AP® Statistics
2002 Sample Student Responses

5. Sleep researchers know that some people are early birds (E), preferring to go to bed by 10 P.M. and arise by 7 A.M., while others are night owls (N), preferring to go to bed after 11 P.M. and arise after 8 A.M. A study was done to compare dream recall for early birds and night owls. One hundred people of each of the two types were selected at random and asked to record their dreams for one week. Some of the results are presented below.

Group	Number of Dreams Recalled During the Week			Proportion Who Recalled	
	Mean	Median	Standard Deviation	No dreams	5 or more dreams
Early birds	7.26	6.0	6.94	0.24	0.55
Night owls	9.55	9.5	5.88	0.11	0.69

(a) The researchers believe that night owls may have better dream recall than do early birds. One parameter of interest to the researchers is the mean number of dreams recalled per week with μ_E representing this mean for early birds and μ_N representing this mean for night owls. The appropriate hypotheses would then be $H_0: \mu_E - \mu_N = 0$ and $H_a: \mu_E - \mu_N < 0$. State two other pairs of hypotheses that might be used to test the researchers' belief. Be sure to define the parameter of interest in each case.

$H_0: p_E - p_N = 0$ (where the proportion of early birds who remembered no dreams equals the proportion of night owls who remembered no dreams)

$H_a: p_E - p_N > 0$ (where the proportion of early birds who remembered no dreams is greater than the proportion of night owls who remembered no dreams)

$H_0: p_E - p_N = 0$ (where the proportion of early birds who remembered 5 or more dreams equals the proportion of night owls who remembered 5 or more dreams)

$H_a: p_E - p_N < 0$ (where the proportion of early birds who remembered 5 or more dreams is less than the proportion of night birds who remembered 5 or more dreams)

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(b) Use the data provided to carry out a test of the hypotheses about the mean number of dreams recalled per week given in the statement of part (a). Do the data support the researchers' belief?

A two-sample difference of means t-test is appropriate for this data. We can safely use this test because the two samples are independent, the samples are random samples, the population standard deviations are unknown, and both sample sizes are greater than 30.

$H_0: \mu_E - \mu_N = 0$ (where the mean number of dreams remembered by early birds equals the mean number of dreams remembered by night owls)

$H_a: \mu_E - \mu_N < 0$ (where the mean number of dreams remembered by early birds is less than the mean number of dreams remembered by night owls).

$$\bar{x}_E = 7.26 \qquad s_E = 6.94 \qquad \bar{x}_N = 9.55 \qquad s_N = 5.88 \qquad df = 192.799 \text{ (from Calculator)}$$
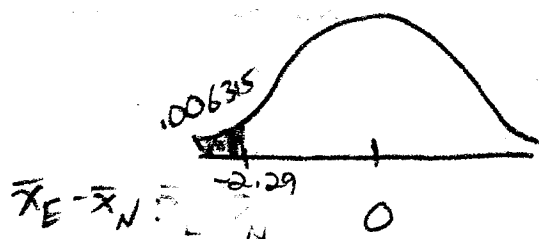
$$P\left(\bar{x}_E - \bar{x}_N < 7.26 - 9.55\right) = P\left(t < \frac{(\bar{x}_E - \bar{x}_N) - 0}{\sqrt{\frac{s_E^2}{n_E} + \frac{s_N^2}{n_N}}}\right)$$

$$= P\left(t < \frac{(7.26 - 9.55) - 0}{\sqrt{\frac{(6.94)^2}{100} + \frac{(5.88)^2}{100}}}\right) = P\left(t < -2.5176\right) = .006315$$

.006315

$\bar{x}_E - \bar{x}_N = -2.29$

If the null hypothesis is true, the probability of obtaining two sample means with a difference (early birds − night owls) of less than −2.29 is .006315. We have very strong evidence against the null hypothesis and in support of the researchers belief that night owls have better recall than early birds.

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5. Sleep researchers know that some people are early birds (E), preferring to go to bed by 10 P.M. and arise by 7 A.M., while others are night owls (N), preferring to go to bed after 11 P.M. and arise after 8 A.M. A study was done to compare dream recall for early birds and night owls. One hundred people of each of the two types were selected at random and asked to record their dreams for one week. Some of the results are presented below.

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$H_0: M_E - M_N = 0$

$H_A: M_E - M_N < 0$

This hypothesis compares the median number of dreams recalled during the week with M_E representing the median for early birds and M_N representing the median for night owls.

$H_0: P_E - P_N = 0$

$H_A: P_E - P_N < 0$

This hypothesis compares the proportion who recalled 5 or more dreams during the week with P_E representing the proportion of early birds and P_N being that of the night owls.

(b) Use the data provided to carry out a test of the hypotheses about the mean number of dreams recalled per week given in the statement of part (a). Do the data support the researchers' belief ?

$H_0: \mu_E - \mu_N = 0$

$H_A: \mu_E - \mu_N < 0$

The difference in the mean number of dreams of early birds (μ_E) and night owls (μ_N) is equal to zero or less than zero.

assumptions : 1) The samples are random for all early birds and night owls.

2) The standard deviation of the populations are unknown.

3) The distribution is assumed to be normal.

Therefore, I will use t-procedures for differences in population means with 99 (≈8⁰) degrees of freedom.

$\alpha = .05$
$t_\alpha = 1.664$
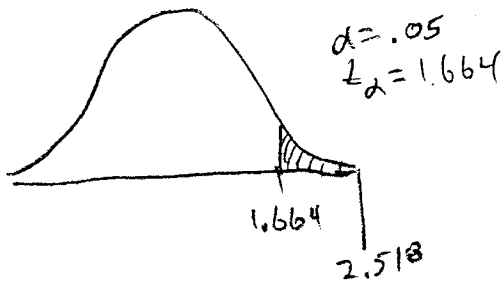
test statistic $= \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$= \dfrac{(9.55 - 7.26) - 0}{\sqrt{\dfrac{5.88^2}{100} + \dfrac{6.94^2}{100}}}$

1.664

2.518

$= \dfrac{2.29}{.358} = 2.518$

initial decision - reject H_0

Final conclusion - There is sufficient evidence to indicate that the difference in mean number of dreams recalled during the week of early birds and night owls is less than zero. That is, night owls recall more dreams per week on average than do early birds. The p-value at $t = 2.518$ of .0063 supports this conclusion since if H_0 were true, we would observe results this extreme only .63% of the time.

• Therefore, the data does support the researchers belief that night owls have better dream recall than do early birds.

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