Question 5

Intent of Question

The primary goals of this question were to assess students' ability to (1) identify and check appropriate conditions for inference; (2) identify and carry out the appropriate inference procedure; (3) determine the sample size necessary to meet certain specifications in planning a study.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions

One-sample (or large-sample) interval for p (the proportion of the vaccine-eligible people in the United States who actually got vaccinated) or $\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

Conditions: 1. Random sample
2. Large sample ($n\hat{p} \geq 10$ and $n(1-\hat{p}) \geq 10$)

The stem of the problem indicates that a random sample of vaccine-eligible people was surveyed. The number of successes (978 vaccine-eligible people who received the vaccine), and failures (1,372 vaccine-eligible people who did not receive the vaccine), are both much larger than 10, so the large-sample interval procedure can be used.

Step 2: Correct mechanics

$$\left(\frac{978}{2,350}\right) \pm 2.57583 \sqrt{\frac{0.41617(1-0.41617)}{2,350}}$$
$$0.41617 \pm 2.57583 \times 0.01017$$
$$0.41617 \pm 0.02619$$
$$(0.38998, 0.44236)$$

Step 3: Interpretation

Based on the sample, we are 99 percent confident that the proportion of the vaccine-eligible people in the United States who actually got vaccinated is between 0.39 and 0.44. Because 0.45 is not in the 99 percent confidence interval, it is not a plausible value for the population proportion of vaccine-eligible people who received the vaccine. In other words, the confidence interval is inconsistent with the belief that 45 percent of those eligible got vaccinated.

Question 5 (continued)

Part (b):

The sample-size calculation uses 0.5 as the value of the proportion in order to provide the minimum required sample size to guarantee that the resulting interval will have a margin of error no larger than 0.02.

$$n \geq \frac{(2.576)^2(0.5)(0.5)}{(0.02)^2} = \left(\frac{2.576}{2(0.02)}\right)^2 = 4{,}147.36$$

Thus, a sample of at least 4,148 vaccine-eligible people should be taken in Canada.

Scoring

Each step in part (a) is scored as essentially correct (E), partially correct (P), or incorrect (I); and part (b) is scored as essentially correct (E), partially correct (P), or incorrect (I).

Step 1 of part (a) is scored as follows:

Essentially correct (E) if the one-sample z-interval for a proportion is identified (either by name or formula) *AND* both conditions (of random sampling and sample size) are stated and checked.

Partially correct (P) if the response identifies the correct procedure *BUT* adequately addresses only one of the two required conditions, *OR* if the response does not identify the correct procedure *BUT* adequately addresses both required conditions.

Incorrect (I) for any of the following:

- The response identifies the correct procedure *BUT* does not adequately address either required condition,
- The correct procedure is not identified *AND* at most one of the required conditions is adequately addressed,
- An incorrect procedure is identified.

Notes

- If the formula is of the correct form, even if incorrect numbers appear in it, then the procedure may be considered correctly identified.
- Stating only that $n\hat{p}$ and $n(1-\hat{p})$ both are greater than or equal to 10 is only a statement of the sample size condition and is not sufficient for checking the condition. The response must use specific values from the question to check the condition.
- If a response includes additional inappropriate conditions, such as $n \geq 30$ or requiring a normal population, then the response can earn no more than a P for this step. However, stating and checking a condition about the size of the sample relative to the size of the population is not required but is also not inappropriate.
- Any statement of hypotheses, definitions of parameters, statements of populations, etc. should be considered extraneous. However, if these statements are included and incorrect, this should be considered poor communication in terms of holistic scoring.

Question 5 (continued)

Step 2 of part (a) is scored as follows:

Essentially correct (E) if a 99 percent confidence interval is correctly computed.

Partially correct (P) for any of the following:

- If a correct method (confidence interval for a proportion) is used, *BUT* an incorrect critical z-value or a t-value is used.
- 0.45 is used for the value of $\hat{p}$.
- There are errors in the calculation of the interval (unless such errors follow from an incorrect procedure in step 1).

Incorrect (I) if an incorrect method is used, such as a t-interval for a population mean, *OR* if the resulting interval is unreasonable, such as an interval with integer endpoints.

Step 3 of part (a) is scored as follows:

Essentially correct (E) if the response notes that 0.45 is not in the 99 percent confidence interval *AND* states that this is evidence against the belief that 45 percent of vaccine-eligible people had received flu-vaccine.

Partially correct (P) if a reasonable statement about the belief that 45 percent of vaccine-eligible people is made, in context, but there is no clear connection made to the confidence interval, *OR* a clear connection to the confidence interval is made, but the response includes one or both of the following two omissions:

1. The response is not in context.
2. The response does not mention the confidence level of 99 percent.

Incorrect (I) if the response fails to meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if an appropriate sample size is calculated and supporting work is shown.

Partially correct (P) if supporting work is shown, *BUT* the response includes one or both of the following errors:

1. 0.41617 (the sample proportion) or 0.45 is used instead of 0.5.
2. An incorrect critical z-value is used — unless the same incorrect value was used in part (a).

Incorrect (I) if the response fails to meet the criteria for E or P.

Notes

- In this situation, the formula for margin of error used to compute sample size is only an approximation of the margin of error. Because of this, we will not insist that the computed sample size be rounded up; that is, 4,147 is scored as E, as long as supporting work is shown.
- If the critical value of 2.575 is used, then the sample size should be $n \geq 4{,}144.14$ or $n = 4{,}145$ (or 4,144).
- If the final recommended sample size is not an integer, then the response can earn no more than a P.

Question 5 (continued)

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 2½ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

5. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people had received flu vaccine.

(a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.

One sample Z confidence interval for proportion is appropriate to the test.
Conditions for constructing the confidence interval are met since Random Sampling is given, and $np = 978$, $n(1-p) = 2350 - 978 = 1372$ is both above 10, so the distribution will be approximately normally distributed.

Calculator gives, 99 percent confidence interval $(0.39, 0.4424)$

$ME = 0.026192$

Based on this sample, we are 99 percent confident that the true proportion of vaccine-eligible people who had received flu vaccine is between $(0.39, 0.4424)$

Since 0.45 is not contained in the interval, we can reject the null hypothesis that 45 percent of vaccine-eligible people received flu vaccine. In other words, we have statistically significant evidence that the proportion of vaccine-eligible people who had received flu vaccine is not 45 percent

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If you need more room for your work for part (a), use the space below.

(b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 ?

Margin of Error in 99 percent confidence interval

$$= 2.576 \times \sqrt{\frac{0.5 \times 0.5}{n}} \leq 0.02$$

$$4147.36 \leq n$$

So, at least 4148 sample size can be used to guarantee that the margin of error will be less than or equal to 0.02.

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5B1 5B1

5. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people had received flu vaccine. $\frac{978}{2,350} =$

(a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.

$n = 2,350$

$x = 978$

$\frac{978}{2,350} = .41617$

- The sample is random.
- 2350 is more than 10% of the population

→ The conditions are met so we can perform a one proportion z interval test.

→ I am 99% confident that the proportion of vaccine-eligible people who had received flu vaccine are between .38998 to .44236. 38.9% to 44.2%

→ The interval of people that receive this flu is less than 45% so the belief is wrong.

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If you need more room for your work for part (a), use the space below.

(b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 ?

$$0.02 \leq 2.57 \times \sqrt{\frac{(0.41617)(0.58383)}{n}}$$

$$n = 4012.023 \rightarrow$$

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5. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people had received flu vaccine.

   (a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.

Use 1 proportion Z interval to find the 99% confidence interval.

$\hat{p} = 0.41617$ C Lower = 0.389979 C Upper = 0.442362

Within the 99% confidence interval we conclude that the proportion of vaccine-eligible people who had recieved flu vaccine is between 39% to 44%. 45 percent is not included in between 39% to 44% so we cant say that 45% of vaccine-eligible people recieved flu vaccine.

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If you need more room for your work for part (a), use the space below.

(b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 ?

0.2475.

$$(2.57583)\frac{0.45}{n} = 0.507978$$

at least a number of 200.

Question 5

Sample: 5A
Score: 4

Part (a) correctly identifies the procedure to be a "[o]ne sample z confidence interval for [a] proportion." The response notes that random sampling is given in the statement of the problem and includes appropriate checks of the sample-size conditions based on the observed counts of successes and failures. Step 1 of the solution to part (a) was scored as essentially correct. In step 2 of part (a) the mechanics are shown and are correct for the calculation of the interval identified in step 1, so step 2 of the solution to part (a) was scored as essentially correct. After stating that the interval is a 99 percent confidence interval, the response notes that "0.45 is not contained in the interval," and so it is not reasonable to believe that 45 percent of the vaccine-eligible people had received flu vaccine. The solution to step 3 of part (a) was scored as essentially correct. The solution to part (b) incorporates 2.576, the correct critical value for a 99 percent confidence interval based on a normal distribution, and also includes 0.5 as the value of the proportion. The conservative choice of 0.5 guarantees that the margin of error based on the resulting sample size calculation will be no greater than 0.02. Part (b) was scored as essentially correct. Because the three steps in part (a) were scored as essentially correct and part (b) was also scored as essentially correct, the response earned a score of 4.

Sample: 5B
Score: 3

This response notes that "[t]he sample is random" but also that "2350 is more than 10% of the population." This sample-size restriction is both incomplete (because it does not restrict the numbers of successes and failures) and incorrect (because it should read "less than" rather than "more than"). The procedure is identified as a "one proportion z interval test." Because of the incorrect sample-size condition, step 1 of part (a) was scored as partially correct. Because the correct "one proportion z interval test" procedure is named, and the correct interval is provided, step 2 of part (a) was scored as essentially correct. The response then compares 0.45 to the resulting confidence interval by indicating that the interval of people that receive this flu vaccine is less than 45 percent. The response includes context and a direct comparison of 0.45 to the interval, so step 3 of part (a) was scored as essentially correct. The response states that "the belief is wrong," which is not necessarily correct, although the evidence indicates that 0.45 is not a plausible value for the true proportion. In part (b) the response substitutes the observed proportion 0.41617 into the formula for the margin of error. The observed proportion is not as conservative as using the value 0.5, so the resulting sample size does not guarantee that the margin of error will be no greater than 0.02. Part (b) was scored partially correct. Because one step in part (a) was scored as essentially correct, and part (b) was scored as essentially correct, while two steps in part (a) were scored as partially correct, the response earned a score of 3.

Sample: 5C
Score: 2

The solution to part (a) includes no consideration of conditions for inference, so step 1 of part (a) was scored as incorrect. The correct procedure ("1 proportion z interval") is named, and the correct interval is calculated. Step 2 of part (a) was thus scored as essentially correct. The response notes that 45 percent is not included in the interval and that "we can't say that 45% of vaccine-eligible people received flu vaccine." This response was scored essentially correct for step 3 of part (a). The solution in part (b) does not provide a formula, includes 0.45 in the numerator of the standard error instead of the more statistically conservative value 0.5, and does not include the square root in the standard error. Therefore, part (b) was scored as incorrect. Because two steps in part (a) were scored as essentially correct, one step in part (a) was scored as incorrect, and part (b) was scored as incorrect, the response earned a score of 2.