AP® STATISTICS
2011 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) determine a conditional probability from a table of data; (2) use a table of data to determine whether or not two events are independent; (3) demonstrate an understanding of the concept of independence by constructing a graph that displays independence between two variables.

Solution

Part (a):

Of the 200 male registered voters in Franklin Township, 48 are registered for Party Y. Therefore the conditional probability that a randomly selected voter is registered for Party Y, given that the voter is a male, is $\frac{48}{200} = 0.24$.

Part (b):

No, the events "is a male" and "is registered for Party Y" are not independent. One justification of this conclusion is to note that the conditional probability of the event "is registered for Party Y" given the event "is a male" — which was computed in part (a) — is not equal to the probability of the event "is registered for Party Y," as shown below.

$$P(\text{is registered for Party Y} \mid \text{is a male}) = 0.24$$

$$P(\text{is registered for Party Y}) = \frac{168}{500} = 0.336$$

Because $0.24 \neq 0.336$, the two events are not independent.

Part (c):

The marginal proportions of voters registered for each of the three political parties (without regard to gender) are given below.

$$\text{Party W: } \frac{88}{500} = 0.176$$

$$\text{Party X: } \frac{244}{500} = 0.488$$

$$\text{Party Y: } \frac{168}{500} = 0.336$$

Because party registration is independent of gender in Lawrence Township, the proportions of males and females registered for each party must be identical to each other and also identical to the marginal proportion of voters registered for that party. Using the order Party W, Party X, and Party Y, the graph for Lawrence Township is displayed below.

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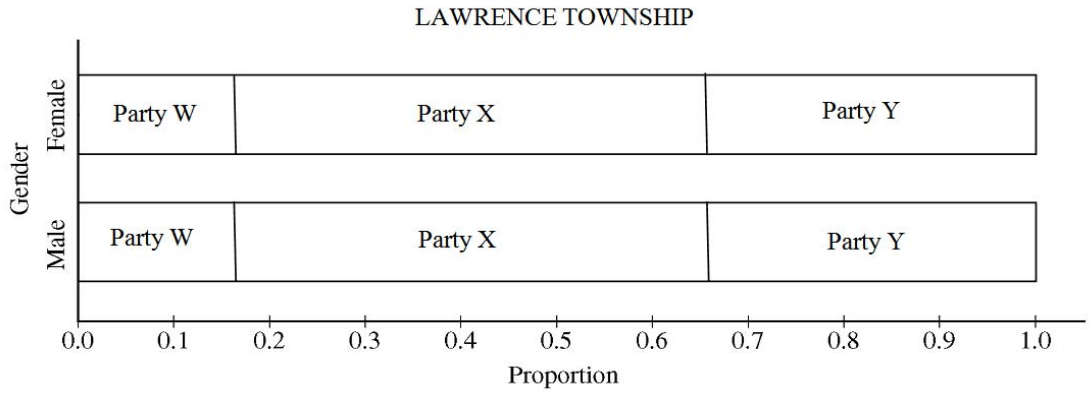
Question 2 (continued)



Scoring

Parts (a), (b) and (c) are scored as essentially correct (E), partially correct (P) or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response has the correct conditional probability *AND* shows the work.

Partially correct (P) if the response has the correct reverse conditional probability (of being a male given that he is registered for Party Y),
OR
if the response has the correct conditional probability *BUT* does not show work.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if the response identifies two values whose inequality implies a lack of independence between the events *AND* includes the following three components:
1. Correct computations of the two values.
2. An explicit statement of whether the two values are equal or unequal.
3. An appropriate conclusion about the independence of the events.

Partially correct (P) if the response identifies two values whose inequality implies a lack of independence between the events but includes only two of the three components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (c) is scored as follows:

Essentially correct (E) if the response shows the same conditional distribution of party registration for both males and females *AND* includes the following two components:
1. Correct proportions for each party.
2. Correct labels (Party W, Party X, Party Y).

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Question 2 (continued)

Partially correct (P) if the response shows the same conditional distribution of party registration for both males and females *AND* includes only one of the two components listed above.

Incorrect (I) if the response fails to meet the criteria for E or P.

Note: For all three parts, an incorrect statement that indicate a serious misunderstanding of statistical concepts, even if unrelated to the rest of the response, lowers the score one level (that is, from E to P, or from P to I). An example of this is a response that indicates confusion between independent events and disjoint events.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

2A 1

2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

$P(\text{male and registered for party y}) = P(\text{male}) \cdot P(\text{registered for party y} \mid \text{male})$

$P(\text{registered for party y} \mid \text{male}) = \frac{P(\text{male and party y})}{P(\text{male})}$

$= \frac{48/500}{200/500} = \frac{.096}{.4} = .24$

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent? Justify your answer based on probabilities calculated from the table above.

$P(B \mid A) = P(B)$

$P(\text{registered for party y} \mid \text{male}) = P(\text{registered for party y})$

NOT INDEPENDENT

$.24 = 168/500$

$.24 \neq .336$

$P(A \text{ and } B) = P(A) \cdot P(B)$

$P(\text{registered for party y and male}) = P(\text{registered for party y}) \cdot P(\text{male})$

NOT INDEPENDENT

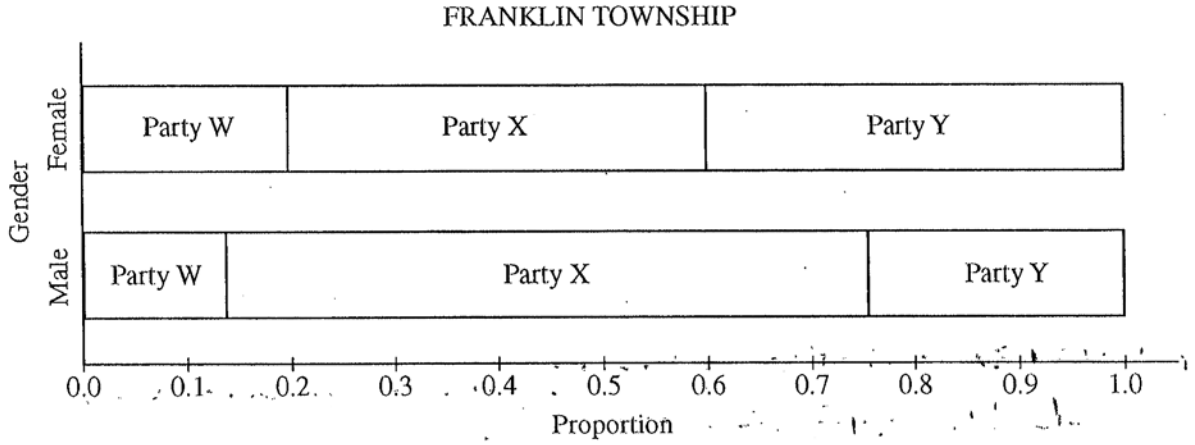
$48/500 = (168/500) \cdot (200/500)$

$.096 = (.336)(.4)$

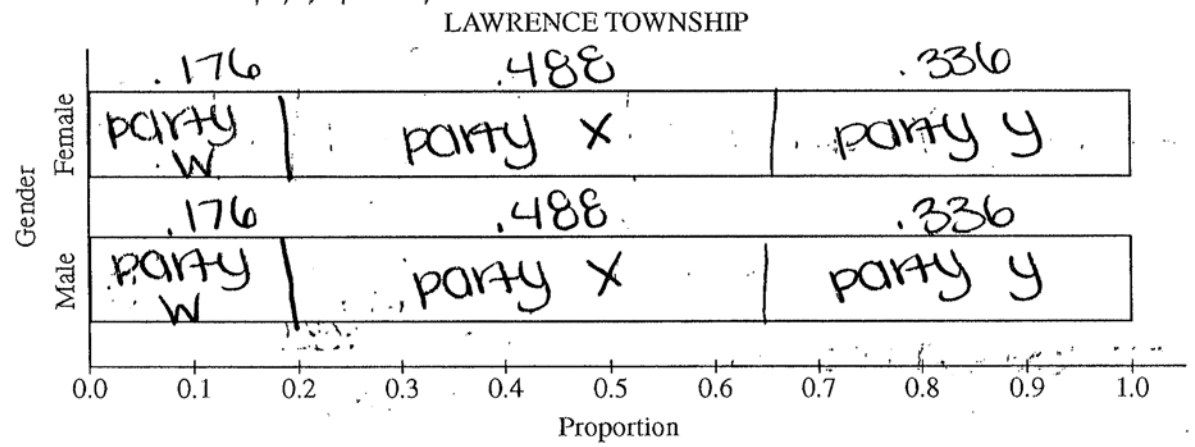
$.096 \neq .1344$

GO ON TO THE NEXT PAGE.

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.



In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



since gender and party registration are independent, gender does not matter so the probability are the same for each gender.

W = 88/500 = .176

X = 244/500 = .488

Y = 168/500 = .336

GO ON TO THE NEXT PAGE.

2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

$\frac{\text{Party Y male}}{\text{total males}} = \frac{48}{200} = .24$

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent? Justify your answer based on probabilities calculated from the table above.

events are independent if $P(A) \times P(B) = P(A \cap B)$.
let event A be "is a male" and event B be "is registered for Party Y"

$P(A) = \frac{200}{500} = .4$ $\quad P(B) = \frac{168}{500} = .336$

$P(A \cap B) = \frac{48}{500} = .096$

$P(A) \times P(B) = (.4)(.336) = .1344$

$.1344 \neq .096$. the events are not independent.

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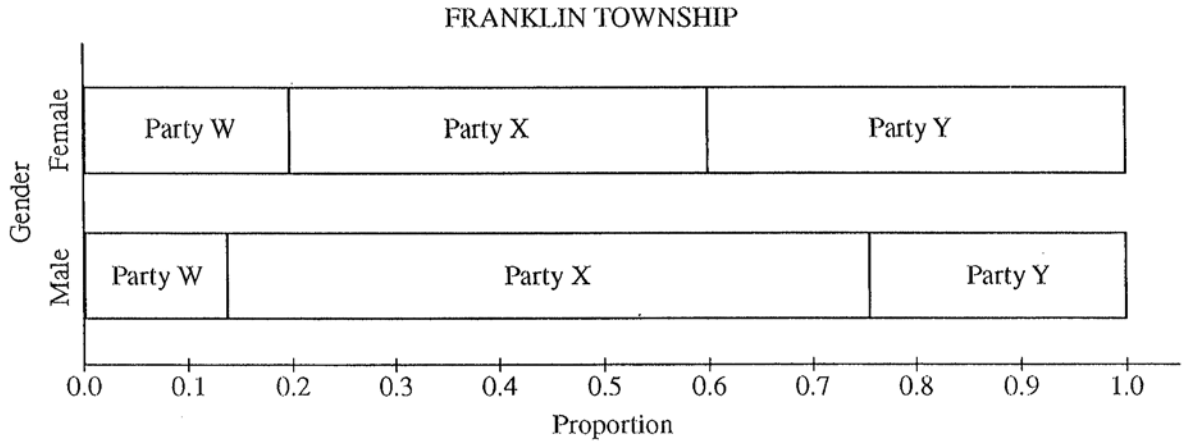
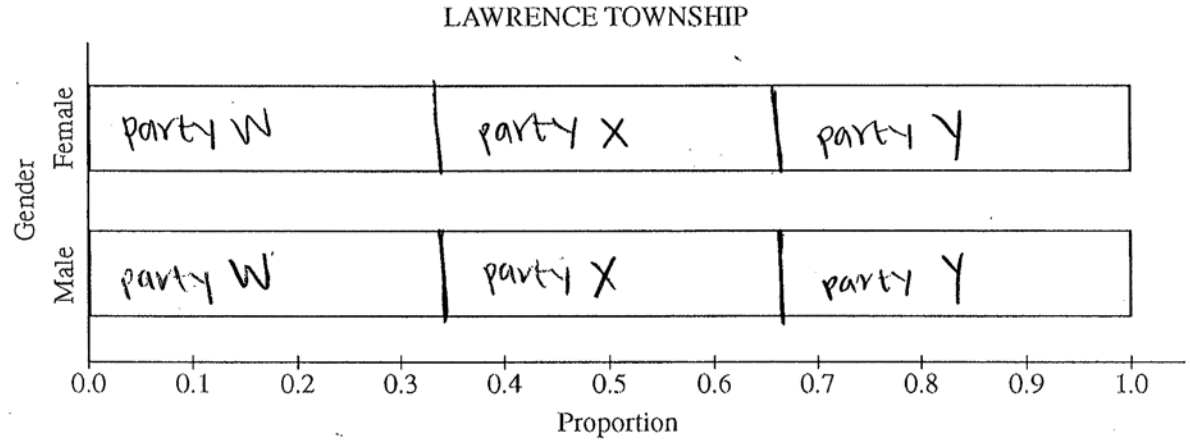
(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.



In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



if party registration is independent of gender, then each party should have the same proportion of voters based on gender.

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2 c 1

2. The table below shows the political party registration by gender of all 500 registered voters in Franklin Township.

PARTY REGISTRATION–FRANKLIN TOWNSHIP

	Party W	Party X	Party Y	Total
Female	60	120	120	300
Male	28	124	48	200
Total	88	244	168	500

(a) Given that a randomly selected registered voter is a male, what is the probability that he is registered for Party Y?

Given that the voter is male, the probability that he is registered for party y is $\frac{48}{200}$. Since there are 200 males, out of these 200, 48 are registered for party y, therefore $\frac{48}{200}$ or 24% of male voters are registered for Y.

(b) Among the registered voters of Franklin Township, are the events "is a male" and "is registered for Party Y" independent? Justify your answer based on probabilities calculated from the table above.

Based on the probabilities above, "being a male" and "being registered for Party y" are not independent of each other.

200 voters are male out of 500 total voters, so $\frac{200}{500}$ voters are male, or 40% of voters are male

168 voters are registered for Party y out of 500 total voters, so $\frac{168}{500}$ voters are registered for party y, or 33.6% voters are registered party y.

These two probabilities are not the same, meaning they are not independent of one another.

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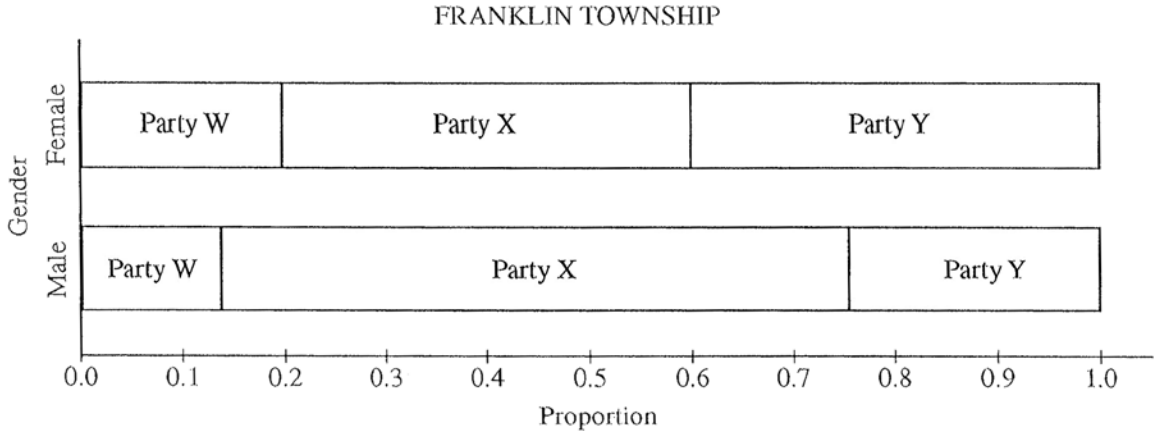
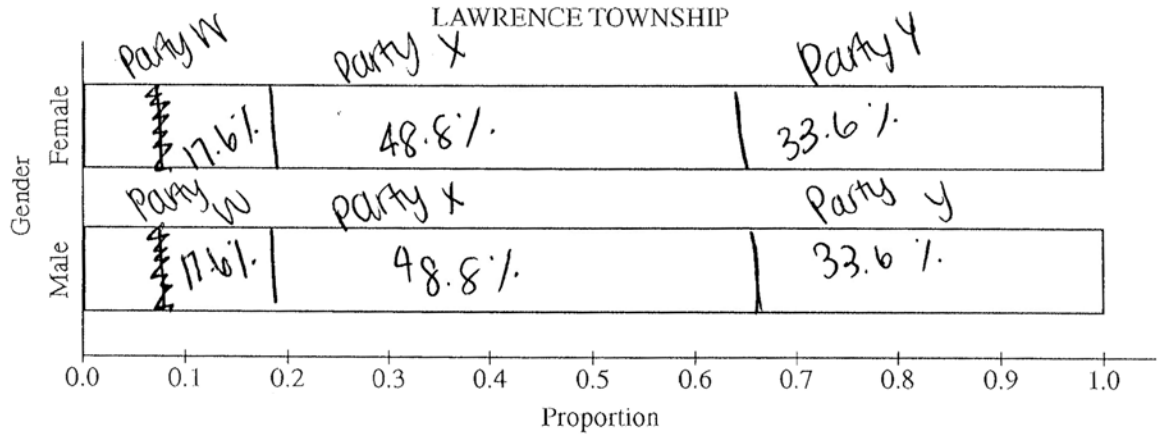
2C2

(c) One way to display the data in the table is to use a segmented bar graph. The following segmented bar graph, constructed from the data in the party registration–Franklin Township table, shows party-registration distributions for males and females in Franklin Township.



In Lawrence Township, the proportions of all registered voters for Parties W, X, and Y are the same as for Franklin Township, and party registration is independent of gender. Complete the graph below to show the distributions of party registration by gender in Lawrence Township.



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AP® STATISTICS
2011 SCORING COMMENTARY

Question 2

Overview

The primary goals of this question were to assess students' ability to (1) determine a conditional probability from a table of data; (2) use a table of data to determine whether or not two events are independent; (3) demonstrate an understanding of the concept of independence by constructing a graph that displays independence between two variables.

Sample: 2A
Score: 4

In part (a) the student begins by writing the correct general formula for a joint probability and then applies the formula to the problem by replacing A with the event "male" and B with the event "registered for party y." The student divides both sides of the equation by $P(\text{male})$ and derives the correct formula for a conditional probability. In the last line of work, the student correctly computes the conditional probability using the data given in the table. Part (a) was scored as essentially correct. Part (a) would also have been scored as essentially correct had the student simply begun at the fourth line with the correct formula for a conditional probability. In part (b) the student begins with the equation "$P(\text{B} \mid \text{A}) = P(\text{B})$" and again replaces A and B with the appropriate events. The work that follows makes it clear that the student plans to check whether that equality holds for the events A and B that were identified in part (a). The computations are correct, and the response states that the conditional and the marginal probabilities are unequal. The correct conclusion, "NOT INDEPENDENT," appears on the left. This approach alone would have been sufficient for the response to part (b) to be scored as essentially correct. However, the student continues by performing a different (but equivalent) check for independence; that is, two events must be independent if their joint probability is equal to the product of their marginal probabilities. Once again the computations are correct, the response states that the two quantities are unequal, and the correct conclusion is made. This alone would also have been sufficient for the response to be scored as essentially correct. Because both approaches were done correctly, part (b) was scored as essentially correct. (Had either of the approaches contained a serious error, the other approach would *not* have overridden the one containing the error; the response would have then been scored according to the approach containing the error.) In part (c) the student draws line segments in the same places in the bars for both males and females, indicating an understanding of the concept of independence. The line segments are in the correct places, and party labels are included. Part (c) was scored as essentially correct. Although neither computations nor explanations were necessary for the response to part (c) to be scored as essentially correct, it is a strength of this response that the student shows the computations that indicate where the dividing line segments in the graph should be placed, gives the proportions in the graph itself, and includes a written statement that very clearly indicates an understanding of the concept of independence: "since gender and party registration are independent, gender does not matter so the probability [*sic*] are the same for each gender." Because all three parts were scored as essentially correct, the response earned a score of 4.

Sample: 2B
Score: 3

In part (a) the student correctly computes the conditional probability of a randomly selected registered voter being registered for Party Y given that the voter is male, using the data given in the table. Part (a) was scored as essentially correct. In part (b) the student states the generality that two events, A and B, are independent if the product of their marginal probabilities is equal to their joint probability. This is an appropriate way to determine whether two events are independent. The student then applies that rule in

Question 2 (continued)

context to the given problem by letting A be the event "'is a male'" and letting B be the event "'is registered for Party Y.'" The student correctly computes the two marginal probabilities, $P(A)$ and $P(B)$, and the joint probability, $P(A \cap B)$. Finally, the student states that because the earlier condition for independence is not met — the product of the marginal probabilities is not equal to the joint probability — then "the events are not independent." This conclusion is correct and justified by the student's reasoning. Part (b) was scored as essentially correct. In part (c) the student draws line segments in the same places for males and females, which indicates some understanding of the concept of independence. However, the student places the dividing lines in the wrong places and incorrectly states, "if party registration is independent of gender, than [*sic*] each party should have the same proportion of voters." This explains the erroneous placement of the dividing lines between party proportions at 1/3 and 2/3. Part (c) was scored as partially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 2C
Score: 2

In part (a) the student correctly computes the conditional probability that a randomly selected registered voter is registered for Party Y given that the voter is a male, using the data given in the table. The clear communication is a strength of this response. Part (a) was scored as essentially correct. In part (b) the student correctly computes the marginal proportion of all registered voters who are registered for Party Y and the marginal proportion of all registered voters who are male, referring to these proportions as "probabilities," which is acceptable. However, the student asserts that because the two marginal probabilities are unequal, the events "'being a male' and 'being registered for Party Y' are not independent." Marginal probabilities alone cannot be used to determine whether two events are independent, so the comparison is invalid. Although the computations and conclusion are correct, the critical part of the response — a comparison by which the independence of two events can be assessed — is incorrect. Part (b) was scored as incorrect. In part (c) the student places line segments in the same places on the bars for females and males, indicating an understanding of the concept of independence. Although the student does not show the computations that led to the proportions of voters registered for Parties W and X (the computations for Party Y were shown in the previous part), it is still a strength of the response that the correct proportions are given in the graph along with party labels. Part (c) was scored as essentially correct. Because two parts were scored as essentially correct and one part was scored as incorrect, the response earned a score of 2.