AP® Statistics
2004 Sample Student Responses
Form B

A1

STATISTICS
SECTION II
Part B
Question 6

Spend about 25 minutes on this part of the exam.
Percent of Section II grade—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. In order to monitor the populations of birds of a particular species on two islands, the following procedure was implemented.

   Researchers captured an initial sample of 200 birds of the species on Island A; they attached leg bands to each of the birds, and then released the birds. Similarly, a sample of 250 birds of the same species on Island B was captured, banded, and released. Sufficient time was allowed for the birds to return to their normal routine and location.

   Subsequent samples of birds of the species of interest were then taken from each island. The number of birds captured and the number of birds with leg bands were recorded. The results are summarized in the following table.

	Island A	Island B
Number Captured in Subsequent Sample	180	220
Number with Leg Bands in Subsequent Sample	12	35

Assume that both the initial sample and the subsequent samples that were taken on each island can be regarded as random samples from the population of birds of this species.

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(a) Do the data from the subsequent samples indicate that there is a difference in proportions of the banded birds on these two islands? Give statistical evidence to support your answer.

Simple random sample stated in the question.

$\hat{P}_A = \frac{12}{180} = 0.0\overline{6} = \frac{1}{15}$, $\hat{P}_B = \frac{35}{220} = 0.1591 = \frac{7}{44}$.

$\hat{P}_A$: sample proportion of birds in island A with leg bands.

$\hat{P}_B$: sample proportion of birds in island B with leg bands

$n_A\hat{P}_A = 180(\frac{1}{15}) = 12 > 10$ $\quad n_B\hat{P}_B = 220(\frac{7}{44}) = 35 > 10$.

$n_A(1-\hat{P}_A) = 168 > 10$ $\quad n_B(1-\hat{P}_B) = 185 > 10$ conditions met.

*2 sample proportion z test

SD (standard deviation of difference) $= \sqrt{\frac{\hat{P}_A(1-\hat{P}_A)}{n_A} + \frac{\hat{P}_B(1-\hat{P}_B)}{n_B}} = 0.03088$.

$H_0: P_B = P_A$

$H_a: P_B \neq P_A$.

$z = \frac{\hat{P}_B - \hat{P}_A}{SD} = \frac{\frac{7}{44} - \frac{1}{15}}{0.03088} = 2.193$.

This corresponds to a p-value of $2(1 - 0.9786)$ [2 sided test]

from tables $= 0.0028$,

which is a very small value

The the 1% of level, the p-value is $< 1\%$ ($\because 0.28\% < 1\%$)

$\Rightarrow$ I reject H_0 and conclude, at the 1% level, that there is a difference in the proportion of banded birds on these two islands.

A3

(b) Researchers can estimate the total number of birds of this species on an island by using information on the number of birds in the initial sample and the proportion of banded birds in the subsequent sample. Use this information to estimate the total number of birds of this species on Island A. Show your work.

island A: 200 birds banded, only $\frac{1}{15}$ = a sample had bands $\Rightarrow$ 200 represents $\frac{1}{15}$ of the population

$\Rightarrow$ total number of birds of this species on A = 200(15) = 3000

(c) The analyses in parts (a) and (b) assume that the samples of birds captured in both the initial and subsequent samples can be regarded as random samples of the population of birds of this species that live on the respective islands. This is a common assumption made by wildlife researchers. Describe two concerns that should be addressed before making this assumption.

- one concern to check is the effect of placing bands on birds on their survival rate. If banded birds are more likely to be killed by predation by other animals than the second sample is influenced by the bands.

- Another concern is that only birds that are easily caught (such as sick or physically poor birds) can be caught easily, which would not result in a simple random samples as other superior birds won't be included in the sample.

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B1

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   Subsequent samples of birds of the species of interest were then taken from each island. The number of birds captured and the number of birds with leg bands were recorded. The results are summarized in the following table.

n = 200

	Island A	Island B
Number Captured in Subsequent Sample	180	220
Number with Leg Bands in Subsequent Sample	12	35

n = 250

Assume that both the initial sample and the subsequent samples that were taken on each island can be regarded as random samples from the population of birds of this species.

(a) Do the data from the subsequent samples indicate that there is a difference in proportions of the banded birds on these two islands? Give statistical evidence to support your answer.

I did a 2 proportion z test and found a z value of -2.8557 and a p value of .00429. At the 5% level of significance, I would reject the null hypothesis that $\hat{p}_1 = \hat{p}_2$ because the p value, .00429, was less than .05, meaning that the z fell within the critical region, thus the proportion of banded birds on island A is not equal to the proportion of banded birds on island B.

$H_0: p_1 = p_2$

$H_1: p_1 \neq p_2$

$\hat{p}_1 = .0667$

$\hat{p}_2 = .159$

$n_1 = 180$

$n_2 = 220$

- $n_1 \hat{p}_1 \geq 10$ (11.9 ≥ 10)
- $n_1(1-\hat{p}_1) \geq 10$ (168 ≥ 10)
- $n_2 \hat{p}_2 \geq 10$ (35 ≥ 10)
- $n_2(1-\hat{p}_2) \geq 10$ (185 ≥ 10)

B_3

(b) Researchers can estimate the total number of birds of this species on an island by using information on the number of birds in the initial sample and the proportion of banded birds in the subsequent sample. Use this information to estimate the total number of birds of this species on Island A. Show your work.

$\hat{p}_1 = \frac{12}{180} = .0667$

$n_1 = 200$ $\quad 200 \times .0667 = 13.33$

$\frac{200}{x} = \frac{12}{180} \therefore \frac{(200)180}{12} = x$

the total population is ≥ 3000 birds of that species on the island.

(c) The analyses in parts (a) and (b) assume that the samples of birds captured in both the initial and subsequent samples can be regarded as random samples of the population of birds of this species that live on the respective islands. This is a common assumption made by wildlife researchers. Describe two concerns that should be addressed before making this assumption.

$\frac{33}{220} = \frac{200}{x}$ $\quad 166.\bar{6}$

- Perhaps the birds that they were able to capture were the weak or run down birds only, then they may not indeed be regarded as a random sample and are not representative of the entire population.

Also, perhaps the initial capture made the banded birds more susceptible to recapture, or something along those lines.

C1

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C2

(a) Do the data from the subsequent samples indicate that there is a difference in proportions of the banded birds on these two islands? Give statistical evidence to support your answer.

Since we are attempting to determine the dependency of two proportions, a χ^2 test would be most appropriate.

H_0: The proportion of banded birds is not different

H_a: The proportion of banded birds is different

observed values →

	A	B	
w/o leg band	168	185	353
with leg Band	12	35	47
	180	220	400

expected values →

	A	B
w/o band	158.85	194.15
with band	21.15	25.85

$$\chi^2 = \frac{(168-158.85)^2}{158.85} + \frac{(185-194.15)^2}{194.15} + \frac{(12-21.15)^2}{21.15} + \frac{(35-25.85)^2}{25.85}$$

$= 8.1555$, $df = 1$

$P = 0.0042929$

There is strong evidence to reject H_0.

∴ There is strong reason to believe that the samples indicate a difference of proportion of banded birds in Island A and Island B.

(b) Researchers can estimate the total number of birds of this species on an island by using information on the number of birds in the initial sample and the proportion of banded birds in the subsequent sample. Use this information to estimate the total number of birds of this species on Island A. Show your work.

The proportion of banded birds in the sample is 0.0625.

As the sample size (192) is sufficiently large, the central limit theorem allows us to assume an approximately normal distribution of sample proportions.

$\hat{p} \sim N\left(0.0625, \sqrt{\frac{p(1-p)}{n}}\right)$

$\sim N(0.0625, 0.01715)$

200 birds account for 0.060008 to 0.064991 of the population.

$\therefore$ A 95% confidence interval for the population proportion will be (0.060008, 0.064991)

Population $\sim$ (200 ÷ 0.060008, 200 ÷ 0.064991)

$= (3077.349, 3332.889)$

or to state a single number, 200 ÷ 0.0625 = 3200

(c) The analyses in parts (a) and (b) assume that the samples of birds captured in both the initial and subsequent samples can be regarded as random samples of the population of birds of this species that live on the respective islands. This is a common assumption made by wildlife researchers. Describe two concerns that should be addressed before making this assumption.

First is that birds may migrate or die of natural causes if the time between samplings is too long. Therefor the second sample must be taken as soon as the birds are allowed time to return to their routine.

A second concern is that birds that have experienced captivity once may be easier to capture a second time. Therefor the birds must be banded and released as soon as possible from the first sampling, and care must be taken against a convenience sample in the second.