2018

AP CollegeBoard

AP Statistics

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 3

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compute a probability based on a weighted mixture of two populations; (2) compute a conditional probability; and (3) recognize a binomial random variable and compute the probability associated with it.

Solution

Part (a):

Let L denote left-handed, M denote multiple birth, and S denote single birth.
The probability that a randomly selected child born in the region is left-handed is:

$$P(L) = P(M)P(L \mid M) + P(S)P(L \mid S) = (0.035)(0.22) + (0.965)(0.11) = 0.0077 + 0.10615 = 0.11385.$$

Part (b):

From part (a), $P(L) = 0.11385.$ Therefore,

$$P(M \mid L) = \frac{P(L \text{ and } M)}{P(L)} = \frac{(0.035)(0.22)}{0.11385} = \frac{0.0077}{0.11385} \approx 0.0676.$$

Part (c):

Let X represent the number of children who are left-handed in a random sample of 20 children from the region. X has a binomial distribution with $n = 20$ and $p = 0.11385$ (found in part (a)). Using the binomial distribution,

$$P(X \geq 3) = 1 - P(X \leq 2)$$
$$= 1 - \binom{20}{0}(0.11385)^0(0.88615)^{20} - \binom{20}{1}(0.11385)^1(0.88615)^{19} - \binom{20}{2}(0.11385)^2(0.88615)^{18}$$
$$\approx 1 - 0.598 \approx 0.402.$$

Question 3 (continued)

Scoring

Parts (a), (b), and (c) each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the probability is computed correctly, *AND* work is shown that includes correct numerical values using a formula, end results from a tree diagram, or some other appropriate strategy.

Partially correct (P) if the response provides a reasonable strategy for finding the probability, such as a formula or tree diagram, but uses one or more inappropriate values;

OR

if the response gives the correct probability but not enough work is shown to determine how that probability was found.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: A reasonable strategy needs to include summing the results of two multiplications.

Part (b) is scored as follows:

Essentially correct (E) if the probability is computed correctly, with work shown that includes appropriate numerical values for both the numerator and denominator.

Partially correct (P) if the response includes a numerator and denominator in calculating the conditional probability, with one appropriate term (numerator or denominator) and the other inappropriate.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: Appropriate values include incorrectly calculated values from part (a).

Question 3 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following five components:

1. Uses a calculation based on the binomial distribution to find the probability of the number of children in the sample who are left-handed.
2. Specifies appropriate values for n and p.
3. Uses correct endpoint value for the probability.
4. Uses correct direction to calculate the probability of at least three left-handed children.
5. Correctly calculates a binomial probability consistent with the previous work.

Partially correct (P) if the response satisfies component 1 and only two or three of the other four components;
OR
if components 2, 3, 4, and 5 are met, and the response does not explicitly indicate the binomial distribution is used by name or formula.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- "Appropriate" values include incorrectly calculated values from part (a) or a recalculated probability from part (b).
- An unlabeled numerical value in a calculator statement cannot be used to satisfy a component.
- A response which calculates $P(X \leq 3)$ satisfies component 3 but does not satisfy component 4.
- A normal approximation to the binomial is not appropriate because $np = 20 \times 0.11385 = 2.277 < 5$.

  A response using the normal approximation can score at most P. To earn a score of P, the response must include all of the following:
  - a correct mean and standard deviation based on the binomial parameters
  - clear indication of boundary and direction with a z-score or diagram
  - the probability computed correctly

Notes for all parts:

- If the resulting probability or part of the calculation of the probability uses a value that is not between 0 and 1, inclusive, the score for that part is lowered by one level (that is, from E to P, or from P to I).
- An arithmetic or transcription error in a response can be ignored if correct work is shown. For example, $0.0077 + 0.10615 = 0.1385$.

Question 3 (continued)

4 **Complete Response**

Three parts essentially correct

3 **Substantial Response**

Two parts essentially correct and one part partially correct

2 **Developing Response**

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 **Minimal Response**

One part essentially correct

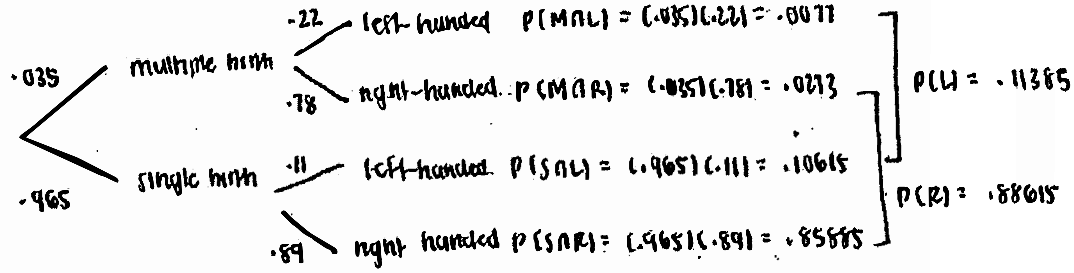
OR

No parts essentially correct and two parts partially correct

3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.

(a) What is the probability that a randomly selected child born in the region is left-handed?

Let M = multiple L = left
S = single R = right



P (left handed) = [(.035)(.22)] + [(.965)(.11)]

P (left-handed) = .1139

(b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?

* refer to tree above

$$P(M|L) = \frac{P(M \cap L)}{P(L)}$$

$$P(M|L) = \frac{.0077}{.1139}$$

P(M|L) = .0676

GO ON TO THE NEXT PAGE.

3A2

(c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

$n=20 \quad p(L) = .1134$

$p(X=K) = \binom{n}{K} p^K (1-p)^{n-K}$

$p(X \geq 3) = 1 - [(p(X=0)) + p(X=1) + p(X=2)]$

$p(X=0) = \binom{20}{0} (.1134)^0 (.8861)^{20} = .0891$

$p(X=1) = \binom{20}{1} (.1134)^1 (.8861)^{19} = .2284$

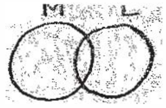
$p(X=2) = \binom{20}{2} (.1134)^2 (.8861)^{18} = .2796$

$p(X \geq 3) = 1 - (.0891 + .2284 + .2796)$

$p(X \geq 3) = .4024$

Unauthorized copying or reuse of any part of this page is illegal.

GO ON TO THE NEXT PAGE.



P(M∩L) = .22   P(S∩L) = .11

3B1

.035 M   P(M)

3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed. P(L)

(a) What is the probability that a randomly selected child born in the region is left-handed?

$P(L) = .22 + .11 = .33$

The probability a randomly selected child is left handed is .33.

(b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?

$$P(M|L) = \frac{P(M \cap L)}{P(L)} = \frac{.22}{.33} = .6667$$

The probability a child is left handed given they're from a multiple birth is .6667

GO ON TO THE NEXT PAGE.

3B2

(c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

trials prob X

$P(L \geq 3) = 1 - P(L<2) = \text{binomcdf}(20, .067, 2)$

2.26×10^{-7}

The probability that out of 20 children in the region at least three are left-handed is 2.26×10^{-7}

GO ON TO THE NEXT PAGE.

3. Approximately 3.5 percent of all children born in a certain region are from multiple births (that is, twins, triplets, etc.). Of the children born in the region who are from multiple births, 22 percent are left-handed. Of the children born in the region who are from single births, 11 percent are left-handed.

(a) What is the probability that a randomly selected child born in the region is left-handed?

$\hat{p}_A = .22$

$\hat{p}_B = .11$ $\quad 0.22 + 0.11 = 0.33$

The probability that a randomly selected child born in this region is left handed is 0.33 or 33%.

(b) What is the probability that a randomly selected child born in the region is a child from a multiple birth, given that the child selected is left-handed?

$P(A|B)$ $\quad P(A) = .035$, $P(B) = .33$ $\quad P(A \cap B) = .035(0.33) = .012$

Where P(A) is the probability that the child is from a multiple birth and P(B) is the probability that the child is left handed.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.012}{0.33} = .036$$

The probability that a randomly selected child born in the region is a child from a multiple birth given that the child selected is left handed is .036 or 3.6%.

GO ON TO THE NEXT PAGE.

3C2

(c) A random sample of 20 children born in the region will be selected. What is the probability that the sample will have at least 3 children who are left-handed?

$\binom{20}{3}$ 20 nCr 3

$1140(.33)^3(1-.33)^{17} = .045$

The probability that a random sample of 20 children born in the region will result in at least 3 children who are left handed is .045 or 4.5%.

Unauthorized copying or reuse of any part of this page is illegal.

GO ON TO THE NEXT PAGE.

Question 3

Overview

The primary goals of this question were to assess a student's ability to (1) compute a probability based on a weighted mixture of two populations; (2) compute a conditional probability; and (3) recognize a binomial random variable and compute the probability associated with it.

Sample: 3A
Score: 4

In part (a) the response has a well-labeled tree diagram with the events and corresponding probabilities on the branches. The multiplication is shown at the end of each branch using correct notation, and the symbols are defined above the tree. The response clearly indicates which branches have been used to find $P(L)$ at the end of the tree and then summarizes the arithmetic below the tree. Part (a) was scored as essentially correct. In part (b) the same symbols used in part (a) are found. The formula for the conditional probability is given using the symbols defined in part (a). The probabilities, as found in the tree and corresponding to the symbols, are given in the numerator and denominator of the fraction. The final probability is correctly calculated. Part (b) was scored as essentially correct. In part (c) the response indicates how the probability of x equal to k is found. This statement satisfies component 1. The response clearly indicates the value for n and the probability of being left-handed as found in part (a). These statements satisfy component 2. The response gives the statement $p(x \geq 3) = 1 - [(p(x = 0)) + p(x = 1) + p(x = 2)]$, which includes the correct value for the endpoint of the cumulative distribution and the direction used to calculate the probability, satisfying components 3 and 4, respectively. Although not necessary, each of these calculations is shown. The final probability is correctly calculated, satisfying component 5. Part (c) was scored as essentially correct. Because the three parts were scored as essentially correct, the response earned a score of 4.

Sample: 3B
Score: 2

In part (a) the response does not weight the conditional probability of left-handed children of multiple births by the probability that a child is from a multiple birth nor does the response weight the conditional probability of the left-handed children of single births by the probability the child is from a single birth. Because this is not a reasonable strategy to find a weighted probability, part (a) was scored as incorrect. In part (b) the correct formula for a conditional probability is given. The incorrect value for $P(M \cap L)$ from part (a) is used and is identified at the top of the page. The incorrect value for $P(L)$ from part (a) is used. Because the response clearly identifies the quantities and carries the quantities from part (a), the quantities were scored as appropriate as specified in the note for part (b) in the scoring guidelines. The statement "The probability a child is left handed given they're from a multiple birth" was overlooked in the scoring. Part (b) was scored as essentially correct. In part (c) the response uses the calculator notation for the function binomcdf(20, .667, 2) and labels the parameters. Component 1 is satisfied with the function statement. Labeling 20 as trials satisfies the n portion of component 2. However, the value labeled as "prob" is the value labeled in part (b) as "probability a child is left handed given they're from a multiple birth." In part (a) the response defines the "probability a randomly selected child is left-handed is .33." In part (c) the response specifies binomcdf(20,.667,2) and labels the parameters, satisfying component 1. The probabilities defined in parts (a) and (b) are used and component 2 is satisfied. Component 3 is satisfied with the statement $P(L \geq 3)$. There is poor notation in the equation as $1 - P(L < 2)$ should equal $1 - (\text{binomcdf}(20, .667, 2)$, but that was overlooked. The calculator notation does indicate the correct endpoint for the complement and the correct direction. Component 4 is satisfied. The calculation is incorrect for the given p, so component 5 is not satisfied. Because component 1 and two of the other four components are satisfied, part (c)

Question 3 (continued)

was scored as partially correct. Because one part was scored as essentially correct, one part was scored as partially correct, and one part was scored as incorrect, the response earned a score of 2.

Sample: 3C
Score: 1

In part (a) the response does not weight the conditional probability of left-handed children of multiple births by the probability that a child is from a multiple birth nor does the response weight the conditional probability of the left-handed children of single births by the probability the child is from a single birth. Because this is not a reasonable strategy to find a weighted probability, part (a) was scored as incorrect. In part (b) the probability $P(B)$ calculated in part (a) is used. The probability that a child is left-handed and from a multiple birth is incorrect because these events are not independent. Because the numerator of the conditional probability is incorrect, part (b) was scored as partially correct. In part (c) the binomial distribution is attempted with the formula given, satisfying component 1. Because the combination $\binom{20}{3}$ is standard notation, component 3 is satisfied, and the portion of component 2 requiring the identification of n is satisfied. The portion of component 2 requiring p is satisfied because the probability from part (a) is used in the binomial formula. The probability of $x = 3$ is calculated correctly, satisfying component 5. However, no direction is given, and component 4 is not satisfied. Because component 1 and three of the other four components are satisfied, part (c) was scored as partially correct. Because two parts were scored as partially correct, and one part was scored as incorrect, the response earned a score of 1.