

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 4

#### **Intent of Question**

The primary goals of this question were to assess a student's ability to (1) describe why the median might be preferred to the mean in a particular context; (2) compare the relative merits of two sampling plans; and (3) describe a consequence of nonresponse in a particular study.

#### **Solution**

##### **Part (a):**

The median is less affected by skewness and outliers than the mean. With a variable such as income, a small number of very large incomes could dramatically increase the mean but not the median. Therefore, the median would provide a better estimate of a typical income value.

##### **Part (b):**

Method 2 is better than Method 1. A sample obtained from Method 1 could be biased because of the voluntary nature of the response. It is plausible that class members with larger incomes might be more likely to return the form than class members with smaller incomes. The mean income for such a sample would overestimate the mean income of all class members. With Method 2, despite the smaller sample size, the random selection is likely to result in a sample that is more representative of the entire class and produce an unbiased estimate of mean yearly income of all class members.

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### Question 4 (continued)

#### **Scoring**

This question is scored in three sections. Part (b) has two components: (1) identifying a relevant characteristic for each sampling method; (2) indicating the effect of the biased method on the estimate of the mean income. Section 1 consists of Part (a); section 2 consists of part (b), component 1; and section 3 consists of part (b), component 2. Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Describes how skewness or outliers affect the mean or do not affect the median.
2. Makes a conjecture about a relevant characteristic of the distribution of incomes, such as skewness or an outlier.

Partially correct (P) if the response includes only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

*Notes:*

- For Component 1, examples of responses that are acceptable include:
  - The mean is affected by skewness (outliers).
  - The median is not affected by skewness (outliers).
  - The mean is greater (less) than the median when there is right (left) skewness or outliers.
- For Component 1, examples of responses that are not acceptable include:
  - Don't use the mean for skewed distributions or distributions with outliers.
  - Use the median for skewed distributions or distributions with outliers.
  - Responses that include an incorrect statement about means and/or medians, such as for right skewed distributions, the median will be higher than the mean.
- It is possible to satisfy *both* components with a single sentence, such as, "If there was a billionaire in the sample, the mean would be higher than the median."
- If a response argues that using the *mean* is a more appropriate way to estimate the typical income, then reduce the score in section 1 by one level (that is, from E to P or from P to I).

**Section 2** is scored as follows:

Essentially correct (E) if the response chooses Method 2 *AND* includes the following two components:

1. Identifies a relevant characteristic of Method 1.
2. Identifies a relevant characteristic of Method 2.

Partially correct (P) if the response chooses Method 2 *AND* includes only one of the two components listed above

*OR*

if the response includes both components but does not choose a method.

Incorrect (I) if the response chooses Method 1 *OR* otherwise does not meet the criteria for E or P.

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### Question 4 (continued)

Notes:

- Responses that do not explicitly choose Method 2 can still earn an E for section 2 if the choice is clearly implied. The choice of Method 2 is clearly implied if the response only discusses negative characteristics of Method 1 and only discusses positive characteristics of Method 2, such as, Method 1 is biased but Method 2 uses a random sample.
- Responses that *compare* the two methods can satisfy *both* components, such as, saying that Method 1 is more biased or that nonresponse will be less of a problem with Method 2.
- Responses that refer to the nonresponse bias as *voluntary response bias*, *response bias*, *undercoverage* can still earn an E.
- Discussions of conditions for inference should be considered extraneous and ignored.

**Section 3** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Indicates the incomes of responders may be different from the incomes of nonresponders.
2. Indicates the biased sampling method may produce a misleading estimate/conclusion about the mean income, including direction, for example, “The sample mean is likely to be higher than the mean of the population.”

Partially correct (P) if the response provides only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- A single sentence can satisfy the first component of section 2 and the first component of section 3. (For example, “In method 1, rich people are more likely to respond.”)
- For component 2, either direction is acceptable but the direction must be consistent with the identified bias. Saying only that Method 2 will be more accurate or more representative does not satisfy component 2.
- If a response addresses possible nonresponse bias in Method 2, the response can still satisfy both components of section 3.
- Responses that focus on the larger sample size in Method 1 can satisfy component 2 if such responses describe the effect as reducing the variability of the estimate. (For example, “I would use Method 1 since the larger sample size would give less variability of the mean.”)
- Responses that focus on untruthful survey answers can satisfy component 2 if the effect on the estimate is appropriate. (For example, “People contacted in Method 2 might say they make more money than they actually do. This would make the estimated mean too high.”)

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**Question 4 (continued)**

**4 Complete Response**

All three sections essentially correct

**3 Substantial Response**

Two sections essentially correct and one section partially correct

**2 Developing Response**

Two sections essentially correct and one section incorrect

*OR*

One section essentially correct and one or two sections partially correct

*OR*

Three sections partially correct

**1 Minimal Response**

One section essentially correct and two sections incorrect

*OR*

Two sections partially correct and one section incorrect

4. As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be \$189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was \$189,952!"

- (a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

Since income is probably right skewed (less likely to get a high paying job), the mean is greater than the median. Therefore, the median is a better predictor of the estimate of the typical income because it is not affected by outliers, but the mean is affected by outliers.

- (b) The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.

Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.

Method 2: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.

Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

I would use method 2 for estimating the average yearly income of all 6,826 members of the class of 1988 because method 2 is a simple random sample, so the estimate from method 2 will have less bias and will be closer to the true average yearly income. Since method 1 is a voluntary response, only people who are motivated <sup>enough</sup> will complete the online form. I believe that this will cause the estimate from method 1 to be higher than the true average yearly income because people with high paying jobs want to brag about their success and are more likely than people with low paying jobs to complete the online form.

4. As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be \$189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was \$189,952!"

- (a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

if you used the median of the reported incomes, alumni who have no job or a small income could less affect the statistic.

- (b) The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.

Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.

Method 2: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.

Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

I would select method 2 because it would reduce bias in the survey. Method 1 is not as favorable because there is a risk of voluntary bias. Alumni who have a low salary may be embarrassed to respond as well as alumni with higher salaries may want to respond. Method 2 would give a more accurate answer because the alumni are selected at random; however, there is a smaller sample size.

4. As part of its twenty-fifth reunion celebration, the class of 1988 (students who graduated in 1988) at a state university held a reception on campus. In an informal survey, the director of alumni development asked 50 of the attendees about their incomes. The director computed the mean income of the 50 attendees to be \$189,952. In a news release, the director announced, "The members of our class of 1988 enjoyed resounding success. Last year's mean income of its members was \$189,952!"

- (a) What would be a statistical advantage of using the median of the reported incomes, rather than the mean, as the estimate of the typical income?

The median would be more resistant to outliers compared to the mean as means are NOT resistant to outliers.

- (b) The director felt the members who attended the reception may be different from the class as a whole. A more detailed survey of the class was planned to find a better estimate of the income as well as other facts about the alumni. The staff developed two methods based on the available funds to carry out the survey.

Method 1: Send out an e-mail to all 6,826 members of the class asking them to complete an online form. The staff estimates that at least 600 members will respond.

Method 2: Select a simple random sample of members of the class and contact the selected members directly by phone. Follow up to ensure that all responses are obtained. Because method 2 will require more time than method 1, the staff estimates that only 100 members of the class could be contacted using method 2.

Which of the two methods would you select for estimating the average yearly income of all 6,826 members of the class of 1988? Explain your reasoning by comparing the two methods and the effect of each method on the estimate.

Method 2. Although  $n$  is smaller, method 1 suffers from nonresponse bias and certain income level alumni may be more likely to respond. Method 2 is also large enough to satisfy the conditions for inference.

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## 2014 SCORING COMMENTARY

### Question 4

#### Overview

The primary goals of this question were to assess a student's ability to (1) describe why the median might be preferred to the mean in a particular context; (2) compare the relative merits of two sampling plans; and (3) describe a consequence of nonresponse in a particular study.

#### Sample: 4A

##### Score: 4

In section 1 the response begins by stating that the distribution of income is probably right skewed, which satisfies component 2. This statement is followed by a correct description of how the skewness makes the mean greater than the median, which satisfies component 1. The response continues to provide other facts about how outliers affect the mean and median, which would also satisfy component 1. Because the response correctly describes a statistical advantage of using the median of the reported incomes, section 1 was scored as essentially correct. In section 2 the response correctly chooses Method 2, identifies a relevant characteristic of Method 1 ("method 1 is voluntary response"), and identifies a relevant characteristic of Method 2 ("method 2 is a simple random sample"). Because the response picks the better sampling method and addresses relevant characteristics of both methods, section 2 was scored as essentially correct. In section 3 the response describes how the incomes of responders are different from the incomes of nonresponders ("people with high paying jobs...are more likely...to complete the online form"), satisfying component 1. The response also describes the effect of the bias, with direction ("cause the estimate from Method 1 to be higher than the true average"), satisfying component 2. Because the response correctly describes the effect of nonresponse on the sample and on the estimate, section 3 was scored as essentially correct. Because all three sections were scored as essentially correct, the response earned a score of 4.

#### Sample: 4B

##### Score: 3

In section 1 the response identifies possible low outliers in the distribution of incomes ("alumni who have no job or a small income"), satisfying component 2. In the same sentence, the response also indicates that these low outliers could "less affect" the median, satisfying component 1. Because the response correctly describes a statistical advantage of using the median of the reported incomes, section 1 was scored as essentially correct. In section 2 the response correctly selects Method 2 and addresses both methods with a comparison ("reduce bias"), satisfying both components in the first sentence. The response goes on to discuss both methods separately, which would also satisfy both components. Because the response picks the better sampling method and addresses relevant characteristics of both methods, section 2 was scored as essentially correct. In section 3 the response identifies how the incomes of responders might be different than the incomes of nonresponders ("alumni who have a low salary may be embarrassed to respond...alumni with higher salaries may want to respond"), satisfying component 1. However, the response does not address the effect of the bias on the estimate. Saying only that the answer will be "more accurate" does not provide a direction. Because the response satisfies only one component, section 3 was scored as partially correct. Because two sections were scored as essentially correct and one section was scored as partially correct, the response earned a score of 3.



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**Question 4 (continued)**

**Sample: 4C**

**Score: 2**

In section 1 the response provides a generic statement about how medians are less affected by outliers than means, satisfying component 1. However, because the response does not include any conjecture about the distribution of incomes, component 2 is not satisfied. Because the response satisfies only one component, section 1 was scored as partially correct. In section 2 the response correctly selects Method 2, identifies a relevant characteristic of Method 1 (“suffers from nonresponse bias”), and acknowledges the adequate sample size for Method 2. However, sample size isn’t considered a relevant characteristic such as random sampling or lack of bias. Because the response satisfies only one component, section 2 was scored as partially correct. In section 3 the response describes how the incomes of responders are different from the incomes of nonresponders (“certain income level alumni may be more likely to respond”), satisfying component 1. However, the response does not address the effect of the bias on the estimate. Because the response satisfies only one component, section 3 was scored as partially correct. Because three sections were scored as partially correct, the response earned a score of 2.