AP® STATISTICS
2015 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) use the information provided by a scatterplot to describe the relationship between two quantitative variables; (2) interpret and use the information given by lines displayed on a scatterplot; and (3) use a regression equation to estimate a predicted value of y for a given x value.

Solution

Part (a):

There is a moderately strong, positive, linear relationship between height and arm span so that taller students tend to have longer arm spans.

Part (b):

(i) The line in Graph 2 is the one that is helpful. For each student, the graph illustrates whether arm span is equal to height (points on the line), arm span is greater than height (points above the line), or arm span is less than height (points below the line).

(ii)

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	3	4	5

Part (c):

The predicted arm span is $\hat{y} = 11.74 + 0.8247x = 11.74 + 0.8247(61) = 62.05$ inches.

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes the following four components:
1. The relationship is approximately linear.
2. The relationship is positive.
3. There is a moderate to strong association (or relationship).
4. The response is given in context.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

Question 5 (continued)

Notes:

- Correlation alone is not sufficient to indicate a linear relationship. However, linear correlation is sufficient to indicate a linear relationship.
- Reporting that the correlation coefficient is 0.81, the correlation coefficient is close to 1.0, or some other value or range of values for the correlation coefficient is not sufficient to satisfy component 3.

Part (b) is scored as follows:

Essentially correct (E) if the response contains the following four components:

1. The line in Graph 2 is selected.
2. A reasonable explanation for selecting the line in Graph 2 that links the body shapes to the regions defined by the line is given. The explanation should indicate that square shapes (arm span is equal to height) are represented by points on the line, tall rectangle shapes (arm span less than height) are represented by points below the line, and short rectangle shapes (arm span is greater than height) are represented by points above the line.
3. Correct counting of body types demonstrated by reporting correct frequencies (3, 4, and 5) or reporting correct proportions $\left(\frac{1}{4} = \frac{3}{12}, \frac{1}{3} = \frac{4}{12}, \text{ and } \frac{5}{12}\right)$ for the square, tall rectangle, and short rectangle body types, respectively.
4. The correct frequencies (3, 4, and 5) are reported in the table.

Partially correct (P) if the response includes only two or three of the four components.

Incorrect (I) if the response includes at most one of the four components.

Notes:

- Selecting the regression line on Graph 1 cannot satisfy either component 1 or component 2.
- To satisfy component 1, it is sufficient to refer to the $y = x$ line without explicitly mentioning Graph 2.
- Use of incorrect labels, such as regression line or least-squares regression line, in referring to the $y = x$ line in Graph 2 is an incorrect use of terminology that should be strongly discouraged, but it is ignored in this scoring rubric.
- The explanation for selecting Graph 2 is acceptable if it explicitly includes at least two of the following: (i) square body shapes (arm span equal to height) are represented by points *on* the line, (ii) tall rectangle body shapes (arm span less than height) are represented by points *below* the line, or (iii) short rectangle body shapes (arm span greater than height) are represented by points *above* the line.
- Incorrectly counting the points on the $y = x$ line in Graph 2 is considered a minor error. Frequencies (2, 4, and 5) are accepted for both component 3 and component 4. Reporting corresponding proportions $\left(\frac{2}{11}, \frac{4}{11}, \frac{5}{11}\right)$ or $\left(\frac{2}{12}, \frac{4}{12}, \frac{5}{12}\right)$ in the table satisfies component 3 but does not satisfy component 4.
- Frequencies reported in the table as (2, 5, 4) or (3, 5, 4), interchanging the counts for tall and short rectangle body shapes, satisfies component 3, but does not satisfy component 4. Reporting corresponding proportions, $\left(\frac{2}{11}, \frac{5}{11}, \frac{4}{11}\right)$ or $\left(\frac{3}{12}, \frac{5}{12}, \frac{4}{12}\right)$, in the table does not satisfy either component 3 or component 4.

Question 5 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response contains the following three components:
1. Correct formula for predicting arm span with 61 inserted for x.
2. Correct value for the predicted arm span.
3. Units for the predicted arm span given as inches.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- Any value for the predicted arm span between 61.5 and 62.5 is acceptable; values outside the interval do not satisfy component 2. This allows for inaccuracy in obtaining a value for the predicted arm span directly from the line displayed on Graph 1 and for rounding in applying the prediction formula; but it excludes predictions based on the $y = x$ line and other unreasonable predictions.
- Reporting a prediction equation that is similar to the equation given in the stem, with 61 inserted for x, satisfies components 1 and 2 if the value of the predicted arm span is between 61.5 and 62.5 inches. Otherwise, neither of those two components is satisfied. This could occur if a student enters the data from the graph into a calculator to compute the least squares regression line, instead of using the formula provided in the stem.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and two parts partially correct

OR

One part essentially correct, one part partially correct, and one part incorrect

OR

Three parts partially correct

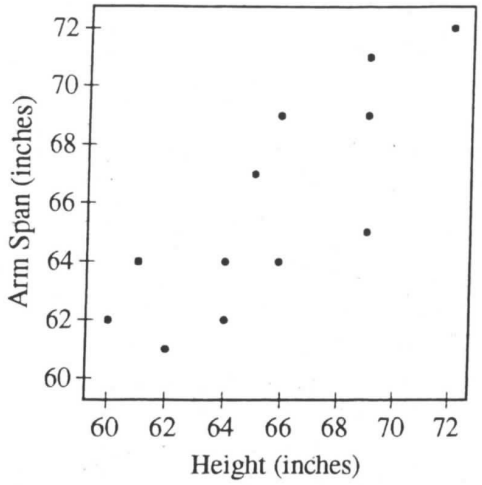
1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

5. A student measured the heights and the arm spans, rounded to the nearest inch, of each person in a random sample of 12 seniors at a high school. A scatterplot of arm span versus height for the 12 seniors is shown.



(a) Based on the scatterplot, describe the relationship between arm span and height for the sample of 12 seniors.

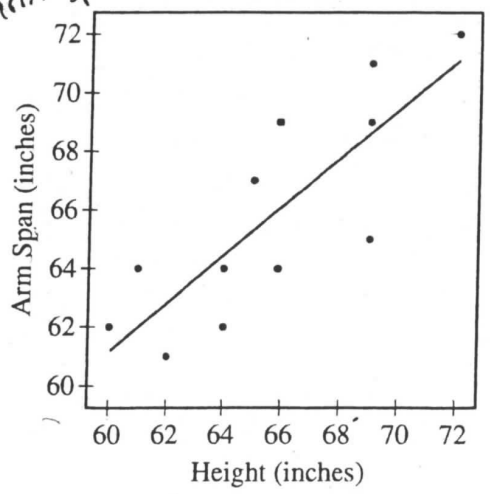
Based on this scatterplot, there appears to be a moderate, positive linear relationship between arm span and height for the sample of 12 seniors.

Let x represent height, in inches, and let y represent arm span, in inches. Two scatterplots of the same data are shown below. Graph 1 shows the data with the least squares regression line $\hat{y} = 11.74 + 0.8247x$, and graph 2 shows the data with the line $y = x$.

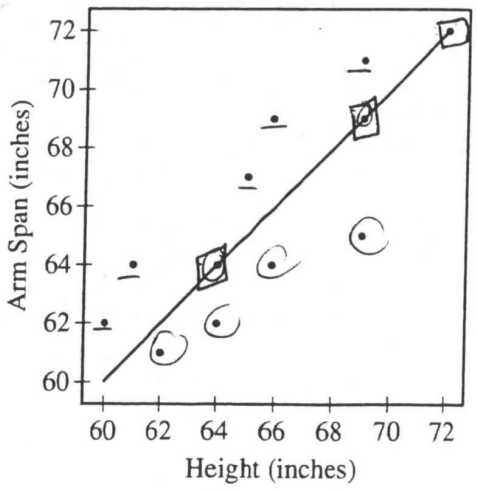
$\hat{y}$ = predicted arm span

x = height.

GRAPH 1



GRAPH 2



GO ON TO THE NEXT PAGE.

(b) The criteria described in the table below can be used to classify people into one of three body shape categories: square, tall rectangle, or short rectangle.

Square	Tall Rectangle	Short Rectangle
Arm span is equal to height.	Arm span is less than height.	Arm span is greater than height.

(i) For which graph, 1 or 2, is the line helpful in classifying a student's body shape as square, tall rectangle, or short rectangle? Explain.

Graph 2 is helpful for identifying a student's body shape as square, tall rectangle or short rectangle. The line shown is for when the arm span and height are the same (a square body type). The points above the line show that arm span is greater than height (a short rectangle) and the points below show that arm span is less than height (a tall rectangle).

(ii) Complete the table of classifications for the 12 seniors.

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	3	4	5

(c) Using the best model for prediction, calculate the predicted arm span for a senior with height 61 inches.

$\hat{y} = 11.74 + .8247(x)$

$\hat{y} = 11.74 + 0.8247(61)$

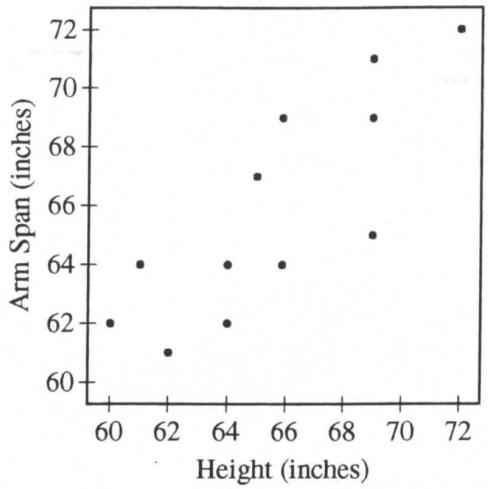
$\hat{y} = 62.0467$ inches.

$\hat{y}$ = predicted arm span

x = height

The predicted arm span for a senior with a height of 61 inches is 62.0467 inches.

5. A student measured the heights and the arm spans, rounded to the nearest inch, of each person in a random sample of 12 seniors at a high school. A scatterplot of arm span versus height for the 12 seniors is shown.

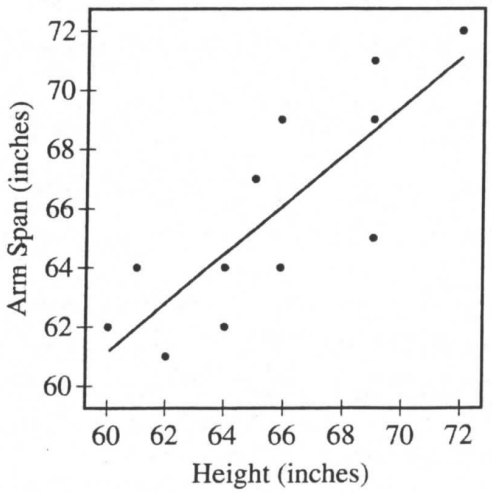


(a) Based on the scatterplot, describe the relationship between arm span and height for the sample of 12 seniors.

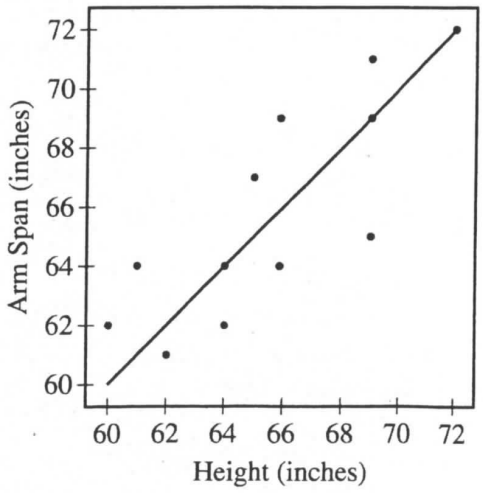
There is a fairly strong, positive, linear relationship between armspan and height of these 12 seniors.

Let x represent height, in inches, and let y represent arm span, in inches. Two scatterplots of the same data are shown below. Graph 1 shows the data with the least squares regression line $\hat{y} = 11.74 + 0.8247x$, and graph 2 shows the data with the line $y = x$.

GRAPH 1



GRAPH 2



(b) The criteria described in the table below can be used to classify people into one of three body shape categories: square, tall rectangle, or short rectangle.

Square	Tall Rectangle	Short Rectangle
Arm span is equal to height.	Arm span is less than height.	Arm span is greater than height.

(i) For which graph, 1 or 2, is the line helpful in classifying a student's body shape as square, tall rectangle, or short rectangle? Explain.

graph 2. By showing the line $y=x$ you can determine by the placement of the point whether the arms or height are longer and/or if the lengths are equal, graph 1 just displays a best fit line.

(ii) Complete the table of classifications for the 12 seniors.

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	3	4	5

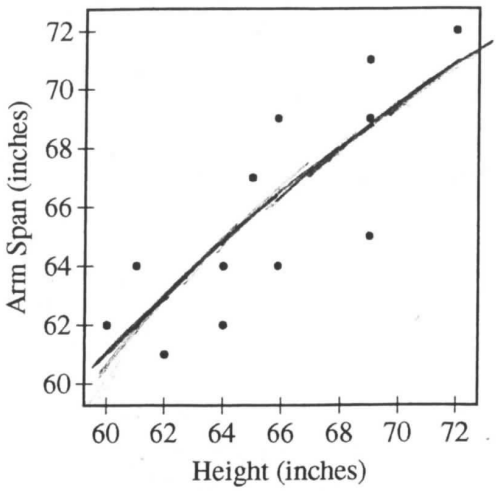
(c) Using the best model for prediction, calculate the predicted arm span for a senior with height 61 inches.

$\hat{y}$ = predicted arm span $\quad \hat{y} = 11.74 + .8247x$

x = height $\quad \hat{y} = 11.74 + .8247(61)$

A girl with a 61" height should have an armspan of 62.0467 inches according to the least squares regression line.

5. A student measured the heights and the arm spans, rounded to the nearest inch, of each person in a random sample of 12 seniors at a high school. A scatterplot of arm span versus height for the 12 seniors is shown.

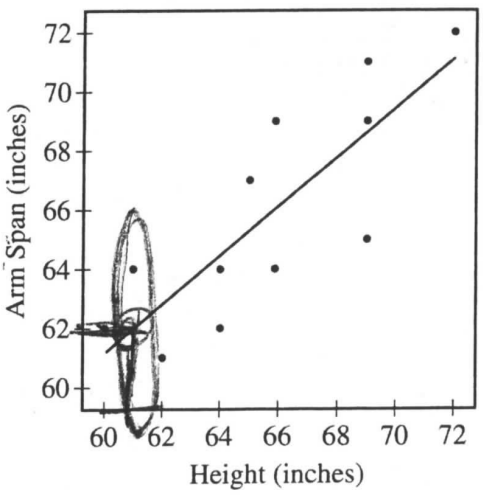


(a) Based on the scatterplot, describe the relationship between arm span and height for the sample of 12 seniors.

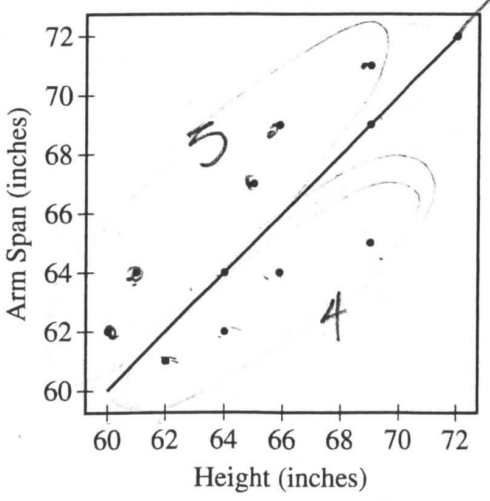
As height(in) increases the length of arm span (in.) increases. There is a positive correlation between height (in.) and length of arm span (in.).

Let x represent height, in inches, and let y represent arm span, in inches. Two scatterplots of the same data are shown below. Graph 1 shows the data with the least squares regression line $\hat{y} = 11.74 + 0.8247x$, and graph 2 shows the data with the line $y = x$.

GRAPH 1



GRAPH 2



GO ON TO THE NEXT PAGE.

(b) The criteria described in the table below can be used to classify people into one of three body shape categories: square, tall rectangle, or short rectangle.

Square	Tall Rectangle	Short Rectangle
Arm span is equal to height.	Arm span is less than height.	Arm span is greater than height.

(i) For which graph, 1 or 2, is the line helpful in classifying a student's body shape as square, tall rectangle, or short rectangle? Explain.

Graph 2 is helpful in classifying a student's body shape because you can clearly see where the arm span is equal to the height and when the arm span is greater/less than the height (in.).

(ii) Complete the table of classifications for the 12 seniors.

Classification	Square	Tall Rectangle	Short Rectangle
Frequency	1/4 / 0.25	1/3 / 0.33	5/12 / 0.41666

(c) Using the best model for prediction, calculate the predicted arm span for a senior with height 61 inches.

Using graph 1 the predicted arm span for a senior with height of 61 inches would be approximately 62 inches.

AP® STATISTICS
2015 SCORING COMMENTARY

Question 5

Overview

The primary goals of this question were to assess a student's ability to (1) use the information provided by a scatterplot to describe the relationship between two quantitative variables; (2) interpret and use the information given by lines displayed on a scatterplot; and (3) use a regression equation to estimate a predicted value of y for a given x value.

Sample: 5A
Score: 4

In part (a) the response correctly describes the form, direction, and strength of the relationship in the context of arm span and height of students. The response contains all four of the required components, and part (a) was scored as essentially correct. The response correctly selects the line in Graph 2 in part (b-i) and clearly describes how the body shape categories are distinguished by that line. Correct frequencies are reported in the table in part (b-ii). Because all four of the required components are satisfied, part (b) was scored as essentially correct. The response in part (c) presents the correct prediction formula with a value of 61 inserted for the height variable. The predicted arm span is correctly evaluated with units of measurement reported in inches. All three of the required components are satisfied, and part (c) was scored as essentially correct. Because all three parts were scored as essentially correct, the response earned a score of 4.

Sample: 5B
Score: 3

The response in part (a) correctly describes the form, direction, and strength of the relationship in the context of arm span and height of students. The response contains all four of the required components, and part (a) was scored as essentially correct. The response in part (b-i) correctly selects the $y = x$ line in Graph 2, but the explanation for selecting that line does not adequately describe how the $y = x$ line distinguishes the three body type categories. A complete explanation must explicitly describe how the $y = x$ line partitions the graph into three regions corresponding to the three body shape categories. This can be done, for example, by indicating that students with arm span equal to height (square body shape) are represented by points on the line, students with arm span less than height (tall rectangle body shape) are represented by points below the line, and students with arm span greater than height (short rectangle body shape) are represented by points above the line. Correct frequencies are reported in the table in part (b-ii). Consequently, the response in part (b) includes three of the four required components and part (b) was scored as partially correct. The response in part (c) presents the correct prediction formula with a value of 61 inches inserted for height. The correct value for the predicted arm span is reported in the concluding sentence of the response to part (c) with units of measurement identified as inches. Part (c) was scored as essentially correct. Because two parts were scored as essentially correct, and one part was scored as partially correct, the response earned a score of 3.

Question 5 (continued)

Sample: 5C
Score: 2

The response in part (a) is stated in the context of a relationship between arm span and height, and it indicates a positive relationship. With respect to the scoring rubric, the word "correlation" by itself is not sufficient to describe a linear relationship. Furthermore, the response does not address the strength of the relationship. Consequently, the response to part (a) satisfies only two of the four required components, and part (a) was scored as partially correct. The response selects Graph 2 in part (b-i) and indicates that the line on Graph 2 allows one to clearly see where arm span is equal to height and where arm span is greater or less than height. The explanation is incomplete because it does not explicitly describe how the line distinguishes the three body shape categories. To be complete, the explanation must explicitly indicate that square body shapes (arm span is equal to height) are represented by points on the line, tall rectangle body shapes (arm span greater than height) are represented by points below the line, and short rectangle body shapes (arm span less than height) are represented by points above the line. In part (b-ii) the proportions reported in the table provide evidence that body shapes were counted correctly. Proportions are not frequencies, however, and the component for reporting frequencies is not satisfied by reporting proportions. Consequently, the response contains two of the four required components and part (b) was scored as partially correct. The response in part (c) reports an acceptable value for the predicted arm span with inches given as the units of measurement, but the response simply refers to Graph 1 instead of showing how the least-squares regression line is used to predict arm span for a given height. The formula for the least-squares regression line, with a value of 61 inserted for the height variable, must be displayed to make this clear and complete the response. The response satisfies only two of the required components and part (c) was scored as partially correct. Because three parts were scored as partially correct, the response earned a score of 2.