AP Statistics
2000 Student Samples

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STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. A random sample of 400 married couples was selected from a large population of married couples.

   - Heights of married men are approximately normally distributed with mean 70 inches and standard deviation 3 inches.
   - Heights of married women are approximately normally distributed with mean 65 inches and standard deviation 2.5 inches.
   - There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.

   (a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.

Propotion of married couple in the population for which the wife is taller than her husband is $20/400 = 0.05$
with standard deviation of $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{0.05 \cdot 0.95}{400}} = 0.0108$
($\hat{p}$ is close enough to p)
z^* for 95 % Confidence interval is 1.96.

$$CI = \bar{X} \pm z^* \cdot \sigma = 0.05 \pm 1.96 \cdot 0.010897$$
$$= (0.02864 ; 0.071358)$$

This means that 95 % of time we construct a CI, the true proportion falls in to the intervals. About 0.02864 ~ 0.071358 of married couples have taller wife.

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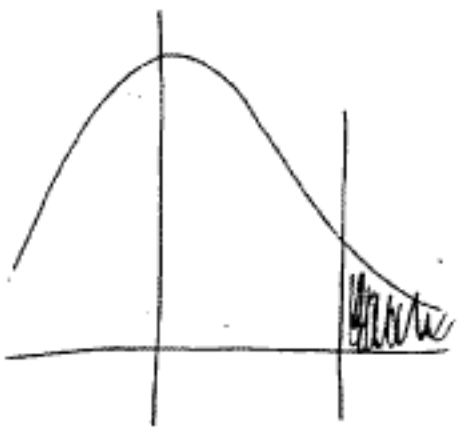
(b) Suppose that a married man is selected at random and a married woman is selected at random. Find the approximate probability that the woman will be taller than the man.

Let M be the selected married man's height and
W be the selected married woman's height.

S_M be the standard deviation for man's height and
S_W be the standard deviation for women's height

mean difference is $W - M = 65 - 70 = -5$

standard diviation of the difference is $\sqrt{S_M^2 + S_W^2} = 3.9051$



probability that the woman is taller than the man is shaded region of the curve on the left.

$$z = \frac{0 - \bar{x}}{\sigma} = \frac{0+5}{3.9051} = 1.2804$$

p-value for $z > 1.2804$ is **.1002**

-5 0

$\sigma = 3.9051$

(c) Based on your answers to (a) and (b), are the heights of wives and their husbands independent? Explain your reasoning.

No. If the heights were independent, the probability found in (b) must be in the CI found in (a). Since probability that the woman is taller than man in general is about .10 & the probability of taller wife is 0.05 and 95% CI for this probability does not contain .10, there is enough evidence to conclude that there is some association b/w married women & married man that they are <u>not</u> independent.

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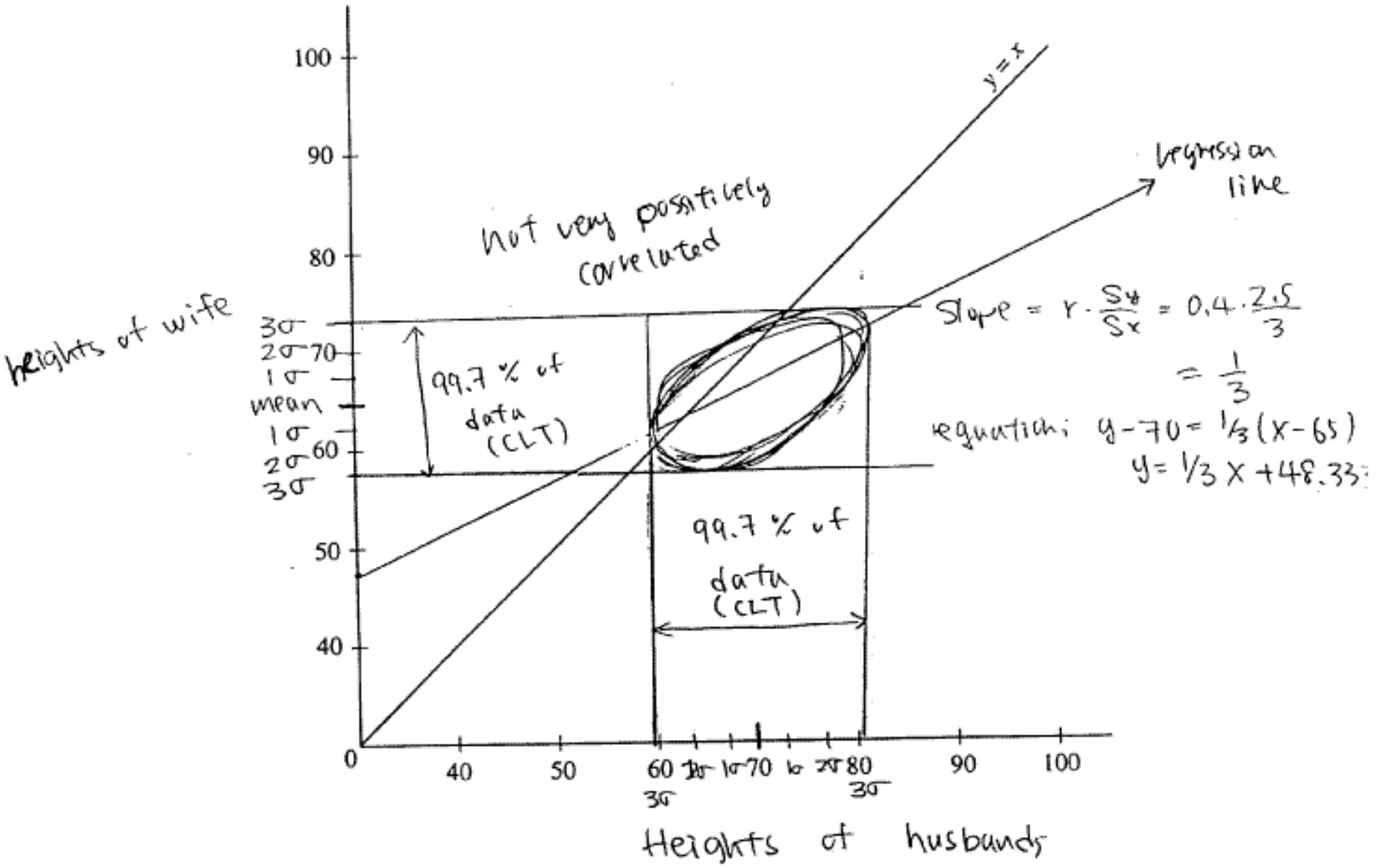
(d) A scatterplot (not shown) of husband's height versus wife's height for the 400 couples in the sample shows an approximately linear relationship with correlation 0.4. On the graph below, sketch an ellipse that could enclose the points on the scatterplot. Be sure to

- label your axes, and
- locate and orient your ellipse correctly with respect to the two axes and the line $y = x$.

Include any information that you think will be helpful in clarifying your sketch.



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   - There were 20 couples in which the wife was taller than her husband, and there were 380 couples in which the wife was shorter than her husband.

   (a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.

$\hat{p} = \frac{20}{400} = 0.05$

$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$0.05 \pm 1.96 \sqrt{\frac{0.05(0.95)}{400}}$

$0.0286 < p < 0.0714$

Assumptions that that the sample be less than 10% the population size and that $n\hat{p}$ and $n(1-\hat{p})$ are greater the 10 ($n\hat{p} = 20$, $n(1-\hat{p}) = 380$) are satisfied. The sample was random so z procedures can be used

In this question a confidence interval of 2.86% to 7.14% means that, at the 95% confidence level, the actual population proportion is in that range. Ninety-five percent of all samples of size 400 will have $\hat{p}$ values in that range.

(b) Suppose that a married man is selected at random and a married woman is selected at random. Find the approximate probability that the woman will be taller than the man.

men's height

woman's height (h)	P(h)	P(H<h)
57	0.0010	[illegible]
58	0.00331	0.0000632
59	0.00[illegible]4	0.000233
60	[illegible]	0.000781
61	[illegible]	0.00230
62	[illegible]	0.00621
63	0.1156	0.01519
64	[illegible]	[illegible]
65	[illegible]	0.06681
66	0.1465	0.1217
67	[illegible]	0.2023
68	[illegible]	0.3055
69	[illegible]	0.4338
70	[illegible]	[illegible]
71	[illegible]	[illegible]
72	[illegible]	0.7977
73	[illegible]	0.8782

$\sum P(h)P(H<h) = 0.1249$

I listed values for the woman's height (h) TO M ± 3σ then found the corresponding probability of that height P(h) & then found the probability of having a man's height (H) less than the same value for h. That column is P(H<h). I then multiplied P(h) and P(H<h) to find the probability of a woman and took the sum.

(c) Based on your answers to (a) and (b), are the heights of wives and their husbands independent? Explain your reasoning.

They are dependent. The probability that any random woman is taller than another random man is about 12.5% while it is only between 2.5% and 7.1% if they are married.

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(d) A scatterplot (not shown) of husband's height versus wife's height for the 400 couples in the sample shows an approximately linear relationship with correlation 0.4. On the graph below, sketch an ellipse that could enclose the points on the scatterplot. Be sure to

- label your axes, and
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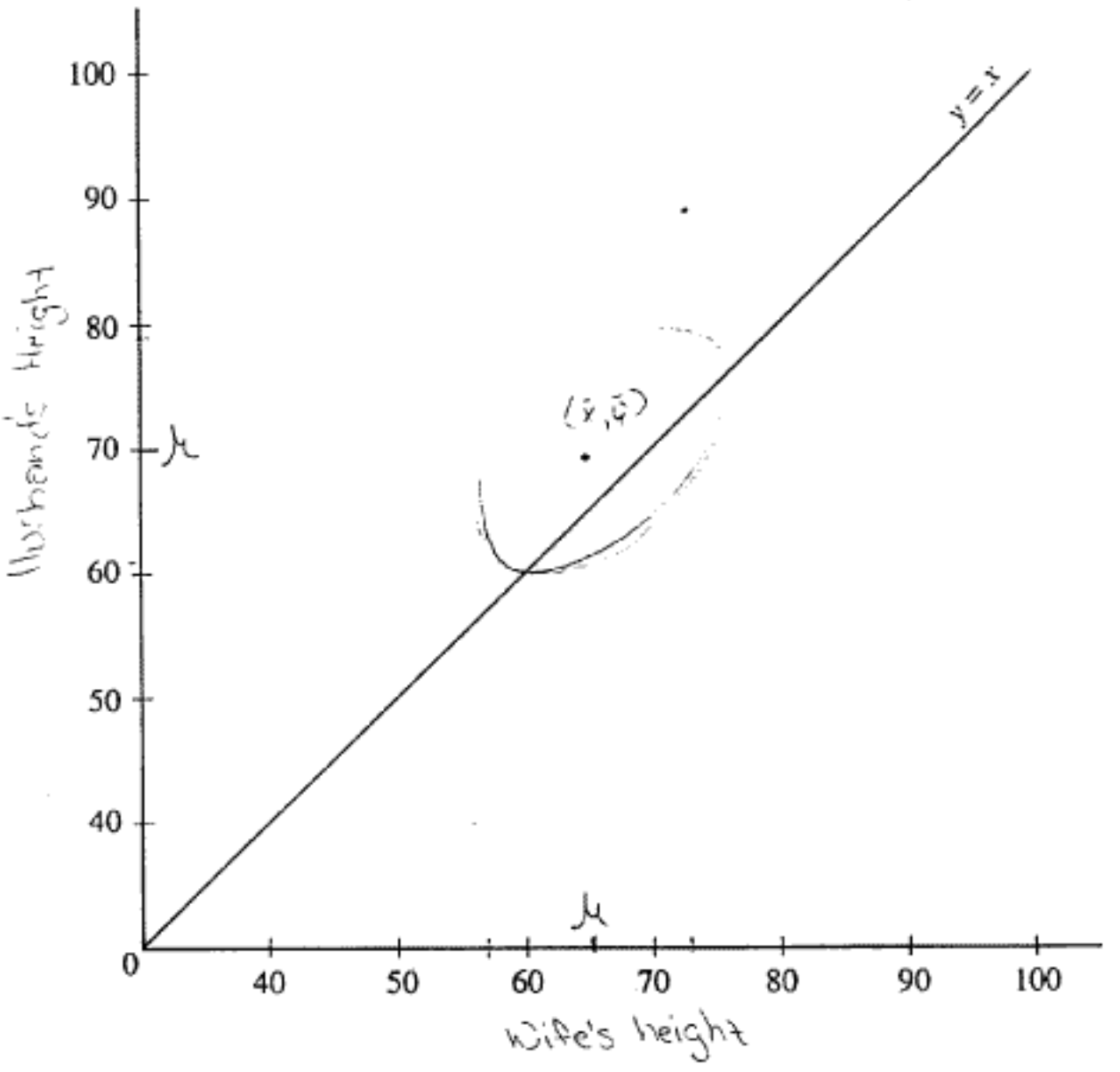

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(a) Find a 95 percent confidence interval for the proportion of married couples in the population for which the wife is taller than her husband. Interpret your interval in the context of this question.

p_1 = proportion of couples in which the wife is taller than the husband. $\hat{p}_1 = \frac{20}{400} = .05$, $n_1 = 400$

$\hat{p}_1 n_1 = (.05)(400) = 20 > 10$, $(1-\hat{p}_1) n_1 = (.95)(400) = 380 > 10$

$10n_1 = 4000$ - assume "a large population" means greater than 4000 (a pretty reasonable assumption, I think).

I_C = statistic $\pm z^* \times SE$, $SE = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}} = \sqrt{\frac{(.05)(.95)}{400}} = .011$

$z^* = 1.960$, $I_C = .05 \pm (1.960)(.0109)$

$= (.0286, .0714)$

We are 95% confident that the true proportion of couples with the wife taller than the husband is between .0286 and .0714.

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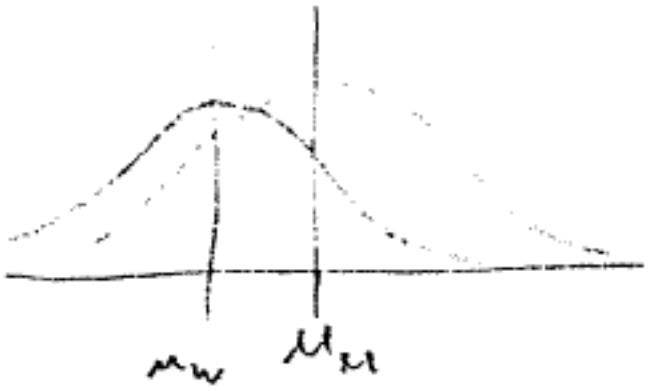
(b) Suppose that a married man is selected at random and a married woman is selected at random. Find the approximate probability that the woman will be taller than the man.

μ_M = true mean of men's heights

μ_W = true mean of women's heights



μ_{W-M} = true mean in difference of heights = $\mu_W - \mu_M$

= 65 - 70 = -5 inches

σ_{W-M} = true std. dev. in difference of heights

$= \sqrt{\sigma_W^2 + \sigma_M^2} = \sqrt{2.5^2 + 3^2} = 3.905$

$P(\bar{x}_W - \bar{x}_M > 0) = P\left(z > \frac{0-(-5)}{3.905}\right) = .1002$

(c) Based on your answers to (a) and (b), are the heights of wives and their husbands independent? Explain your reasoning.

No, they are not independent, because if they were, we would have a larger proportion of couples ~~in which~~ (close to 10%) in which the wife would be taller than the husband. As it is, the proportion is between (probably, anyway) .0286 and .0314, which suggests that the heights are not independent.

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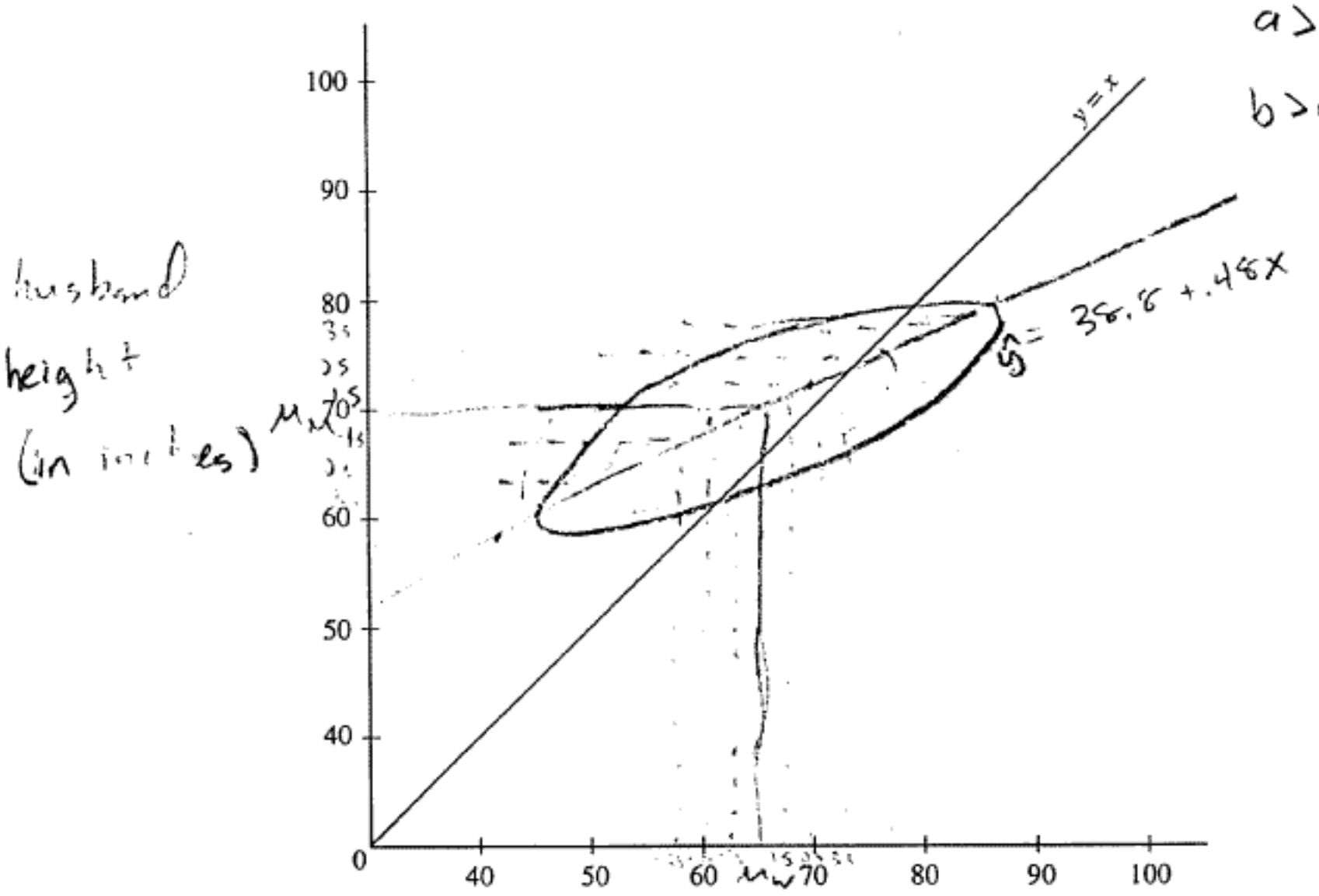
LSRL: $\hat{y} = a + bx$

$a > 0$

$b > r\left(s_M > s_W\right)$

$\left(b = r\frac{s_y}{s_x}\right.$



wife height (in inches)

slope: $b = r\frac{s_M}{s_W} = (.4)\left(\frac{3}{2.5}\right)$

$= .48$

$\bar{y} = a + b\bar{x}$ ⟶ END OF EXAMINATION $70 = a + (.48)(65)$, $a = 38.8$

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