CollegeBoard
Advanced Placement Program

AP® Statistics
2011 Free-Response Questions

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Questions begin on page 6.
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Formulas

(I)   Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2}$$

$$s_p = \sqrt{\frac{\left(n_1 - 1\right)s_1^2 + \left(n_2 - 1\right)s_2^2}{\left(n_1 - 1\right) + \left(n_2 - 1\right)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sum\left(x_i - \bar{x}\right)^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1}\sum\left(\frac{x_i - \bar{x}}{s_x}\right)\left(\frac{y_i - \bar{y}}{s_y}\right)$$

$$b_1 = r\frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\dfrac{\sum\left(y_i - \hat{y}_i\right)^2}{n-2}}}{\sqrt{\sum\left(x_i - \bar{x}\right)^2}}$$

(II)   Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum \left(x_i - \mu_x\right)^2 p_i$$

If X has a binomial distribution with parameters n and p, then:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1-p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ, then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III)   Inferential Statistics

Standardized test statistic: $\dfrac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic $\pm$ (critical value) • (standard deviation of statistic)

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\dfrac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\dfrac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\dfrac{p_1(1-p_1)}{n_1} + \dfrac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$

Chi-square test statistic $= \displaystyle\sum \dfrac{(\text{observed} - \text{expected})^2}{\text{expected}}$

STATISTICS

SECTION II

Part A

Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. A professional sports team evaluates potential players for a certain position based on two main characteristics, speed and strength.

   (a) Speed is measured by the time required to run a distance of 40 yards, with smaller times indicating more desirable (faster) speeds. From previous speed data for all players in this position, the times to run 40 yards have a mean of 4.60 seconds and a standard deviation of 0.15 seconds, with a minimum time of 4.40 seconds, as shown in the table below.

	Mean	Standard Deviation	Minimum
Time to run 40 yards	4.60 seconds	0.15 seconds	4.40 seconds

   Based on the relationship between the mean, standard deviation, and minimum time, is it reasonable to believe that the distribution of 40-yard running times is approximately normal? Explain.

   (b) Strength is measured by the amount of weight lifted, with more weight indicating more desirable (greater) strength. From previous strength data for all players in this position, the amount of weight lifted has a mean of 310 pounds and a standard deviation of 25 pounds, as shown in the table below.

	Mean	Standard Deviation
Amount of weight lifted	310 pounds	25 pounds

   Calculate and interpret the *z*-score for a player in this position who can lift a weight of 370 pounds.

   (c) The characteristics of speed and strength are considered to be of equal importance to the team in selecting a player for the position. Based on the information about the means and standard deviations of the speed and strength data for all players and the measurements listed in the table below for Players A and B, which player should the team select if the team can only select one of the two players? Justify your answer.

	Player A	Player B
Time to run 40 yards	4.42 seconds	4.57 seconds
Amount of weight lifted	370 pounds	375 pounds

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2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent? Justify your answer based on probabilities calculated from the table above.

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.
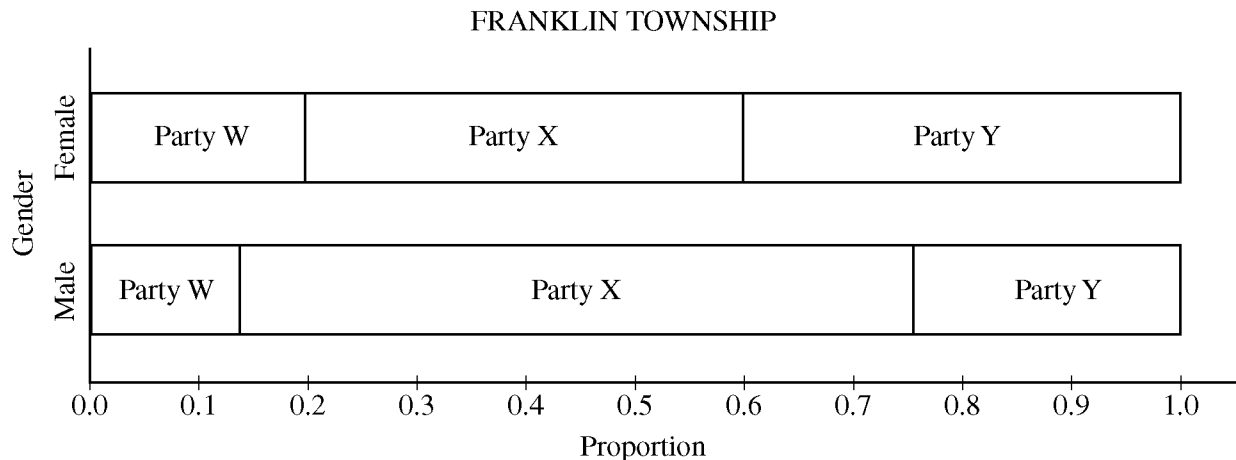
FRANKLIN TOWNSHIP

In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.

LAWRENCE TOWNSHIP

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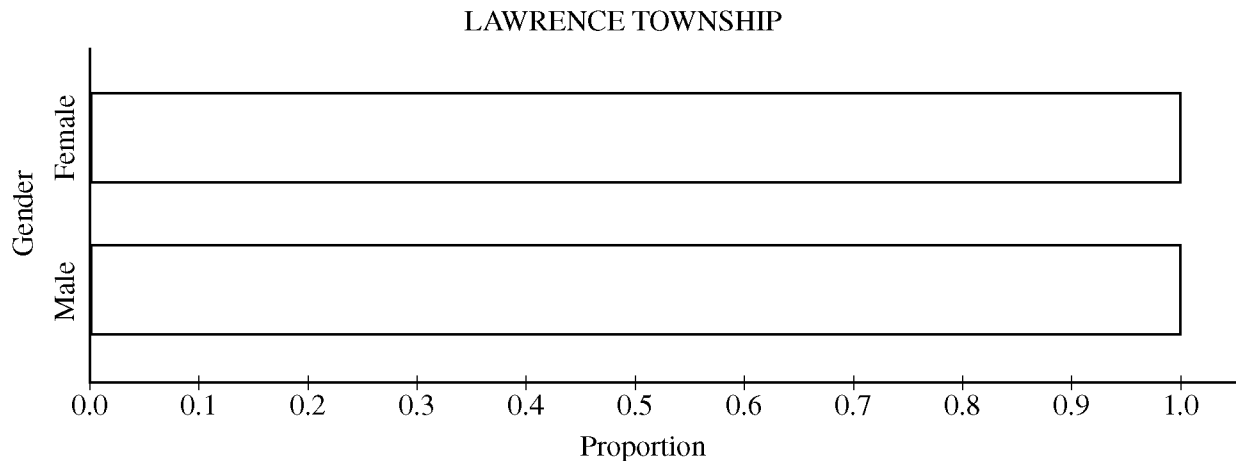
3. An apartment building has nine floors and each floor has four apartments. The building owner wants to install new carpeting in eight apartments to see how well it wears before she decides whether to replace the carpet in the entire building.

The figure below shows the floors of apartments in the building with their apartment numbers. Only the nine apartments indicated with an asterisk (*) have children in the apartment.

11* 12		21 22*		31 32	
1st Floor		2nd Floor		3rd Floor	
14 13		24 23*		34 33	

41 42		51* 52		61 62		* = Children
4th Floor		5th Floor		6th Floor		in the
44 43		54 53		64 63		apartment

71 72		81 82		91 92*
7th Floor		8th Floor		9th Floor	
74* 73*		84* 83		94 93*	

(a) For convenience, the apartment building owner wants to use a cluster sampling method, in which the floors are clusters, to select the eight apartments. Describe a process for randomly selecting eight different apartments using this method.

(b) An alternative sampling method would be to select a stratified random sample of eight apartments, where the strata are apartments with children and apartments with no children. A stratified random sample of size eight might include two randomly selected apartments with children and six randomly selected apartments with no children. In the context of this situation, give one statistical advantage of selecting such a stratified sample as opposed to a cluster sample of eight apartments using the floors as clusters.

4. High cholesterol levels in people can be reduced by exercise, diet, and medication. Twenty middle-aged males with cholesterol readings between 220 and 240 milligrams per deciliter (mg/dL) of blood were randomly selected from the population of such male patients at a large local hospital. Ten of the 20 males were randomly assigned to group A, advised on appropriate exercise and diet, and also received a placebo. The other 10 males were assigned to group B, received the same advice on appropriate exercise and diet, but received a drug intended to reduce cholesterol instead of a placebo. After three months, posttreatment cholesterol readings were taken for all 20 males and compared to pretreatment cholesterol readings. The tables below give the reduction in cholesterol level (pretreatment reading minus posttreatment reading) for each male in the study.

Group A (placebo)

Reduction (in mg/dL)	2	19	8	4	12	8	17	7	24	1

Mean Reduction: 10.20   Standard Deviation of Reductions: 7.66

Group B (cholesterol drug)

Reduction (in mg/dL)	30	19	18	17	20	-4	23	10	9	22

Mean Reduction: 16.40   Standard Deviation of Reductions: 9.40

Do the data provide convincing evidence, at the $\alpha = 0.01$ level, that the cholesterol drug is effective in producing a reduction in mean cholesterol level beyond that produced by exercise and diet?

5. Windmills generate electricity by transferring energy from wind to a turbine. A study was conducted to examine the relationship between wind velocity in miles per hour (mph) and electricity production in amperes for one particular windmill. For the windmill, measurements were taken on twenty-five randomly selected days, and the computer output for the regression analysis for predicting electricity production based on wind velocity is given below. The regression model assumptions were checked and determined to be reasonable over the interval of wind speeds represented in the data, which were from 10 miles per hour to 40 miles per hour.

Predictor	Coef	SE Coef	T	P
Constant	0.137	0.126	1.09	0.289
Wind velocity	0.240	0.019	12.63	0.000

S = 0.237   R-Sq = 0.873   R-Sq (adj) = 0.868

(a) Use the computer output above to determine the equation of the least squares regression line. Identify all variables used in the equation.

(b) How much more electricity would the windmill be expected to produce on a day when the wind velocity is 25 mph than on a day when the wind velocity is 15 mph? Show how you arrived at your answer.

(c) What proportion of the variation in electricity production is explained by its linear relationship with wind velocity?

(d) Is there statistically convincing evidence that electricity production by the windmill is related to wind velocity? Explain.

STATISTICS
SECTION II
Part B
Question 6
Spend about 25 minutes on this part of the exam.
Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. Every year, each student in a nationally representative sample is given tests in various subjects. Recently, a random sample of 9,600 twelfth-grade students from the United States were administered a multiple-choice United States history exam. One of the multiple-choice questions is below. (The correct answer is C.)

In 1935 and 1936 the Supreme Court declared that important parts of the New Deal were unconstitutional. President Roosevelt responded by threatening to

(A) impeach several Supreme Court justices
(B) eliminate the Supreme Court
(C) appoint additional Supreme Court justices who shared his views
(D) override the Supreme Court's decisions by gaining three-fourths majorities in both houses of Congress

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Of the 9,600 students, 28 percent answered the multiple-choice question correctly.

(a) Let p be the proportion of all United States twelfth-grade students who would answer the question correctly. Construct and interpret a 99 percent confidence interval for p.

Assume that students who actually know the correct answer have a 100 percent chance of answering the question correctly, and students who do not know the correct answer to the question guess completely at random from among the four options.

Let k represent the proportion of all United States twelfth-grade students who actually know the correct answer to the question.

(b) A tree diagram of the possible outcomes for a randomly selected twelfth-grade student is provided below. Write the correct probability in each of the five empty boxes. Some of the probabilities may be expressions in terms of k.

TREE DIAGRAM OF OUTCOMES FOR A
RANDOMLY SELECTED TWELFTH-GRADE STUDENT

	Outcome	Probability

Conditional probability = $\boxed{1}$ — Answers correctly — Knows answer and answers correctly — Probability = $\boxed{k}$

Probability = $\boxed{k}$ — Knows correct answer

Conditional probability = $\boxed{0}$ — Answers incorrectly — Knows answer and answers incorrectly — Probability = $\boxed{0}$

Conditional probability = $\boxed{}$ — Answers correctly — Guesses at random and answers correctly — Probability = $\boxed{}$

Probability = $\boxed{}$ — Guesses at random

Conditional probability = $\boxed{}$ — Answers incorrectly — Guesses at random and answers incorrectly — Probability = $\boxed{}$

(c) Based on the completed tree diagram, express the probability, in terms of k, that a randomly selected twelfth-grade student would correctly answer the history question.

(d) Using your interval from part (a) and your answer to part (c), calculate and interpret a 99 percent confidence interval for k, the proportion of all United States twelfth-grade students who actually know the answer to the history question. You may assume that the conditions for inference for the confidence interval have been checked and verified.

STOP

END OF EXAM

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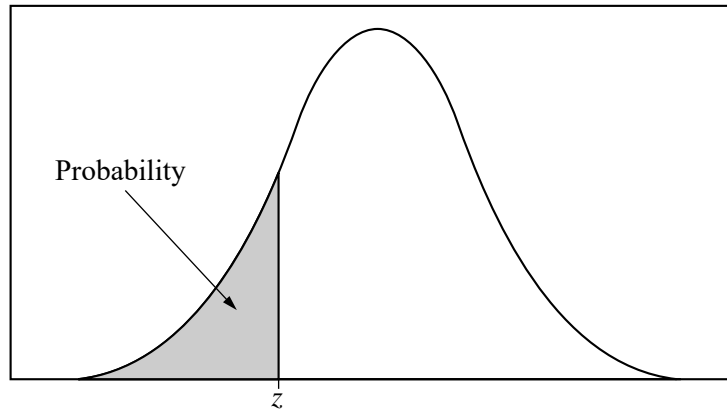
Table entry for z is the probability lying below z.

Probability

z

Table A   Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
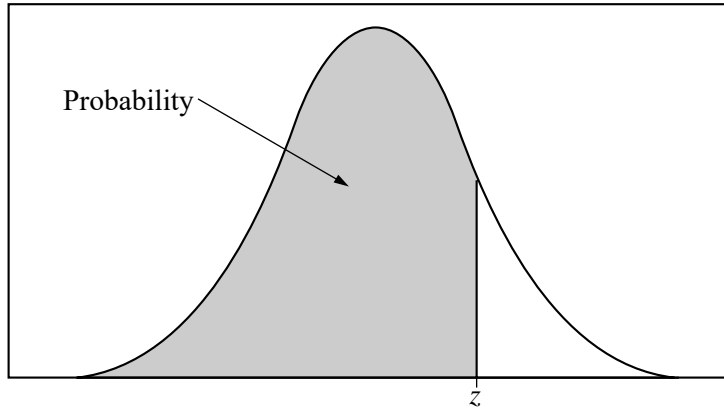
Table entry for z is the probability lying below z.

Table A (*Continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.
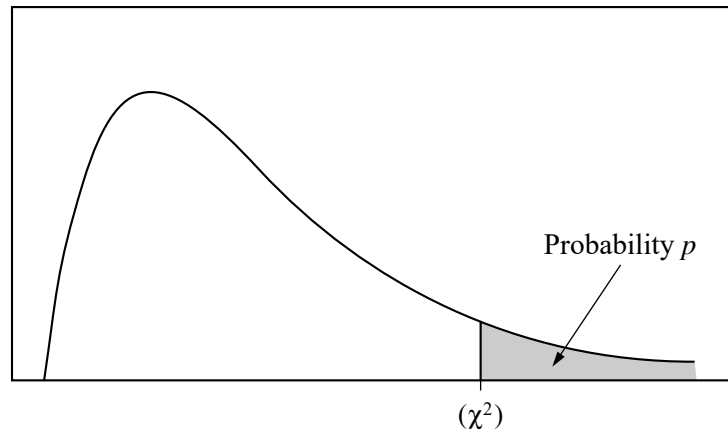
Probability p

t^*
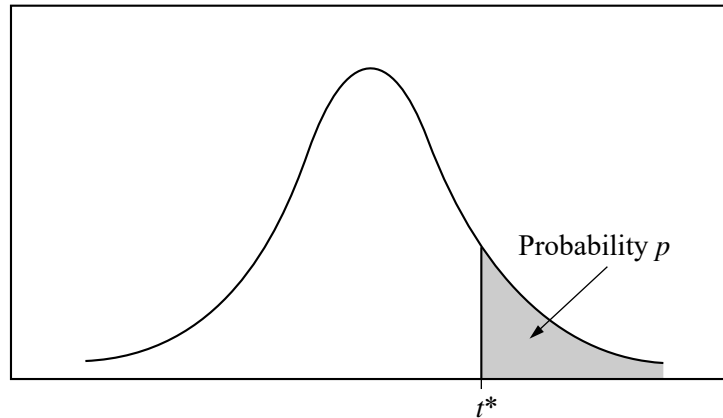
Table B t distribution critical values

df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
							Tail probability p					
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
							Confidence level C					

Table entry for p is the point (χ^2) with probability p lying above it.

Table C χ^2 **critical values**

df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2