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AP Statistics
Sample Student Responses and Scoring Commentary

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Intent of Question

The primary goals of this question were to assess a student's ability to use boxplots to (1) compare multiple sets of data; (2) identify which set of data is most likely to have produced a particular summary value; and (3) determine which variable is most useful for classifying a new observation.

Solution

Part (a):

The median value for the percent of chemical Z in the pottery pieces is similar for all three sites, at about 7 percent. The ranges for the percent of chemical Z are much different for the three sites, with the smallest range of about 2 percent (from 6 percent to 8 percent) at Site II, a range of about 6 percent (from about 4 percent to 10 percent) at Site I, and the largest range of about 8 percent (from about 3 percent to 11 percent) at Site III.

Part (b):

(i) The piece most likely originated at Site III. Although values outside of the range of data observed in the samples would be possible, using the available data results in approximate minimum and maximum sums of the percents for the three chemicals as shown in the table below. Site III is the only site in which 20.5 falls between the sums of the minimum and maximum values.

	Site I		Site II		Site III	
Chemical	Min	Max	Min	Max	Min	Max
X	6	8	5	7	5	7.5
Y	11	15	1.9	4	6	8
Z	4	10	6	8	3	11
Sum	21	33	12.9	19	14	26.5

(ii) Chemical Y would be most useful, because the distribution of the percentages of total weights at the three sites do not overlap. The distributions of chemicals X and Z have substantial overlap.

Scoring

This question is scored in three sections. Section 1 consists of part (a), section 2 consists of part (b-i), and section 3 consists of part (b-ii). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Recognition that the medians or centers are almost the same for the three sites
2. Recognition that the variability (ranges, IQRs, spread) is different across the three sites
3. Context is included

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- In all sections, comments about shape should be ignored because complete shape information is not obtainable from boxplots.
- Responses are not required to give numerical values. If responses provide numerical values, any reasonable approximation from the boxplots is acceptable.
- Because the boxplots are all symmetric, it is acceptable if the response discusses means instead of medians.
- Any discussion of chemical X and chemical Y is considered extraneous.
- Context is satisfied by any of the following references: site, chemical, weight, total weight, X, Y, or Z.

Section 2 is scored as follows:

Essentially correct (E) if the response includes the following three components:
1. Site III is chosen.
2. Sums of the minimum and maximum are computed for the three chemicals at each site.
3. A reasonable numerical justification is given involving sums of a statistical measure across the three chemicals to choose Site III.

Partially correct (P) if the response includes only two of the three components.

Incorrect (I) if the response includes at most one of the three components.

Notes:
- If the response computes only the sum of the minimums for Site I and the sum of the maximums for Site II and recognizes that this is sufficient, the response is scored E.
- If an alternative measure is used that involves sums of the three chemicals, such as the sum of the medians or the sums of the first quartiles and sums of the third quartiles, instead of the minimum and maximum sums, the second component is not satisfied, but the third component might be satisfied.
    - If the response explicitly or implicitly compares the alternate sum to the other two sites (for example, by indicating that the sum is the closest to 20.5 percent or by listing the sums for all three sites) the response is scored P.
    - If the response does not have an implicit or explicit comparison, the response is scored I.
- If either Site I or Site II is identified as the correct choice, no matter how that choice is justified, the response is scored I.
- The approximate sums of the medians are 27.5 for Site I, 16 for Site II, and 20 for Site III.

Section 3 is scored as follows:

Essentially correct (E) if the response chooses chemical Y *AND* gives a reasonable justification based on the fact that the distributions of chemical Y are distinctive across sites.

Partially correct (P) if the response chooses chemical Y *AND* provides justification based on the boxplots, but does not clearly explain that the distributions of chemical Y are distinctive across sites;
    OR
if the response correctly discusses that the distributions of chemical Y are distinctive across sites, but never explicitly chooses chemical Y as the best choice, for instance, by stating only that there is substantial overlap across sites for chemicals X and Z but no overlap for chemical Y.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- To justify that the distributions of chemical Y are distinctive across sites, the justification must address both location and variability of the boxplots; for example, by stating that the boxplots do not overlap for chemical Y.
- If the response chooses chemical X or Z *OR* chooses chemical Y with no reasonable justification, the response is scored I.
- The justification that the distributions of chemical Y are distinctive across sites:
  - The following are acceptable because both location <u>and</u> variability are addressed. Such responses are scored E.
    - The boxplots for chemical Y do not overlap, or the boxplots for chemicals X and Z overlap.
    - All values of Site I are high, all values of Site II are low, and all values of Site III are in the middle.
    - The ranges never intersect.
    - The boxplots share no data.
    - Has completely different percentages at each site.
  - The following are incomplete justifications and are scored P.
    - The boxplots vary.
    - Chemical Y varies the most.
    - Chemical Y has the greatest variation.
    - The variation between/among sites is the largest.
    - The boxplots are different.
    - The medians/means differ.
    - The medians/means are most variable.
    - There is a difference in the percentages of chemical Y for each site.
    - The distribution of percents differs the most among the sites.

4   **Complete Response**

      Three sections essentially correct

3   **Substantial Response**

      Two sections essentially correct and one section partially correct

2   **Developing Response**

      Two sections essentially correct and no sections partially correct

OR

      One section essentially correct and one or two sections partially correct

OR

      Three sections partially correct
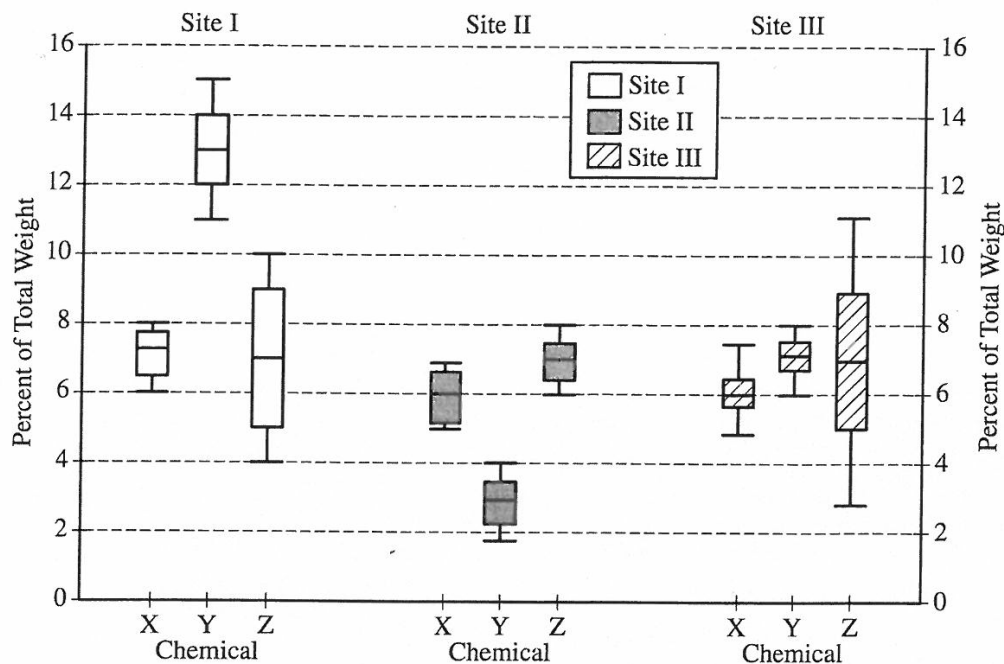
1   **Minimal Response**

      One section essentially correct

OR

      No sections essentially correct and two sections partially correct

4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.

(a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.

Similarities

For all three sites, the <u>median</u> percent of the total weight of ~~chemical Z is~~ the clay ~~that is ch~~ for chemical Z is about 7%. ~~For~~ Sites I and III also have similar <u>interquartile ranges</u> of about 4%.

Differences

For chemical Z, each site differs in the <u>range</u> of the percentages of the total weight that is chemical Z: Site I's range is about 6%, Site II's is about 2%, and Site III's is about 8%.

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(b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.

(i) Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.

Based on the boxplots, Site III is the most likely site where the piece of pottery originated.
For Site I, ~~the range of~~ X+Y+Z is between 21% and 33% so achieving a sum of 20.5% is unlikely.

For Site II, X+Y+Z is between 12.5% and 17%, so a piece of pottery with a sum of 20.5% would be even more unlikely.
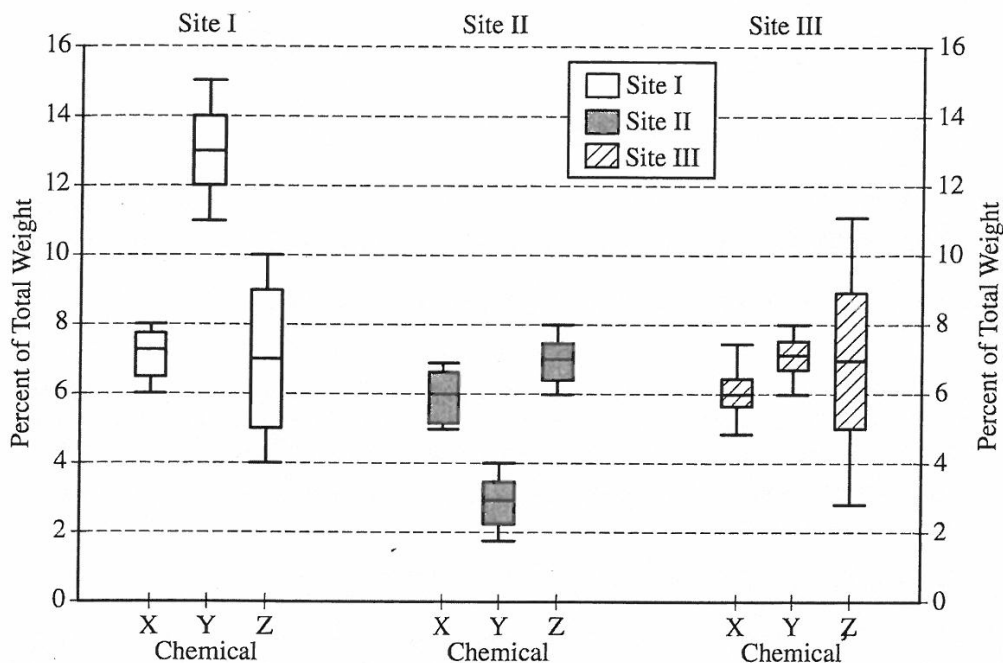
At Site III, X+Y+Z could be between 14% and 26%, making a sum of 20.5% most likely because it fits in the range of previous pottery percentages.

Thus, the pottery most likely originated from Site III.

(ii) Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z—would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

If only one chemical in the piece of ~~pottery could~~ pottery could be analyzed, chemical Y would be most useful in determining the pottery's site of origination. At Site I, chemical Y is ~~betwee~~ 11-15% of the total weight of the pottery, at Site II it is about 1.5-4% of the weight, and at Site III it is ~~be~~ 6-8% of the weight. Since these ranges do not overlap as they do for chemicals X and Z, the site of the pottery's origination can be located based on which range of percentages of the total weight the chemical Y percent fits in.

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4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.

(a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.

The median percent of chemical Z in pottery is very similar across all three sites, at around 7%. However, the range of Z differs between the sites. Site II has a smaller IQR ~~for Z~~ for Z than Site I or Site III, whose IQRs are similar. Site II also has the smallest range for chemical Z, followed by Site I and Site III, which has the largest range. These observations are from the graph of boxplots.

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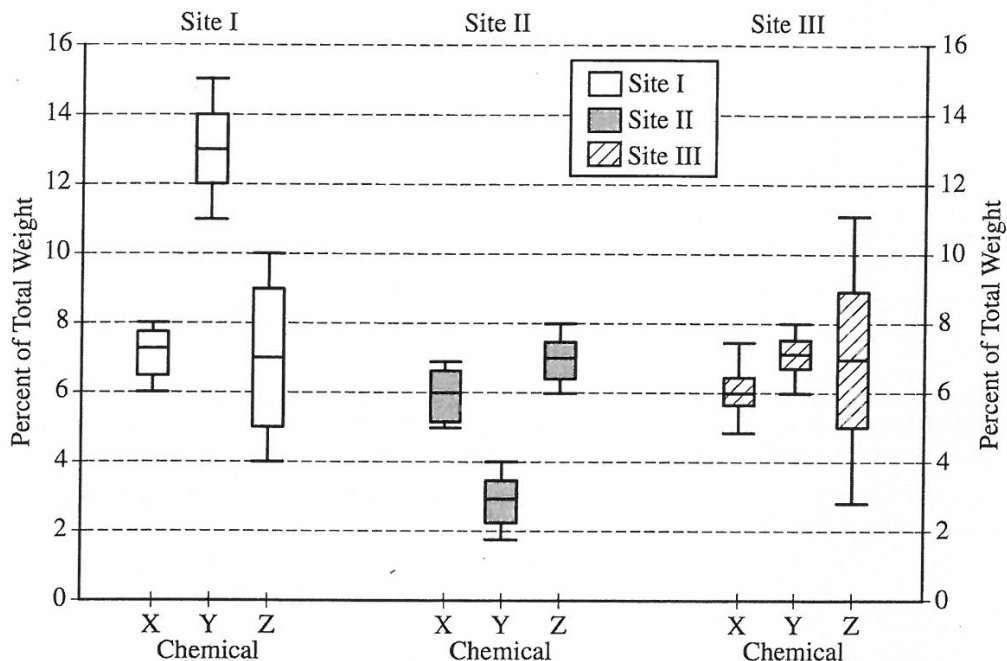(b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.

(i) Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.

Using the median chemical percentages, Site III is the most likely site of this piece of pottery. The sum of the median values in Site III for Chemicals X, Y, and Z, respectively, are around 6, 7, and 7, which sum to 20%. This is the closest to the value of 20.5% using the medians, as Site I would give 7.5 + 13 + 7 = 27.5% and Site II would give around 6 + 3 + 7 = 16% for the percentage sums. These values were obtained from the graph of boxplots.

(ii) Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z—would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

Chemical Y would be the most useful in site identification if only one chemical could be analyzed. The graph of boxplots show that only chemical Y has significantly different possible percentages across the three sites. Y is present in 11-15% in Site I, 2-4% in Site II, and 6-8% in Site III, allowing for easy identification. Chemicals X and Z have overlaps in their ranges across the sites, ruling out definite identification based on those chemicals.

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4. The chemicals in clay used to make pottery can differ depending on the geographical region where the clay originated. Sometimes, archaeologists use a chemical analysis of clay to help identify where a piece of pottery originated. Such an analysis measures the amount of a chemical in the clay as a percent of the total weight of the piece of pottery. The boxplots below summarize analyses done for three chemicals—X, Y, and Z—on pieces of pottery that originated at one of three sites: I, II, or III.

(a) For chemical Z, describe how the percents found in the pieces of pottery are similar and how they differ among the three sites.

The distribution of percents found in the pieces of pottery for chemical Z are all approximately Normal among the three sites, with similar medians as well. However site III has the largest range of percentages for chemical Z, while site II has the smallest range. Site I and III have similar interquartile ranges for chemical Z while site II has a smaller IQR than the other two sites. There are no outlier percentages for chemical Z among the three sites.

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(b) Consider a piece of pottery known to have originated at one of the three sites, but the actual site is not known.

(i) Suppose an analysis of the clay reveals that the sum of the percents of the three chemicals X, Y, and Z is 20.5%. Based on the boxplots, which site—I, II, or III—is the most likely site where the piece of pottery originated? Justify your choice.

It is most likely that the piece of pottery originated in Site III. This is because if we add the approximate median percents of all three chemicals at each of the sites, the sum of the median chemical percents at site III is the closest to 20.5%.

Site I   sum ≈ 7 + 13 + 7 ≈ 27

Site II sum ≈ 6 + 3 + 7 = 16

Site III sum ≈ 6 + 7 + 7 = 20

(ii) Suppose only one chemical could be analyzed in the piece of pottery. Which chemical—X, Y, or Z—would be the most useful in identifying the site where the piece of pottery originated? Justify your choice.

Chemical Y would be the most useful in identifying where a piece of pottery originates because the boxplot of the percents for chemical Y vary the most among the three sites, while the boxplots for chemicals X and Z are more uniform throughout the sites. Since chemical Y covers the extremes of the range of percent of total weight in pottery across all three sites, one could easily determine the origin of an unknown piece of pottery using only chemical Y.

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Overview

The primary goals of this question were to assess a student's ability to use boxplots to (1) compare multiple sets of data; (2) identify which set of data is most likely to have produced a particular summary value; and (3) determine which variable is most useful for classifying a new observation.

Sample: 4A
Score: 4

In section 1 the response clearly displays the intended similarities and differences by labeling each. The response states that the median for chemical Z for all three sites is about 7 percent and satisfies both components 1 and 3. The response adds that the interquartile ranges of chemical Z for sites I and III are similar. The statement is true, but the question asked for a similarity among the three sites (not two). Finding a similarity for just sites I and III would not have been sufficient to satisfy component 1. The response then lists the three different ranges for chemical Z across sites. Simply listing the ranges would not have satisfied component 2 in itself; however, the response clearly labels the listing as a difference, thus, component 2 is satisfied. Because the three components are satisfied, section 1 was scored as essentially correct. In section 2 site III is correctly identified as the most likely site of origin of the piece of pottery, satisfying component 1. The response clearly provides the sum of the minimums and the sum of the maximums of the three chemicals for each site. The calculations that lead to the sums are not required, but the numerical sums must be stated. Component 2 is satisfied. The response justifies the choice of site III with an explanation that site III is the most likely site because 20.5 percent falls within the interval of 14 percent, the minimum sum, to 26 percent, the maximum sum. Component 3 is satisfied. Because three components are satisfied, section 2 was scored as essentially correct. In section 3 chemical Y is correctly chosen as the chemical most useful in identifying the site of origin of the piece of pottery, and the choice has a complete justification. Complete justification is provided because the response states the intervals of the minimums and maximums for each site and then states that these intervals do not overlap. The response refers to these intervals as "ranges," which is an incorrect usage of a statistical term. However, the reference is ignored because range is not in the intent of the question. The response also notes that the corresponding intervals for chemicals X and Z do overlap. The justification is considered complete and has clear communication. Because the correct chemical is chosen with complete justification, section 3 was scored as essentially correct. Because three sections were scored as essentially correct, the response earned a score of 4.

Sample: 4B
Score: 3

In section 1 the response states that the median of chemical Z is similar across all three sites, thus satisfying components 1 and 3. The response continues to explain that there are two differences: the interquartile ranges differ across sites, and the ranges differ across sites. Only one difference about variability across sites is required and either satisfies component 2. Because all three components are satisfied, section 1 was scored as essentially correct. In section 2 the response satisfies component 1 by correctly choosing site III as the mostly likely site of origin of the piece of pottery. Component 2 is not satisfied because the sum of the minimums and the sum of the maximums of the three chemicals for each site are not calculated. An alternative measure involving all three chemicals for each site is used, as the response calculates the sum of the medians for each chemical across the three sites. To justify the choice of site III, the response states that the sum of the medians for site III is closest to 20.5 percent, compared with the sums from sites I and II. Because an alternative measure is used and compared to sites I and II, component 3 is satisfied. Two of the three components in section 2 are satisfied, so section 2 was scored as partially correct.

In section 3 chemical Y is chosen as the most useful chemical to identify the site of origin of the piece of pottery and justification based on the boxplots is provided. For each site, the interval from the minimum to the maximum for chemical Y is provided. The response notes that the intervals are different for chemical Y across the three sites. The response also notes that the corresponding intervals for chemicals X and Z overlap across the sites. The justification is complete and section 3 was scored as essentially correct. Because two sections were scored as essentially correct, and one section was scored as partially correct, the response earned a score of 3.

Sample: 4C
Score: 2

In section 1 the response states that the distribution for chemical Z across all three sites is approximately normal with similar medians. Because the response states, in context, that the medians are similar, components 1 and 3 are satisfied. The comment that the distributions of chemical Z are approximately normal across sites is ignored. Complete information about shape cannot be determined from a boxplot. A symmetric boxplot could come from a normal distribution, a uniform distribution, or a bimodal distribution, for example. In the case of chemical Z the shape of the distribution is unknown. Component 2 is satisfied because the response states that site III has the largest range and site II has the smallest range. Finding a difference between two sites is acceptable for stating a difference across three sites. Because three components are satisfied, section 1 was scored as essentially correct. In section 2 the response correctly identifies site III as the most likely site of origin of the piece of pottery and component 1 is satisfied. The sums of the minimums and the sums of the maximums of the three chemicals for each site are not calculated, and component 2 is not satisfied. An alternative measure, the sums of the medians, is calculated for all three sites. The response compares the sum of the medians for site III to sites I and II and states that 20.5 percent is closest to the sum of the medians for site III. The response uses a reasonable numerical justification that involves the sum of a statistical measure from three chemicals across three sites, so component 3 is satisfied. Because two of the three components for section 2 are satisfied, section 2 was scored as partially correct. In section 3 chemical Y is identified as the most useful chemical in identifying the site of origin of the piece of pottery, but the choice has incomplete justification. The response states that the "percents for chemical Y vary the most among the three sites." Stating that the percents are the most variable does not ensure that the three boxplots do not overlap; that is, the percents could vary and still overlap. The response also refers to chemical Y as having the "extremes of the range of percent of total weight in pottery across all three sites." Range is a single number, but it appears as though this response considers range to be a span of numbers. Boxplots with large ranges could still overlap or not overlap, so it is not clear from the response that there is an understanding that chemical Y is the best choice because the boxplots do not overlap. Because the justification is incomplete for section 3, this section was scored as partially correct. Because one section was scored as essentially correct, and two sections were scored as partially correct, the response earned a score of 2.