2019

AP CollegeBoard

AP® Statistics

Sample Student Responses and Scoring Commentary

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AP® STATISTICS
2019 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) evaluate a percentile of a normal distribution; (2) evaluate a probability for a normal distribution; and (3) compute an expected value for a random variable with two possible outcomes.

Solution

Part (a):

The 25th percentile of the standard normal distribution is -0.6745. Consequently the 25th percentile of a normal distribution with mean 30 months and standard deviation 8 months is $30 + 8(-0.6745) = 24.6$ months.

Part (b):

The probability that a randomly selected customer will need to request a replacement because the battery fails within 24 months from the date of purchase is

$$P(\text{life span} \le 24 \text{ months}) = P\left(Z \le \frac{24 - 30}{8}\right) = P(Z \le -0.75) \approx 0.2266.$$

Part (c):

The company's expected gain for each warranty purchased is

$$\begin{aligned}&(\$50) \times P(\text{life span} > 24 \text{ months}) + (-\$150) \times P(\text{life span} \le 24 \text{ months})\\ &= (\$50) \times (0.7734) + (-\$150) \times (0.2266) \approx \$4.68.\end{aligned}$$

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:

1. Indicates the use of a normal distribution with a mean of 30 and a standard deviation of 8.
2. Sets up a correct approach for finding the 25th percentile of the battery life span distribution.
3. Reports the correct value of the 25th percentile (24.6) or a percentile value that is consistent with components 1 and 2.

Partially correct (P) if the response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Component 1 may be satisfied in either part (a) or (b).
- Incorrect statistical notation in specifying the distribution mean or standard deviation (for example, $\bar{x} = 30$ or $s = 8$) results in Component 1 not being met the first time it appears.

- Component 1 may be satisfied by one of the following:
  - Graphical: Displaying a graph of a normal density function with the horizontal axis clearly labeled using the mean and standard deviation for the battery life span distribution.
  - Calculator syntax: Labeling the mean and standard deviation in an inverse normal cdf calculator statement, for example, invNorm$(0.25, \mu = 30, \sigma = 8)$.
  - z-score: Showing correct components in a standard z-score calculation, for example, $z = \frac{x - 30}{8}$, or $-0.6745 = \frac{x - 30}{8}$, or $x = 30 + 8(-0.6745)$.
  - Notation: Using standard notation for a normal distribution, for example, $N(30, 8)$ or Normal$(30, 64)$.
- Component 2 may be satisfied by one of the following:
  - Graphical: Identifying the lower-tail area corresponding to the probability of 0.25 in a graph of a normal density function.
  - Calculator syntax: Stating the correct percentile in an inverse normal cdf calculator statement, for example, invNorm$(0.25, \mu = 30, \sigma = 8)$.
  - z-score: Equating the z-score of the 25th percentile of the battery life span distribution to -0.6745, for example, $-0.6745 = \frac{x - 30}{8}$ or $x = 30 + 8(-0.6745)$.

Part (b) is scored as follows:

Essentially correct (E) if the response satisfies the following three components:
1. Indicates the use of a normal distribution with a mean of 30 and a standard deviation of 8.
2. Specifies the correct event, including the correct boundary value and direction.
3. Reports the correct probability (0.2266) or a probability consistent with components 1 and 2.

Partially correct (P) if response satisfies only two of the three components.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:
- Component 1 may be satisfied in either part (a) or (b).
  - Incorrect statistical notation in specifying the distribution mean or standard deviation (for example, $\bar{x} = 30$ or $s = 8$) results in Component 1 not being met the first time it appears.
- Component 1 may be satisfied by one of the following:
  - Graphical: Displaying a graph of a normal density function with the horizontal axis clearly labeled using the mean and standard deviation for the battery life span distribution.
  - Calculator syntax: Labeling the mean and standard deviation in a normal cdf calculator statement, for example, normalcdf$(0, \text{upper} = 24, \mu = 30, \sigma = 8)$.
  - z-score: Showing correct components in a standard z-score calculation, for example, $z = \frac{x - 30}{8}$, or $z = \frac{24 - 30}{8}$, or $\frac{24 - 30}{8} = -0.75$.
  - Notation: Using standard notation for a normal distribution, for example, $N(30, 8)$ or Normal$(30, 64)$

Question 5 (continued)

- Component 2 may be satisfied by one of the following:
  - Graphical: A normal density graph labeled with boundary and direction corresponding to the region of interest.
  - Calculator syntax: Labeling the upper bound in a normal cdf calculator statement. The lower bound does not need to be labeled but must be less than or equal to 0, for example, normalcdf(0, upper = 24, $\mu = 30$, $\sigma = 8$).
  - In words: Specifying the correct event, boundary value, and direction, in words: for example, P(battery life span $\leq$ 24 months), or "probability requires battery replacement within 24 months."
  - Random variable: Specifying the boundary value and direction using a random variable, for example, $P(X \leq 24)$ or $P(Z \leq -0.75)$.

Part (c) is scored as follows:

Essentially correct (E) if the response satisfies the following two components:

1. Provides an expected value calculation with two appropriate probabilities that add to 1 and are paired with the correct outcomes.
2. Reports the correct expected value (4.68), or a reasonable expected value consistent with work shown.

Partially correct (P) if the response satisfies only one of the two components.

Incorrect (I) if the response does not satisfy the criteria for E or P.

Notes:

- For component 1, appropriate probabilities that add to 1 are:
  - probabilities consistent with the value computed in part (b); or
  - the correct probabilities; or
  - if there is no probability provided in part (b), probabilities explicitly defined in context in part (c).
- If the correct probability is used, due to rounding considerations, any number between 4.00 and 4.70 is acceptable as a correct expected value for component 2.
- A reasonable expected value for component 2 can be any number between $-\$150$ and $+\$50$ that is consistent with work shown.

Question 5 (continued)

4 Complete Response

Three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct

OR

No parts essentially correct and two parts partially correct

5A1 5A1

5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of $50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.

(a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.

let X = number of months a battery lives

$X \rightsquigarrow N(30, 8)$

invNorm(.25) = $-.67449$

$z = -.67449 = \frac{x - 30}{8}$

$x = 24.6$ months

(b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?

$X \rightsquigarrow N(30, 8)$

$P(x < 24)$

$P(z < \frac{24-30}{8}) = P(z < -.75)$

$= .2266$

5A2

5A2

(c) The company has a gain of $50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of $150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

Let Y = Amount of money gained by the company

Y	+50	-150
P(Y)	.7734	.2266

1-.2266=.7734

50 × .7734 = 38.6686

-150 × .2266 = -33.9941

38.6686 + -33.9941 = $4.67

GO ON TO THE NEXT PAGE.

5B1

5B1

5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of $50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.

(a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.

$z_{0.75} = 0.6745$

$0.6745 = \frac{x - 30}{8}$

$x = 35.3959$ months from date of purchase

(b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?

$X =$ life span of new battery)

$Y \sim N(30, 8)$

$P(X < 24) = 0.2266$

5B2

5B2

(c) The company has a gain of $50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of $150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

Events	$ Gain	probability
buy warrant but do not replacement	50	0.7734
buy warrant do not replacement	-150	0.2266

$50(0.7734) - 150(0.2266) = \boxed{\$4.6745}$

GO ON TO THE NEXT PAGE.

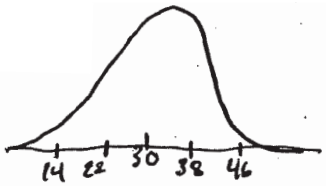
SC1

SC1

5. A company that manufactures smartphones developed a new battery that has a longer life span than that of a traditional battery. From the date of purchase of a smartphone, the distribution of the life span of the new battery is approximately normal with mean 30 months and standard deviation 8 months. For the price of $50, the company offers a two-year warranty on the new battery for customers who purchase a smartphone. The warranty guarantees that the smartphone will be replaced at no cost to the customer if the battery no longer works within 24 months from the date of purchase.

(a) In how many months from the date of purchase is it expected that 25 percent of the batteries will no longer work? Justify your answer.

q7



invnorm (.75, 30, 8) = 35.39.

So in about 38 months from purchase 25% of the batteries will no longer work

(b) Suppose one customer who purchases the warranty is selected at random. What is the probability that the customer selected will require a replacement within 24 months from the date of purchase because the battery no longer works?

$N(\mu = 30, \sigma = 8)$ => normalcdf (-1E99, 24, 30, 8) = 0.2266

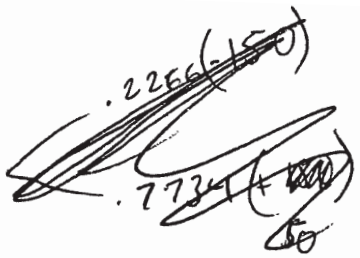
there is a 22.66% chance that the the customer selected will require a Replacement within 24 months from the date of purchase

5C2

5C2

(c) The company has a gain of $50 for each customer who purchases a warranty but does not require a replacement. The company has a loss (negative gain) of $150 for each customer who purchases a warranty and does require a replacement. What is the expected value of the gain for the company for each warranty purchased?

The expected value of gain would be



AP® STATISTICS
2019 SCORING COMMENTARY

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question are to assess a student's ability to (1) evaluate a percentile of a normal distribution; (2) evaluate a probability for a normal distribution; and (3) compute an expected value for a random variable with two possible outcomes.

This question primarily assesses skills in skill category 3: Using Probability and Simulation. Skills required for responding to this question include (3.A) Determine relative frequencies, proportions or probabilities using simulation or calculations and (3.B) Determine parameters for probability distributions.

This question covers content from multiple units, including Unit 1: Exploring One-Variable Data, Unit 4: Probability Rules, Random Variables, and Probability Distributions and Unit 5: Sampling Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 1.10, 4.8, and 5.2, and learning objectives VAR-2.B, VAR-5.C, and VAR-6.A.

Sample: 5A
Score: 4

In part (a) the response correctly identifies the use of a normal distribution with mean 30 and standard deviation 8 using standard statistical notation, satisfying component 1. An equation setting -0.67449 equal to an expression for the z-score of the unknown 25th percentile of the battery life span distribution satisfies component 2. The correct numerical answer for the 25th percentile of the battery life span distribution is provided, satisfying component 3. This response includes all three components; therefore, part (a) was scored as essentially correct.

In part (b) the response correctly identifies the use of a normal distribution with mean 30 and standard deviation 8 using standard statistical notation, satisfying component 1. The correct event, boundary value, and direction are provided using a probability statement in terms of a random variable, "$X < 24$," satisfying component 2; and the correct numerical answer is provided, satisfying component 3. The response includes all three components; therefore, part (b) was scored as essentially correct.

In part (c) the response provides a table with the two correct outcomes paired with the correct probabilities, and a correct calculation of expected value is provided in two pieces that are then added together, satisfying component 1. The correct numerical answer is provided satisfying component 2. The response includes both components; therefore, part (c) was scored as essentially correct.

Because three parts were scored as essentially correct, the response earned a score of 4.

Question 5 (continued)

Sample: 5B
Score: 3

In part (a) the response provides an expression for the *z*-score of the unknown 25th percentile of the battery life span distribution that uses the correct mean and standard deviation, satisfying component 1. The *z*-score expression is set equal to the *z*-score for the 75th percentile of a standard normal distribution, which is an incorrect approach, therefore component 2 is not satisfied. An incorrect numerical answer, but consistent with the work in components 1 and 2, is provided. Because the answer is consistent with prior work, it satisfies component 3. Because this response includes only two of the three components part (a) was scored as partially correct.

In part (b) the response correctly identifies the use of a normal distribution with mean 30 and standard deviation 8 using standard statistical notation, satisfying component 1. The correct event, boundary value, and direction are provided using a probability statement in terms of a random variable, "$X < 24$," satisfying component 2; and the correct numerical answer is provided, satisfying component 3. The response includes all three components; therefore, part (b) was scored as essentially correct.

In part (c) the response provides a table with the two correct outcomes paired with the correct probabilities. A correct expression for calculating the expected value is provided, satisfying component 1. The correct numerical answer is provided, satisfying component 2. The response includes both components; therefore, part (c) was scored as essentially correct.

Because two parts were scored as essentially correct, and one part was scored as partially correct the response earned a score of 3.

Sample: 5C
Score: 2

In part (a) the normal distribution graph provided with tick marks at the mean (30), at the mean ± 1 standard deviation, and at the mean $\pm$ 2 standard deviations satisfies component 1. The response provides a calculator statement "invnorm" with unlabeled arguments .75, 30, 8. The first argument .75 is incorrect, resulting in component 2 not being satisfied. Since the numerical answer provided is consistent with the work provided on component 2, component 3 is satisfied. Because the response includes only two of the three components, part (a) was scored as partially correct.

In part (b) the response correctly identifies the use of a normal distribution with mean 30 and standard deviation 8 using standard statistical notation, satisfying component 1. The calculator syntax " normalcdf(-1E99,24,30,8) " does not satisfy component 2 since the 24 is not labeled as an upper bound. However, boundary and direction are provided using the words "there is a 22.66% chance that the customer selected will require a replacement within 24 months from the date of purchase"; hence, component 2 is satisfied. The correct numerical answer is provided, satisfying component 3. The response includes all three components; therefore, part (b) was scored as essentially correct.

In part (c) the words provided in the response do not identify two probabilities and outcomes, resulting in component 1 not being satisfied. A numerical answer is not provided, resulting in component 2 not being satisfied. Because this response includes neither of the two components part (c) was scored as incorrect.

Because one part was scored as essentially correct, one part was scored as partially correct, and one part was scored as incorrect the response earned a score of 2.