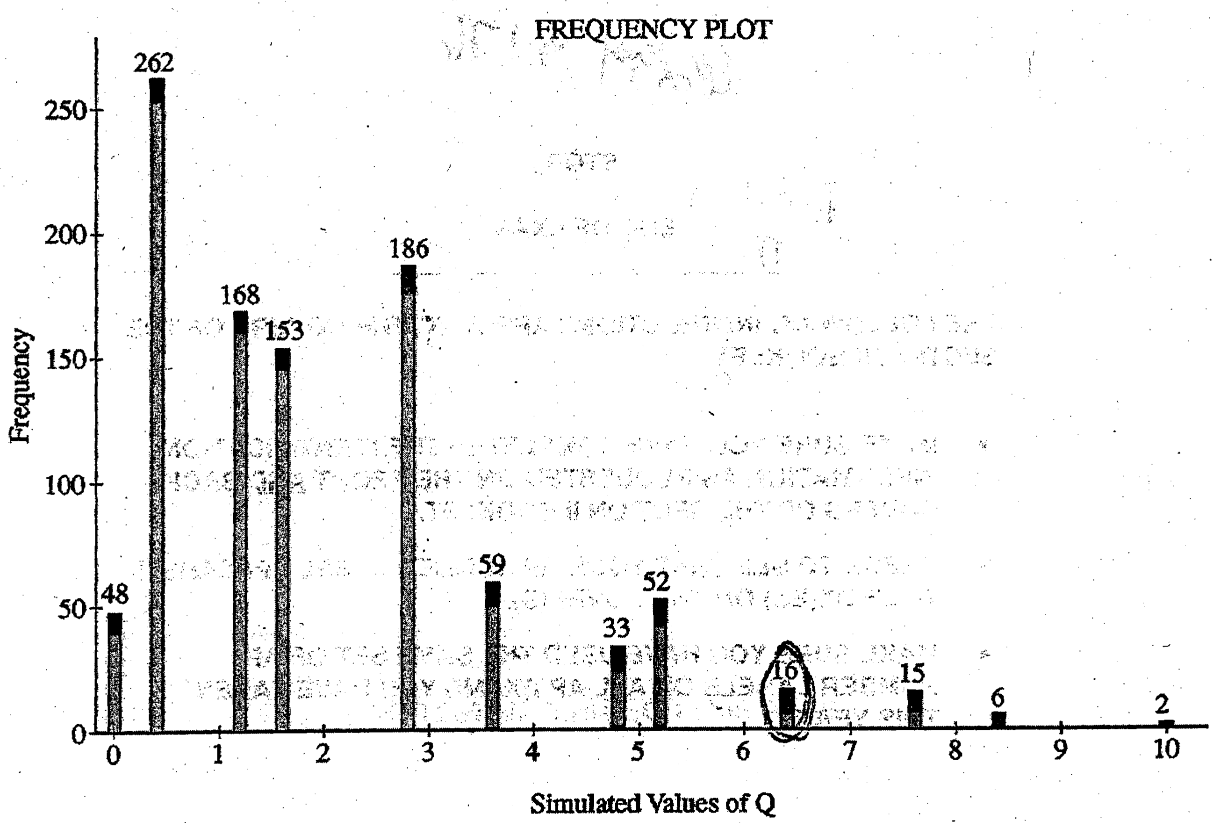

Use these simulated values and the test statistic you calculated in part (d) to determine if the observed data provide evidence of a significant difference in the distributions of hurricane damage amounts among the three coastal regions. Explain.

Q = 6.4 according to my calculations.

On the graph and chart, it shows that this number only occured 16 times in 1000 simulations.

$\frac{16}{1000} = .016$ So the probability of getting 6.4 as the test statistic Q is .016, or extremely rare by chance alone.

Thus, using the H_0 and H_a provided and my obtained data, I would reject H_0 at the $\alpha = .05$ significance level because $.016 < \alpha$. Thus, there is significant evidence that there is a difference in the distributions of hurricane damage amounts among the three regions

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6C1

STATISTICS
SECTION II
Part B
Question 6
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6. Hurricane damage amounts, in millions of dollars per acre, were estimated from insurance records for major hurricanes for the past three decades. A stratified random sample of five locations (based on categories of distance from the coast) was selected from each of three coastal regions in the southeastern United States. The three regions were Gulf Coast (Alabama, Louisiana, Mississippi), Florida, and Lower Atlantic (Georgia, South Carolina, North Carolina). Damage amounts in millions of dollars per acre, adjusted for inflation, are shown in the table below.

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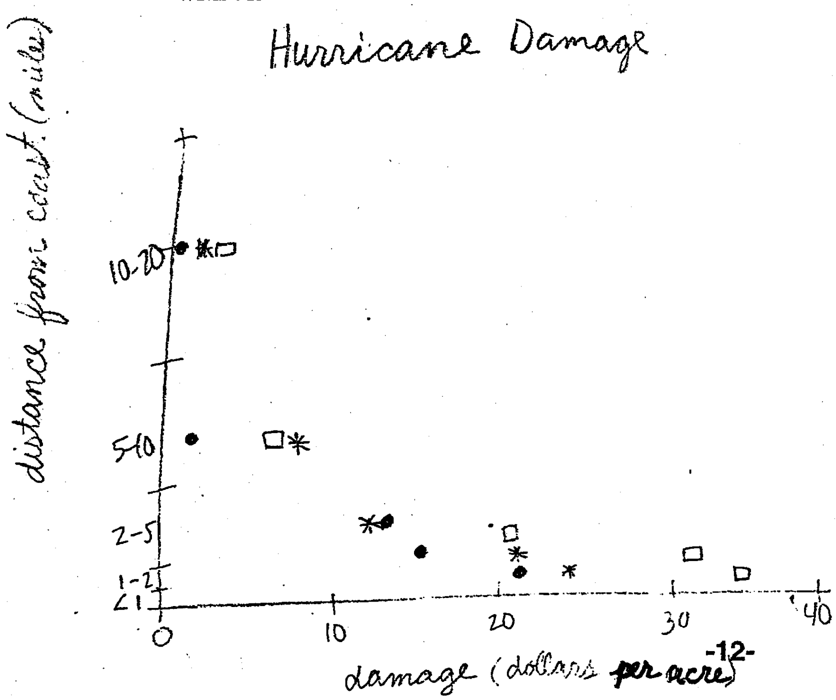

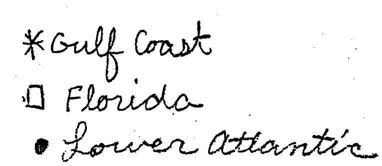

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6 C2

(b) Describe differences and similarities in the hurricane damage amounts among the three regions.

The graphs of all three regions appear to have similar non linear shapes.

The center of damage in Florida is the highest, and the Lower Atlantic the lowes.

Florida appears to have the greatest spread and the Lower Atlantic and Gulf Coast appear to have similar spreads.

There do not appear to be any outliers.

Because the distributions of hurricane damage amounts are often skewed, statisticians frequently use rank values to analyze such data.

(c) In the table below, the hurricane damage amounts have been replaced by the ranks 1, 2, or 3. For each of the distance categories, the highest damage amount is assigned a rank of 1 and the lowest damage amount is assigned a rank of 3. Determine the missing ranks for the 10-to-20-miles distance category and calculate the average rank for each of the three regions. Place the values in the table below.

ASSIGNED RANKS WITHIN DISTANCE CATEGORIES

	Distance from Coast					
	< 1 mile	1 to 2 miles	2 to 5 miles	5 to 10 miles	10 to 20 miles	Average Rank
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GO ON TO THE NEXT PAGE.

(d) Consider testing the following hypotheses.

H_0: There is no difference in the distributions of hurricane damage amounts among the three regions.

H_a: There is a difference in the distributions of hurricane damage amounts among the three regions.

If there is no difference in the distribution of hurricane damage amounts among the three regions (Gulf Coast, Florida, and Lower Atlantic), the expected value of the average rank for each of the three regions is 2. Therefore, the following test statistic can be used to evaluate the hypotheses above:

$$Q = 5\left[\left(\bar{R}_G - 2\right)^2 + \left(\bar{R}_F - 2\right)^2 + \left(\bar{R}_A - 2\right)^2\right]$$

where $\bar{R}_G$ is the average rank over the five distance categories for the Gulf Coast (and $\bar{R}_F$ and $\bar{R}_A$ are similarly defined for the Florida and Lower Atlantic coastal regions).

Calculate the value of the test statistic Q using the average ranks you obtained in part (c).

$$Q = 5\left[(2-2)^2 + (1.2-2)^2 + (2.8-2)^2\right]$$

$$Q = 6.4$$

(e) One thousand simulated values of this test statistic, Q, were calculated, assuming no difference in the distributions of hurricane damage amounts among the three coastal regions. The results are shown in the table below. These data are also shown in the frequency plot where the heights of the lines represent the frequency of occurrence of simulated values of Q.

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7.6	15	992	1.50	99.20
8.4	6	998	0.60	99.80
10.0	2	1000	0.20	100.00

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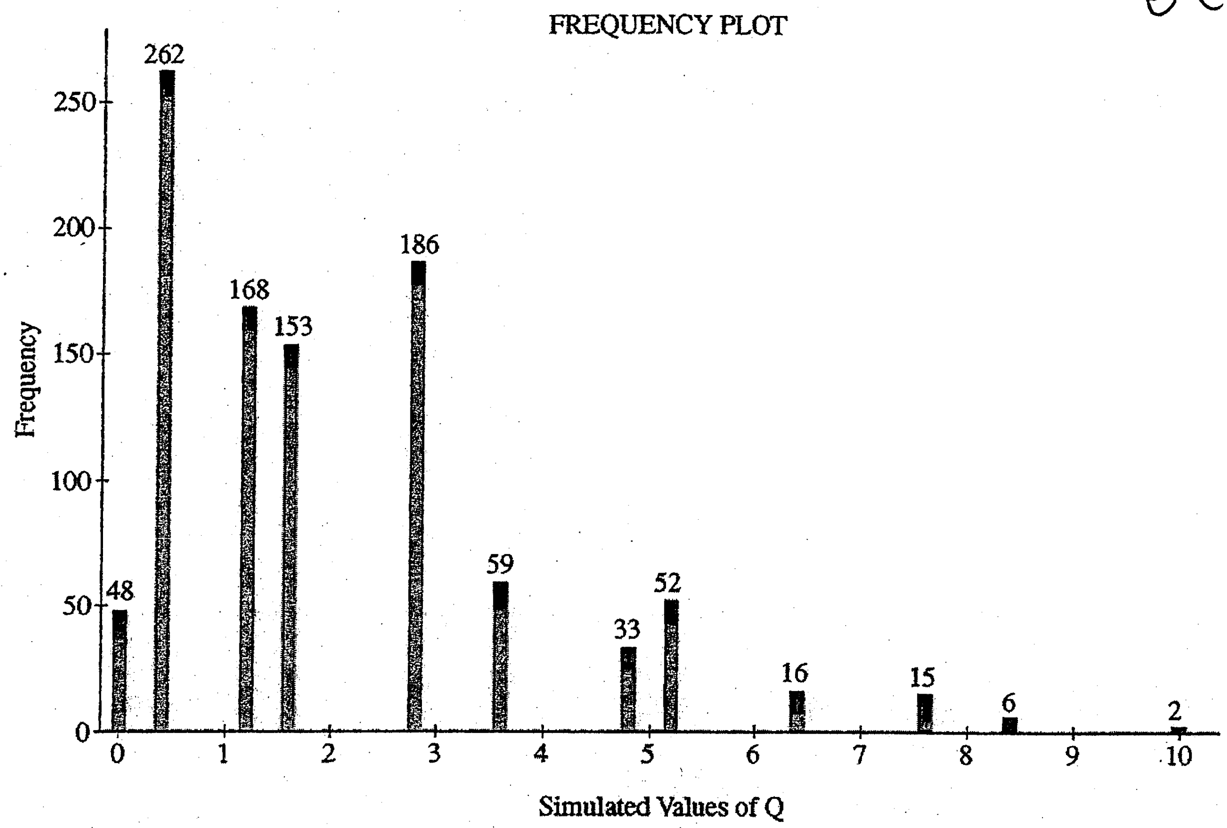
6C4



Use these simulated values and the test statistic you calculated in part (d) to determine if the observed data provide evidence of a significant difference in the distributions of hurricane damage amounts among the three coastal regions. Explain.

This does not provide ~~significant~~ evidence that there is a significant difference in the distributions of hurricane damage amounts among the three regions. Although most of the Q test statistic values appear clustered between zero and three, 6.4 is not that uncommon, and there are many values farther away from the center that it is. This Q = 6.4 does not appear that uncommon to have occurred by random sampling variability with there being no difference among the regions.

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Question 6

Overview

The primary goals of this investigative task were to assess students' ability to (1) produce and comment on a graphical display; (2) calculate a test statistic based on rank data; (3) use simulation results to draw an appropriate conclusion.

Sample: 6A
Score: 4

In part (a) the student draws a well-labeled bar graph in which the bars are grouped by the distance categories, and a key is provided for ease in identifying the three regions. Section 1, consisting of part (a), was scored as essentially correct. In part (b) Florida's having "the most hurricane damage in 4 of the 5 strata" is correctly identified as a significant difference. Although it was not necessary to describe the categorical distances as strata (as provided in the stem of the question), this is a nice detail. Also, the fact that damage decreases as the distance from the coast increases is correctly identified as a major similarity among the regions. Section 2, consisting of part (b), was scored as essentially correct. In parts (c) and (d) the ranks, average ranks and test statistic, Q, are correctly calculated, so section 3, consisting of parts (c) and (d), was scored as essentially correct. In part (e) a simulated p-value of 0.039 is correctly identified for the test statistic value of $Q = 6.4$ and is used to provide an appropriate conclusion in context. Section 4, consisting of part (e), was scored as essentially correct. With all four sections essentially correct, the response earned a score of 4.

Sample: 6B
Score: 3

In part (a) the student draws a well-labeled bar graph, grouping the bars by distance, and a key is provided for ease in identifying the three regions. Section 1, consisting of part (a), was scored as essentially correct. In part (b), although the term "right skewed" is troublesome because of the categorical nature of the distances, the major similarity among the regions is correctly identified by indicating that damages increase closer to each of the coasts. The differences among the three regions are not completely specified for all five distance categories, so section 2, consisting of part (b), was scored as partially correct. In parts (c) and (d) the ranks, average ranks and test statistic, Q, were correctly calculated, and section 3, consisting of parts (c) and (d), was scored as essentially correct. In part (e) no p-value for the test statistic is explicitly identified, but instead 0.016, the simulated probability of getting $Q = 6.4$, is used to provide a conclusion in context. Section 4, consisting of part (e), was scored as partially correct. With two sections essentially correct and two sections partially correct, the response earned a score of 3.

Sample: 6C
Score: 2

In part (a) the graph is a well-labeled plot of the categorical distances on the vertical axis and damage amounts on the horizontal axis, and a key is provided for ease in identifying the three regions. Section 1, consisting of part (a), was scored as essentially correct. In part (b), although the phrase "similar non linear shapes" is correct for the graphs, the response does not convey the nature of relationship (decreasing damages as distance from the coast increases), and none of the statements provides a complete response for differences among the regions for all distances. Section 2, consisting of part (b), was scored as incorrect. In parts (c) and (d) the ranks, average ranks and test statistic, Q, were correctly calculated. Section 3, consisting of parts (c) and (d), was scored as essentially correct. In part (e) the simulated Q values are described, but an

Question 6 (continued)

approximate *p*-value is not identified, and the test statistic $Q = 6.4$ is determined to be “not that uncommon.” Hence, section 4, consisting of part (e), was scored as incorrect. With two sections essentially correct and two sections incorrect, the response earned a score of 2.