

AP[®] STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 1

Intent of Question

The primary goals of this question were to assess students' ability to (1) describe and use a procedure for estimating medians from histograms; (2) use graphical displays to compare two different distributions; (3) use graphical and numerical information to compare the means for two groups.

Solution

Part (a):

The median is the value with half of the P-T ratios at or below it and half of the values at or above it.

For n observations in a group, use $\frac{n+1}{2}$ to find the position of the median in the ordered list of observations.

For states west of the Mississippi ($n = 24$) the median falls between the 12th and 13th value in the ordered list, and both the 12th and 13th values fall in the interval 15–16. For states east of the Mississippi ($n = 26$) the median falls between the 13th and 14th value in the ordered list, and both of these values also fall in the interval 15–16.

From the histogram, cumulative frequencies for the two groups are shown in the table below.

Interval	West	East
12–13	1	2
13–14	$1 + 4 = 5$	$2 + 4 = 6$
14–15	$1 + 4 + 6 = 11$	$2 + 4 + 4 = 10$
15–16	$1 + 4 + 6 + 3 = 14$	$2 + 4 + 4 + 11 = 21$

Thus, the median P-T ratio for both groups is at least 15 students per teacher and at most 16 students per teacher.

Part (b):

The shapes of the two histograms are different. The histogram for states that are west of the Mississippi River is unimodal and skewed to the right, whereas the histogram for states that are east of the Mississippi River is unimodal and nearly symmetric.

As noted in part (a), the medians of the two distributions are about the same, between 15 and 16 for both distributions.

The histograms also show that there is more variability in the P-T ratios for states that are west of the Mississippi River. Although the greatest and least values for each group are not known, the range can be approximated. The range for the west is at most $22 - 12 = 10$, and the range for the east is at most $19 - 12 = 7$.

AP[®] STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 1 (continued)

Part (c):

The medians of the two distributions are about the same, as determined in part (a). The distribution of P-T ratios for states that are west of the Mississippi River is skewed to the right, indicating that the mean will probably be higher than the median. The rough symmetry for the east group indicates that the mean will be close to the median. Thus, the mean for the west group will probably be greater than the mean for the east group.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if a correct estimation method is described and appropriate estimates (values between 15 and 16, inclusive) are provided.

Partially correct (P) for any of the following:

- The response describes a correct estimation method, but the estimates are not provided.
- The response describes a method that conveys the idea of median as the middle value but is not entirely correct (for example, it describes the 12th value rather than the average of the 12th and 13th values), *AND* it provides reasonable estimates.
- The response gives an incomplete description of the method *AND* provides reasonable estimates.
- The response shows work only on the histograms *AND* correct estimates are provided, *BUT* no verbal explanation of the method is given.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if appropriate comparative statements are made for the centers, the shapes, and the spreads of the two groups.

Note: The shape of the east histogram can be described as skewed, *approximately* symmetric, or *approximately* normal. However, if the shape is described as symmetric or normal, part (b) cannot be scored an E.

Partially correct (P) if all three comparative statements are not made, but correct information regarding all three characteristics (center, shape, and spread) is provided for both groups, *OR* if only two of the three comparative statements are made.

Note: If a comparative statement about medians is made in part (a) or in part (c), this can count for a comparison of center in part (b).

Incorrect (I) if at most one comparative statement is made *AND* the response does not include correct information about all three characteristics (center, shape, and spread) for both groups.

AP[®] STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 1 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response indicates that the west group has a higher mean than the east group and provides a justification based on the relationship between means and medians for distributions with different shapes.

Partially correct (P) for any of the following:

- The response indicates that the west group has a higher mean, *BUT* the justification is not based on the relationship between means and medians for distributions with different shapes — for example, the justification is not based on the answers to parts (a) and (b).
- The response includes correct statements about the relative sizes of the mean and median for each group, *BUT* it does not explicitly compare the means of the two groups.
- The response provides a justification based on the relationship between the mean and median for a skewed distribution *BUT* concludes that the mean will be smaller than the median for a right-skewed distribution.

Incorrect (I) if either group is selected with no justification.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

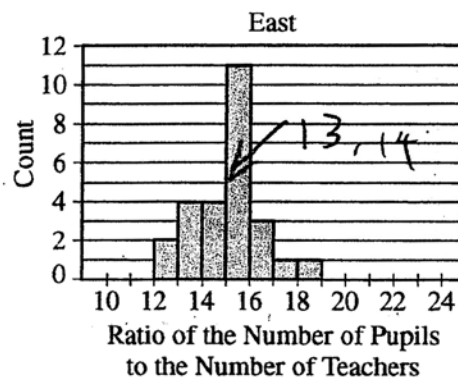
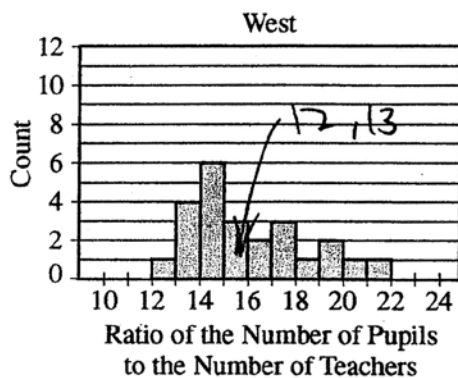
STATISTICS
SECTION II
Part A
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. Records are kept by each state in the United States on the number of pupils enrolled in public schools and the number of teachers employed by public schools for each school year. From these records, the ratio of the number of pupils to the number of teachers (P-T ratio) can be calculated for each state. The histograms below show the P-T ratio for every state during the 2001–2002 school year. The histogram on the left displays the ratios for the 24 states that are west of the Mississippi River, and the histogram on the right displays the ratios for the 26 states that are east of the Mississippi River.



- (a) Describe how you would use the histograms to estimate the median P-T ratio for each group (west and east) of states. Then use this procedure to estimate the median of the west group and the median of the east group.

From the data given, we can see the total size of data (24 and 26). Therefore the median is the mean of the two central values, (since the size is an even number), we can calculate to find them. We can estimate their values from the position they are in the histogram.

$$\text{For } M_W = \frac{12^{\text{th}} + 13^{\text{th}}}{2} = \frac{15.2 + 15.6}{2} = 15.4$$

$$\text{For } M_E = \frac{13^{\text{th}} + 14^{\text{th}}}{2} = \frac{15.2 + 15.3}{2} = 15.25$$

GO ON TO THE NEXT PAGE.

- (b) Write a few sentences comparing the distributions of P-T ratios for states in the two groups (west and east) during the 2001–2002 school year.

The East has a smaller variance than West,
 And it's more centered around the median and
 The West distribution is skewed to the right, ^{mean.}
 While the East is skewed to the left.
 The range of West is larger than East.

- (c) Using your answers in parts (a) and (b), explain how you think the mean P-T ratio during the 2001–2002 school year will compare for the two groups (west and east).

Their median is almost the same,
 But because West is skewed to the right, its
 mean should be larger than its median while
 the mean of the East is smaller,

$$\text{So } \text{Mean}_w > \text{Mean}_e$$

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 in the

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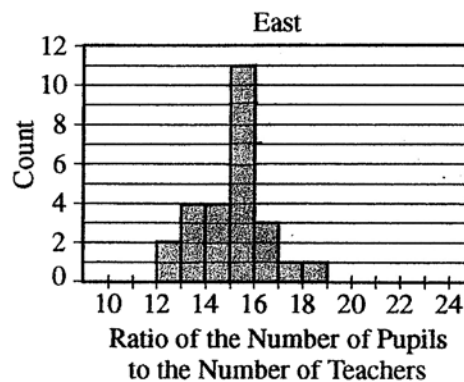
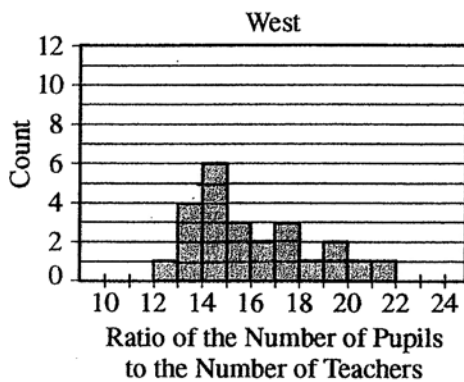
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- (a) Describe how you would use the histograms to estimate the median P-T ratio for each group (west and east) of states. Then use this procedure to estimate the median of the west group and the median of the east group.

The histograms can be divided into small parts which have same area. Every ~~Each~~ small part ~~indicates~~ ^{represents} the same ratio of the ~~the~~ number of pupils to the teachers because they have the same area. Count the small parts and ^{find} the part which is at middle. The value represented by this middle part is ~~off~~ the median of the group.

Like shown in the graph, the west group is ~~is~~ divided into 24 parts, and the 12th and 13th parts represent the ratio between 15 and 16. So the median of the west group is approximately 15.3

Similarly, the east group is divided into 26 parts, and the 13th, 14th parts are both between 15 and 16, so the median of the east group is approximately 15.2

GO ON TO THE NEXT PAGE.

- (b) Write a few sentences comparing the distributions of P-T ratios for states in the two groups (west and east) during the 2001–2002 school year.

Shape: the west group is right-skewed, and the east group is ~~from~~ approximately normal ~~distribution~~ distribution.

Center: the two groups have similar median, but the mean of the west group is ~~larger~~ smaller than that of the east group.

Spread: The west group has a larger standard deviation ~~of~~ than that of the east group, so west group is more spread-out.

- (c) Using your answers in parts (a) and (b), explain how you think the mean P-T ratio during the 2001–2002 school year will compare for the two groups (west and east).

~~side~~
Since the two groups have approximately same median, and the west group is right-skewed while the east group is approximately normal, so the mean of west group is smaller than that of the east group.

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1C1

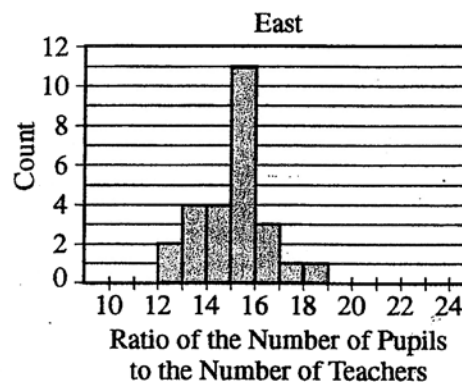
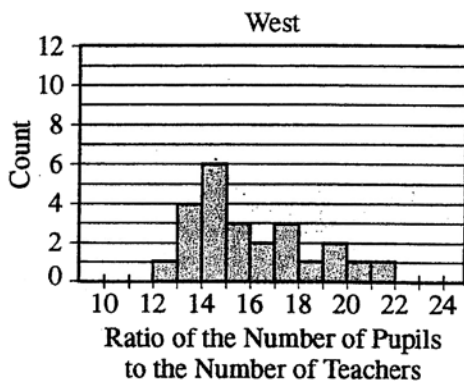
**STATISTICS
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- (a) Describe how you would use the histograms to estimate the median P-T ratio for each group (west and east) of states. Then use this procedure to estimate the median of the west group and the median of the east group.

When use the histograms to estimate the median P-T ratio each group of states. First, I will make sure the median number at what situation. For example 12th or 13th. Then, I will find out this number from the histogram:

West Group has 24 states, so median = $\frac{12^{\text{th}} + 13^{\text{th}}}{2} = \therefore \text{median} = 15$

East Group has 26 states, so median = $\frac{13^{\text{th}} + 14^{\text{th}}}{2} = \therefore \text{median} = 15$

GO ON TO THE NEXT PAGE.

- (b) Write a few sentences comparing the distributions of P-T ratios for states in the two groups (west and east) during the 2001–2002 school year.

West group has widely range than the East group, and the standard deviation of west group may greater than the East group. But these two group have the same median.

- (c) Using your answers in parts (a) and (b), explain how you think the mean P-T ratio during the 2001–2002 school year will compare for the two groups (west and east).

I think the mean P-T ratio during the 2001 ~ 2002 school year in west group is greater than the mean in east group

GO ON TO THE NEXT PAGE.

AP[®] STATISTICS
2011 SCORING COMMENTARY (Form B)

Question 1

Sample: 1A

Score: 4

In part (a) a correct method for using the histograms to estimate the two group medians is described, and reasonable estimates are given. Although the response would have been stronger if units had been included when reporting the median (pupils per teacher), part (a) was scored as essentially correct. Part (b) includes comparative statements about spread and shape. The comparative statements about center that appear in part (c) were considered as well. Thus, the response contains comparative statements about center, shape, and spread, and part (b) was scored as essentially correct. The response in part (c) uses correct reasoning about the relationship between the mean and the median for skewed distributions to conclude that the mean for the west group is greater than the mean for the east group. Part (c) was also scored as essentially correct. Because three parts were scored as essentially correct, the response earned a score of 4.

Sample: 1B

Score: 3

In part (a) a correct method for using the histograms to estimate the two group medians is described, and reasonable estimates are given. Part (a) was scored as essentially correct. In part (b) comparative statements are made about center, shape, and spread. Although the response mistakenly states that “the mean of the west group is smaller than that of the east group,” this was overlooked in scoring part (b) because a correct comparative statement about medians is given and because the comparison of means is the focus of part (c), so these types of errors are considered there. Part (b) was scored as essentially correct. In part (c) the response makes note of the fact that the medians are approximately equal but mistakenly indicates that the mean will be less than the median in right-skewed distributions. This leads to the incorrect (but consistent) conclusion that the mean for the west group is smaller than the mean for the east group. Because of this error, part (c) was scored as partially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 1C

Score: 2

Although the wording is a bit awkward, a reasonable method for estimating the median is described in part (a). Reasonable estimates of the group medians are also given, so part (a) was scored as essentially correct. In part (b) comparative statements about centers and spreads of the two distributions are made, but distribution shapes are not considered. Because only two of the three required comparative statements are made, part (b) was scored as partially correct. In part (c) the response states that the mean is greater for the west group than for the east group, but no justification is provided. Therefore, part (c) was scored as incorrect. Because one part was scored as essentially correct, one part was scored as partially correct, and one part was scored as incorrect, the response earned a score of 2.