2021

AP® CollegeBoard

AP® Statistics

Sample Student Responses and Scoring Commentary

Inside:

Free Response Question 3

☑ Scoring Guideline

☑ Student Samples

☑ Scoring Commentary

Question 3: Focus on Probability and Sampling Distributions — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) **(i)** Let the random variable of interest X represent the number of gift cards that a particular employee receives in a 52-week year. Because each employee has probability $\frac{1}{200} = 0.005$ of being selected each week to receive a gift card and each week's selection is independent from every other week, X has a binomial distribution with $n = 52$ repeated independent trials and probability of success $p = 0.005$ for each trial. **(ii)** The probability that a particular employee receives at least one gift card in a 52-week year is: $P(X \geq 1) = 1 - P(X = 0)$ $= 1 - \binom{52}{0}(0.005)^0(0.995)^{52}$ $= 1 - 0.7705$ $= 0.2295$	**Essentially correct (E)** if the response satisfies the following four components: 1. Defines the random variable as the number of gift cards that a particular employee receives in a 52-week year 2. Describes the distribution as binomial with $n = 52$ and $p = 0.005$ 3. Identifies the event of interest (i.e., identify the correct boundary AND direction for the event) in the calculation of the probability in part (a-ii) 4. Provides supporting work to identify the correct probability of 0.2295 (or 0.230, if rounded) OR a probability consistent with components 2 and 3 **Partially correct (P)** if the response satisfies only two or three of the four components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response that states $X \sim B(52, 0.005)$ satisfies component 2.
- A response that states the random variable is distributed by a distribution that is not binomial (e.g., normal or uniform) and then uses the binomial calculation does not satisfy component 2.
- Stating that gift cards are distributed randomly is not a distribution and does not, in itself, satisfy component 2. Component 2 can still be satisfied if the response goes on to use the binomial distribution.
- In order to satisfy component 2 using calculator function notation, the sample size and probability parameter must be clearly identified.
  - The following satisfy component 2:
    - binomcdf(n or trials = 52, $p = 0.005$, 1, 52)

        - 1 – binomcdf(n or trials = 52, $p = 0.005$, 0)
        - 1 – binompdf(n or trials = 52, $p = 0.005$, 0)
    - The following do not satisfy component 2 because the parameter or sample size is not clearly labeled:
        - binomcdf(52, 0.005, lower bound = 1, upper bound = 52)
        - 1 – binomcdf(52, $p = 0.005$, upper bound or $x = 0$)
        - 1 – binompdf(n or trials = 52, 0.005, $x = 0$)
- In order to satisfy component 3, the supporting work must identify the event of interest, i.e., $X \geq 1$, the boundary is 1, and the direction is greater than or equal to, or at least.
    - Possible ways to do this include:
        - Probability notation, e.g. $P(X \geq 1)$, $1 - P(X = 0)$
        - Summing probabilities, e.g. $\sum_{k=1}^{52} \binom{52}{k} (0.005)^k (0.995)^{52-k}$
        - Description in words $P(\text{employee receives at least one gift card})$, $1 - P(\text{employee receives no gift cards})$
        - Graphical, a bar graph of binomial probabilities with appropriate bars shaded
        - Using calculator function syntax with clearly labeled parameters (e.g. $p = 0.005$, $n = 52$) and clearly labeled event boundaries (e.g., lower bound = 1, upper bound = 52)
    - The following satisfy component 3:
        - binomcdf(n or trials = 52, $p = 0.005$, lower bound=1, upper bound = 52)
        - 1 – binomcdf(n or trials = 52, $p = 0.005$, upper bound or $x = 0$)
        - 1 – binompdf(n or trials = 52, $p = 0.005$, $x = 0$)
    - The following do not satisfy component 3 because the event boundaries are not clearly labeled.
        - binomcdf(n or trials = 52, $p = 0.005$, 1, 52)
        - 1 – binomcdf(n or trials = 52, $p = 0.005$, 0)
        - 1 – binompdf(n or trials = 52, $p = 0.005$, 0)
- A response will satisfy component 3 if the probability is computed for a geometric distribution with the first success within the first 52 weeks ($X \leq 52$) (e.g. response that states geometcdf(0.005, x or upper bound = 52)).
- Because $np = (52)(0.005) = 0.26$ is less than 5, the normal approximation to the binomial distribution is not an appropriate method to calculate the probability, and a response that uses this method does not satisfy component 4. However, a response that uses the normal approximation to the binomial distribution may satisfy component 3 if it displays the correct mean and standard deviation of the binomial distribution AND provides a clear indication of the appropriate collection of possible outcomes included in the event using a diagram or a **z**-score, e.g., $1 - P\left(Z \leq \frac{0 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$, $P\left(Z \geq \frac{1 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$, or $P\left(Z \geq \frac{0.5 - (52)(0.005)}{\sqrt{(52)(0.005)(0.995)}}\right)$.

Model Solution	Scoring
(b) The expected value for the number of gift cards a particular employee will receive in a 52-week year is $np = 52(0.005) = 0.26.$ If the random process of selecting one employee each week to receive a gift card is repeated for a very large number of years, each employee can expect to receive about 0.26 gift cards per year, on average, or about one gift card every four years.	**Essentially correct (E)** if the response satisfies the following two components: 1. Correctly calculates the expected value AND provides supporting work for the calculation of the expected value 2. Provides a reasonable interpretation of the expected value that includes *at least two* of the following three aspects: • The concept of repeating the selection process over a long period of time • The concept of an average or mean • The context of receiving gift cards **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A response may satisfy component 1 if the reported expected value is consistent with the distribution of the random variable identified in the response to part (a-i), AND supporting work for the calculation of the expected value in part (b) is shown.
- Examples of supporting work that satisfies component 1 include:
  - $np = 52(0.005) = 0.26$ or $np = 52(0.005)$
  - $np = \frac{52}{200}$
  - $52(0.005) = 0.26$
  - $np = 0.26,$ if the values of n and p are reported in the response to part (a)
- A response that incorrectly calculates the expected value may still satisfy component 2 using the incorrect expected value in the interpretation.

Model Solution	Scoring
(c) No, Agatha's experience does not constitute strong evidence that the selection process was not truly random. In fact, it is quite likely $(\text{probability} = (0.995)^{52} \approx 0.7705)$ that a particular employee will fail to receive a gift card for an entire 52-week year.	**Essentially correct (E)** if the response satisfies the following three components: 1. Indicates that Agatha does not have a strong argument that the selection process was not truly random 2. Provides a relevant probability or expected value 3. Provides an explanation that correctly links the probability or expected value to the decision **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- Examples that satisfy component 2:
  - The probability that Agatha will receive at least one gift card in a 52-week year is 0.2295, or the value computed in part (a-ii).
  - The probability that Agatha will fail to receive a gift card for an entire 52-week year is 0.7705, or the complement of the value computed in part (a-ii).
  - The expected value computed in part (b).
  - Stating AT MOST 52 out of 200 employees will win a gift card (or AT LEAST 148 will not win).
- A response that indicates that Agatha does have a strong argument that the selection process was not truly random (or responds "yes") that is adequately supported by an explanation based on an incorrectly calculated probability in part (a-ii) OR an incorrectly calculated expected value in part (b) is scored E.
- If a response gives two arguments, treat them as parallel solutions and score the weaker solution.

Scoring for Question 3	Score
Complete Response Three parts essentially correct	**4**
Substantial Response Two parts essentially correct and one part partially correct	**3**
Developing Response Two parts essentially correct and no part partially correct *OR* One part essentially correct and one or two parts partially correct *OR* Three parts partially correct	**2**
Minimal Response One part essentially correct and no part partially correct *OR* No part essentially correct and two parts partially correct	**1**

Begin your response to QUESTION 3 on this page.

3. To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week.

(a) Consider the probability that a particular employee receives at least one gift card in a 52-week year.

(i) Define the random variable of interest and state how the random variable is distributed.

X= the number of gift cards an employee recieves.

The random variable is Binomial with $p = 0.005$ & $n = 52$.

B = Binary (Recieves or doesn't recieve)

I = "Each week's selection is independent from every other week"

N = fixed number = 52

S → set probability → 0.005

(ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.

$$p(x \geq 1) = 1 - p(x \leq 0)$$

$$= 1 - \binom{52}{0}(0.005)^0(1-.995)^{52}$$

$$= 1 - 0.7705$$

$$= 0.2295$$ probability of recieving at least one gift card in a 52-week year.

Continue your response to **QUESTION 3** on this page.

(b) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.

$E(x) = np = 52(.005)$

$E(x) = 0.26$

If many, many random samples of 52-week years are choosen, then there will approx. an average of .26 gift cards a particular employee will recieve.

(c) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

$p(x=0) = \binom{52}{0}(0.005)^0(1-.995)^{52}$

$= .7705$

~~There is approx~~

there is approx. a .7705 chance of not getting a gift card for an entire 52-week, this is a likely occurance to occur so Agatha doesn't have a strong arguement that the selection process is not truely random.

Begin your response to QUESTION 3 on this page.

3. To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week.

(a) Consider the probability that a particular employee receives at least one gift card in a 52-week year.

(i) Define the random variable of interest and state how the random variable is distributed.

The random variable of interest is the number of gift cards received in a 52-week year.

It is binomially distributed based on these conditions:

B(52, 0.005)

① random selection has occurred
② each week's selection is indepent
③ fixed 200 observation
④ probability of success is the same (always equally likely)
⑤ you either win it or you don't (one outcome out of two)

(ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.

$P(X \geq 1) = 1 - P(X < 1)$

$= 1 - P(X = 0)$

$= 1 - \binom{52}{0}(0.005)^0(1-0.005)^{52}$

$= 1 - 0.7705488893$

$= 0.2294511107$

The probability that a particular employee recieves at least one gift card in a 52-week year is 0.2294511107.

Continue your response to **QUESTION 3** on this page.

(b) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.

$B(52, 0.005)$

$\mu_x = np$

$\mu_x = 52\left(\frac{1}{200}\right) = \underline{0.26}$

The expected value demonstrates that a particular employee will receive approx. 0.26 gift cards in a 52-weak year.

(c) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

No, because the probability of never receiving a gift card for an entire 52-week year is

$\binom{52}{0}(0.005)^0(1-0.005)^{52} = 0.770548893.$

77.05% is very probable considering all 200 employees are equally likely and could be selected twice.

Begin your response to QUESTION 3 on this page.

3. To increase morale among employees, a company began a program in which one employee is randomly selected each week to receive a gift card. Each of the company's 200 employees is equally likely to be selected each week, and the same employee could be selected more than once. Each week's selection is independent from every other week. = no replacement

52 trials

(a) Consider the probability that a particular employee receives at least one gift card in a 52-week year.

(i) Define the random variable of interest and state how the random variable is distributed.

~~Whether an employee receives a gift card~~

random variable of interest = an employee receives a gift card

Is distributed by random.

(ii) Determine the probability that a particular employee receives at least one gift card in a 52-week year. Show your work.

P(at least one gift card)

= 1 − P(no gift card)

$= 1 - \left(\frac{199}{200}\right)^{52} = 0{,}2294$

Continue your response to **QUESTION 3** on this page.

(b) Calculate and interpret the expected value for the number of gift cards a particular employee will receive in a 52-week year. Show your work.

expected value = np = $52 \cdot \frac{1}{200} = 0.26$

(c) Suppose that Agatha, an employee at the company, never receives a gift card for an entire 52-week year. Based on her experience, does Agatha have a strong argument that the selection process was not truly random? Explain your answer.

No, because the probability of not receiving any gift cards in a 52-week year is 0.7705. Since this probability is quite large, Agatha does not have a strong argument that the selection process was not truly random.

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student's ability to (1) define a random variable and identify its distribution; (2) identify the value of a binomial probability; (3) identify and interpret the expected value of a binomial random variable; and (4) use the expected value of a random variable or the probability of a specific event to counter a claim.

This question primarily assesses skills in skill category 3: Using Probability and Simulation. Skills required for responding to this question include (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, (3.B) Determine parameters for probability distributions, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 4.10, and 4.11, and learning objectives UNC-3.B, UNC-3.C, and UNC-3.D.

Sample: 3A
Score: 4

The response earned the following: part (a) – E; part (b) – E; part (c) – E.

In part (a) the response satisfies component 1 in part (a-i). The response satisfies component 2 in part (a-i) by stating "Binomial" and the correct values of n and p. Component 3 is satisfied in part (a-ii) in three different ways, any of which alone would satisfy component 3. One way is using the probability statement "$P(X \geq 1)$"; a second way is using the probability statement "$1 - P(X = 0)$"; the third way is by stating "at least one gift card" in the response. The response computes the correct value of the probability that a particular employee receives at least one gift card in a 52-week year and shows work in part (a-ii), satisfying component 4. Note: The response does state $1 - \binom{52}{0}(0.005)^0(1 - .995)^{52}$; the $1 - .995$ is incorrect, but was considered a transcription error during scoring. Part (a) was scored essentially correct (E).

In part (b) the response correctly calculates the expected value and provides supporting work, satisfying component 1. The response provides the correct interpretation of the expected value with all three aspects, satisfying component 2. The response includes the first aspect, repeated selection over a long period of time, by stating, "if many, many random samples of 52-week years are choosen." The response includes the second and third aspects, by stating "on average" and the context of gift cards. Part (b) was scored essentially correct (E).

In part (c) the response states that "Agatha doesn't have a strong argument," satisfying component 1. The response provides the correct probability that a particular employee does not receive a gift card, ".7705," satisfying component 2. Note: The same transcription error, carried from part (a-ii), appears here. The response states that a particular employee not getting a gift card is "a likely occurrence," which links the decision to the probability satisfying component 3. Part (c) was scored essentially correct (E).

Question 3 (continued)

Sample: 3B
Score: 3

The response earned the following: part (a) – E; part (b) – P; part (c) – E.

In part (a) the response correctly defines the random variable in part (a-i) satisfying component 1. Component 2 is satisfied in two ways because the response correctly states the distribution of the random variable is binomial with the correct values of n and p. In part (a-i) the response states " $B(52, 0.005)$ " In part (a-ii) the response uses the binomial formula with the correct values of n and p. The response correctly identifies the event of interest by giving the correct boundary and direction using probability statements in part (a-ii), satisfying component 3. The response computes the correct value of the probability that a particular employee receives at least one gift card in a 52-week year in part (a-ii), satisfying component 4. Part (a) was scored essentially correct (E).

In part (b) the response correctly calculates the expected value and provides supporting work, satisfying component 1. The response does not satisfy component 2 giving only the third aspect of expected value interpretation. Part (b) was scored partially correct (P).

In part (c) the response satisfies component 1. The response provides the correct probability that a particular employee does not receive a gift card, satisfying component 2. The response states that a particular employee not getting a gift card is "very probable," linking the decision to the probability, which satisfies component 3. Part (c) was scored essentially correct (E).

Sample: 3C
Score: 2

The response earned the following: part (a) – P; part (b) – P; part (c) – E.

In part (a) the response does not correctly define the random variable and component 1 is not satisfied. Component 2 is not satisfied as no distribution is stated and $1-\left(\frac{199}{200}\right)^{52}$ does not indicate the binomial distribution. The response correctly identifies the correct boundary and direction with the statement "P(at least one gift card)" in part (a-ii). Additionally, the response identifies boundary and direction a second time with the statement "$1 - P(\text{no gift card})$" in part (a-ii), satisfying component 3. The response computes the correct value of the probability with supporting work in part (a-ii), satisfying component 4. Part (a) was scored partially correct (P).

In part (b) the response correctly calculates the expected value and provides supporting work, satisfying component 1. The response does not provide an interpretation, and so component 2 is not satisfied. Part (b) was scored partially correct (P).

In part (c) the response satisfies component 1. The response provides the correct probability that a particular employee does not receive a gift card, satisfying component 2. The response states that this probability is "quite large" linking the decision to the probability, which satisfies component 3. Part (c) was scored essentially correct (E).