AP® Statistics
2003 Sample Student Responses
Form B

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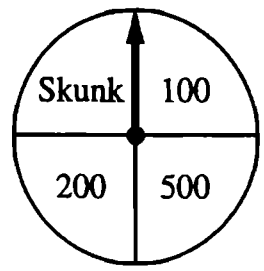
A



5. Contestants on a game show spin a wheel like the one shown in the figure above. Each of the four outcomes on this wheel is equally likely and outcomes are independent from one spin to the next.

   - The contestant spins the wheel.
   - If the result is a skunk, no money is won and the contestant's turn is finished.
   - If the result is a number, the corresponding amount in dollars is won. The contestant can then stop with those winnings or can choose to spin again, and his or her turn continues.
   - If the contestant spins again and the result is a skunk, all of the money earned on that turn is lost and the turn ends.
   - The contestant may continue adding to his or her winnings until he or she chooses to stop or until a spin results in a skunk.

   (a) What is the probability that the result will be a number on all of the first three spins of the wheel?

$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = .422$

   (b) Suppose a contestant has earned \$800 on his or her first three spins and chooses to spin the wheel again. What is the expected value of his or her total winnings for the four spins?

800

$800 + 500 = 1300$
$800 + 200 = 1000$
$800 + 100 = 900$
0

$$\frac{1300 + 1000 + 900 + 0}{4} = 800$$

GO ON TO THE NEXT PAGE.

(c) A contestant who lost at this game alleges that the wheel is not fair. In order to check on the fairness of the wheel, the data in the table below were collected for 100 spins of this wheel.

Result	Skunk	$100	$200	$500
Frequency	33	21	20	26

Based on these data, can you conclude that the four outcomes on this wheel are not equally likely? Give appropriate statistical evidence to support your answer.

H_0: $P_1 = P_2 = P_3 = P_4 = .25$ (where P_1 is probability of getting a skunk, P_2 = probability of getting \$100, P_3 = probability of getting \$200, and P_4 = probability of getting \$500)

H_a: at least one of the population proportions differ from its hypothesized value.

Test statistic $\chi^2 = \sum \frac{[n_i - E(n_i)]^2}{E(n_i)}$ with a degree of freedom of 3

Rejection Region → set $\alpha = .05$. 7.81

reject H_0 if $\chi^2 > 7.81$

Assumptions

The samples are independently & randomly selected, and therefore a multinomial experiment

*Expected value test · (at least 5)

$100 \times .25 = 25$ $E(n_1) = E(n_2) = E(n_3) = E(n_4) = 25$

all expected values are greater than 5. so assumption is satisfied.

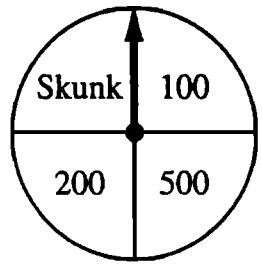
Calculations

$\chi^2 = \sum \frac{[n_i - E(n_i)]^2}{E(n_i)} = 4.24$ H_0 not rejected

Conclusions.

since the critical value of the test statistic does not fall in the rejection region, we don't have enough evidence at a significance level of $\alpha = .05$ to claim that the four outcomes on the spin wheel (skunk, \$100, \$200, & \$500) are not equally likely.

B



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- The contestant may continue adding to his or her winnings until he or she chooses to stop or until a spin results in a skunk.

(a) What is the probability that the result will be a number on all of the first three spins of the wheel?

probability of getting a number : $3/4 = .75$

$(.75)(.75)(.75) = .421875$

The probability would be approx 42.19 % to get a number on all the first three spins of the wheel.

(b) Suppose a contestant has earned $800 on his or her first three spins and chooses to spin the wheel again. What is the expected value of his or her total winnings for the four spins?

Outcome	$100	$500	$200	Skunk
Probability	.25	.25	.25	.25

Expected value = $(100 \times .25) + (500 \times .25) + (200 \times .25) + (0 \times 2.5)$

= $200 will be his / her total winning for the next spins.

(c) A contestant who lost at this game alleges that the wheel is not fair. In order to check on the fairness of the wheel, the data in the table below were collected for 100 spins of this wheel.

Result	Skunk	$100	$200	$500
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Based on these data, can you conclude that the four outcomes on this wheel are not equally likely? Give appropriate statistical evidence to support your answer.

	Observed	Expected	$X^2=\sum\left(\frac{(O-E)^2}{E}\right)$
Skunk	33	25	2.56
$100	21	25	.64
$200	20	25	1
$500	26	25	.04

(Goodness of fit)

Check assumptions for X^2 - test:

① all in Counts ✓

② no more than 20% of the expected values are < 5 ✓

H_0: The four outcomes are not different

H_a: The four outcomes are different.

$X^2 = 4.24$

P-value = .2366875

$df = n - 1 = 3$

In conclusion, the p-value calculated was .2366875. This doesn't give me sufficient evidence to reject null hypothesis. due to a not significant / small p-value. Therefore, based on these data, it seems like the four outcomes have no difference. It is a fair wheel.