2023

AP®

AP® Statistics
Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 1

☑ **Scoring Guidelines**

☑ **Student Samples**

☑ **Scoring Commentary**

Question 1: Focus on Exploring Data   **4 points**

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) The histogram of dissolved oxygen concentration in Alaskan streams with water temperatures colder than 8°C is unimodal and skewed left with a median between 11 and 12 mg/l. The first quartile is in the bin from 10-11 mg/l and the third quartile is in the bin from 12-13 mg/l, so the IQR is approximately 2 mg/l. There do not appear to be any high outliers, but there are several potential low outliers because the values in the 2-3, 4-5, and 5-6 bins are all certainly more than 1.5 IQR below the first quartile.	**Essentially correct (E)** if the description of the distribution satisfies component 1 *AND* at least three of components 2-5: 1. Includes context of dissolved oxygen concentration 2. Shape: The distribution is skewed left 3. Center: The center of the distribution is between 11 mg/l and 12 mg/l 4. Spread: Refers to at least one measure of variation (i.e., the range is between $14 - 2 = 12$ mg/l and $13 - 3 = 10$ mg/l; all values are between 2 mg/l and 14 mg/l or the IQR is approximately 2 mg/l) 5. Unusual features: potential outliers or a gap between 3 mg/l and 4 mg/l **Partially correct (P)** if the response satisfies component 1 and two components out of components 2-5 *OR* if the response satisfies at least three out of components 2-5 but does not satisfy component 1. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- **Context**
  - Component 1 can be satisfied with a reference to dissolved oxygen concentration, dissolved oxygen, amount of oxygen, or mg/l.
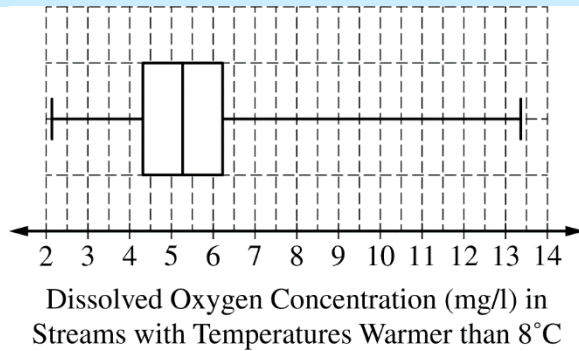
- **Shape:**
  - Component 2 cannot be satisfied if a response describes the histogram as normal or approximately normal.
- **Center:**
  - A response that addresses center using definitive language such as "the mean (median) of the distribution is 11.5" cannot satisfy component 3.
  - A response that addresses center using approximate language such as "the median of the distribution is approximately 11.5" must, for any single measure of center, specify a numeric value that is between 11 mg/l and 12 mg/l, inclusive, to satisfy component 3.
- **Spread:**
  - A response recognizing all values in the sample fall between 2 mg/l and 14 mg/l, satisfies component 4 only for these exact endpoints and need not appeal to a specific measure of spread such as the range.
  - A response that uses interval language must use it correctly. For example, "the observations range from 2 mg/l and 14 mg/l," satisfies component 4 because it correctly indicates that all observations are between 2 mg/l and 14 mg/l, inclusive. However, a statement such as "the range is between 2 mg/l and 14 mg/l," is incorrect because the range is a single number, i.e., $14 - 2 = 12$ mg/l.
  - A response that appeals to a specific measure of spread using approximate language, such as "the IQR is approximately 2," must specify a numeric value within the bounds appropriate to that measure of spread shown in the following table.

Statistic	Min (mg/l)	Max (mg/l)
Range	10	12
IQR	1.5	2.5
Standard Deviation	1.7	1.8

- **Unusual Features:**
  - Component 5 cannot be satisfied if a response indicates that the distribution has an unusual feature other than potential outliers or a gap.
  - Definitive language such as "there is an outlier" does not satisfy component 5.

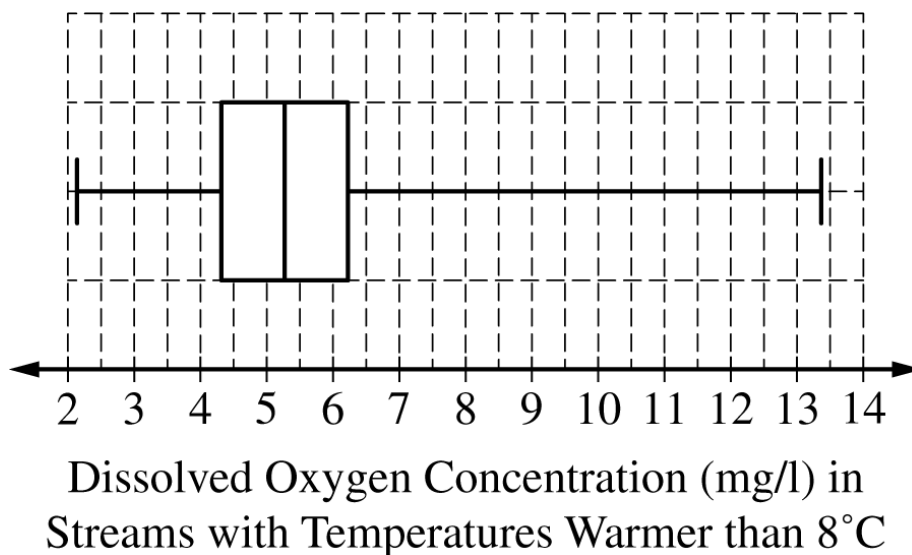| Model Solution	Scoring
(b) Dissolved Oxygen Concentration (mg/l) in Streams with Temperatures Warmer than 8˚C	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. Constructs the box beginning at Q1 2. Constructs the box ending at Q3 3. Locates the median within the box 4. Extends the lines on the boxplot to the minimum value 5. Extends the lines on the boxplot to the maximum value **Partially correct (P)** if the response includes a boxplot with only three of the five components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:
- A response that shows a graph or plot other than a boxplot should be scored incorrect (I).
- To be correctly placed:
  - The minimum must be between the tick marks for 2 and 2.5
  - The first quartile must be between the tick marks for 4 and 4.5
  - The median must be between the tick marks for 5 and 5.5
  - The third quartile must be between the tick marks for 6 and 6.5
  - The maximum must be between the tick marks for 13 and 13.5

Dissolved Oxygen Concentration (mg/l) in Streams with Temperatures Warmer than 8˚C

Model Solution	Scoring
(c) If the researchers' belief is correct, then streams with water temperature colder than 8°C are healthier for wildlife. The distribution of dissolved oxygen concentration for colder streams has a higher center because its median (between 11 mg/l and 12 mg/l) is larger than the median for warmer streams (5.43 mg/l). The shape of the distribution of dissolved oxygen concentration for colder streams is different from the shape of the distribution for warmer streams. The distribution of values of dissolved oxygen concentration for colder streams is skewed to the left but the distribution of values for warmer streams is skewed to the right. Both distributions have a similar spread because they both have similar IQR values — approximately 2 mg/l for the colder streams and 1.73 mg/l for the warmer streams.	**Essentially correct (E)** if the response satisfies component 1 *AND* at least two of components 2-4: 1. States colder streams are healthier for wildlife 2. Directly compares the centers of the two distributions 3. Indicates warmer streams are skewed right and colder streams are skewed left 4. Directly compares the spreads **Partially correct (P)** if the response satisfies component 1 *AND* one of components 2-4. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:
- While both distributions have potential outliers, it is not necessary to compare them for this part of the question.
- A response based on an incorrect part (a) or part (b) may be scored E or P if it justifies the choice consistent with their answers to part (a) or part (b).
- In order to satisfy component 4, a response may compare at least one of the values of the approximate range, IQR, or standard deviation with the value of the same statistic from the other distribution.

Scoring for Question 1	Score
Complete Response Three parts essentially correct	4
Substantial Response Two parts essentially correct and one part partially correct	3

Developing Response
 Two parts essentially correct and no part partially correct
     OR
 One part essentially correct and one or two parts partially correct
     OR
 Three parts partially correct
	2

Minimal Response One part essentially correct and no part partially correct *OR* No part essentially correct and two parts partially correct	1

Question 1

Begin your response to **QUESTION 1** on this page.

STATISTICS

SECTION II

Total Time—1 hour and 30 minutes

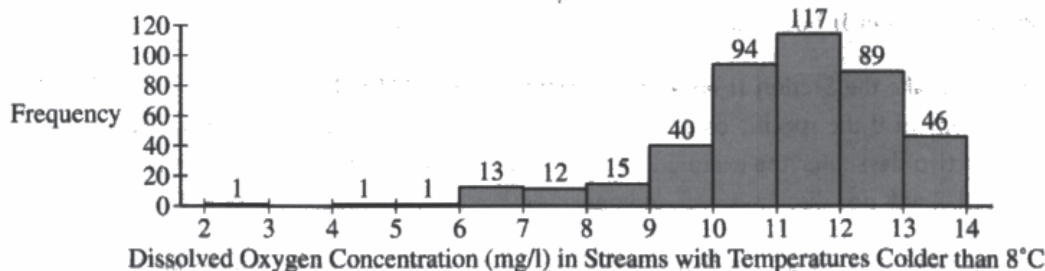**6 Questions**

Part A

Suggested Time—1 hour and 5 minutes

5 Questions

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. As part of a study on the chemistry of Alaskan streams, researchers took water samples from many streams with temperatures colder than 8°C and from many streams with temperatures warmer than 8°C. For each sample, the researchers measured the dissolved oxygen concentration, in milligrams per liter (mg/l).

Dissolved Oxygen Concentration (mg/l) in Streams with Temperatures Colder than 8°C

(a) The researchers constructed the histogram shown for the dissolved oxygen concentration in streams from the sample with water temperatures <u>colder</u> than 8°C. Based on the histogram, describe the distribution of dissolved oxygen concentration in streams with water temperatures <u>colder</u> than 8°C.

The distribution of dissolved oxygen concentration in streams with water temperatures colder than 8°c is unimodal and skewed left. The distribution has a range of 12 mg/l. The median of the distribution of dissolved oxygen concentration in streams with water colder than 8°c is m between 11 mg/l and 12 mg/l. The sample that has a dissolved oxygen concentration in between 2 mg/l and 3 mg/l appears to be a possible outlier of the distribution.
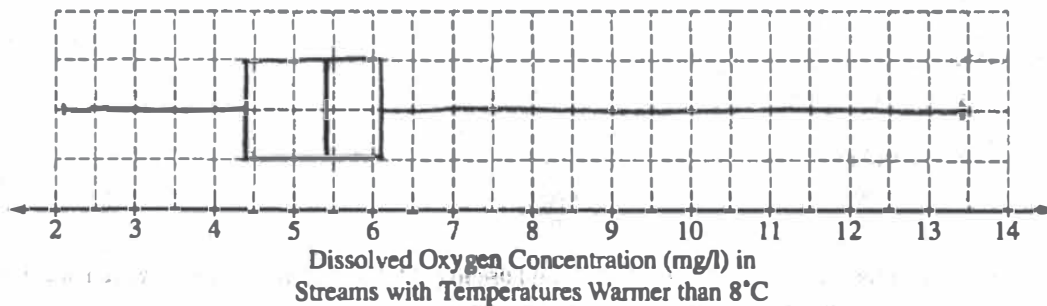
Page 2   **GO ON TO THE NEXT PAGE.**

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/2

Question 1

Continue your response to **QUESTION 1** on this page.

Min	Q1	Median	Q3	Max	Mean	Std. Dev.
2.10	4.39	5,43	6.12	13.45	5;54	1.64

← warmer

(b) The researchers computed the summary statistics shown in the table for the dissolved oxygen concentration in streams from the sample with water temperatures warmer than 8°C. Use the summary statistics to construct a box plot for the dissolved oxygen concentration in streams with water temperatures warmer than 8°C. Do not indicate outliers.

Dissolved Oxygen Concentration (mg/l) in
Streams with Temperatures Warmer than 8°C

(c) The researchers believe that streams with higher dissolved oxygen concentration are generally healthier for wildlife. Which streams are generally healthier for wildlife, those with water temperature colder than 8°C or those with water temperature warmer than 8°C? Using characteristics of the distribution of dissolved oxygen concentration for temperatures colder than 8°C and characteristics of the distribution of dissolved oxygen concentration for temperatures warmer than 8°C, justify your answer.

Streams with water temperature colder than 8°C are healthier for wildlife than streams with water temperature warmer than 8°C. This is because the distribution of oxygen concentration for streams with temperatures colder than 8°C has a higher center than the distribution of oxygen concentration for streams with temperatures warmer than 8°C, as the former has a median between 11mg/l and 12 mg/l, while the latter has a substantially lower median of 5.43 mg/l. Furthermore, the distribution for colder streams is skewed left, meaning most of its data is clustered around higher oxygen temperatures. The distribution for warmer streams, on the other hand, is skewed right, meaning most of its data is clustered around lower oxygen temperatures.

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Question 1

Begin your response to **QUESTION 1** on this page.

STATISTICS

SECTION II

Total Time—1 hour and 30 minutes

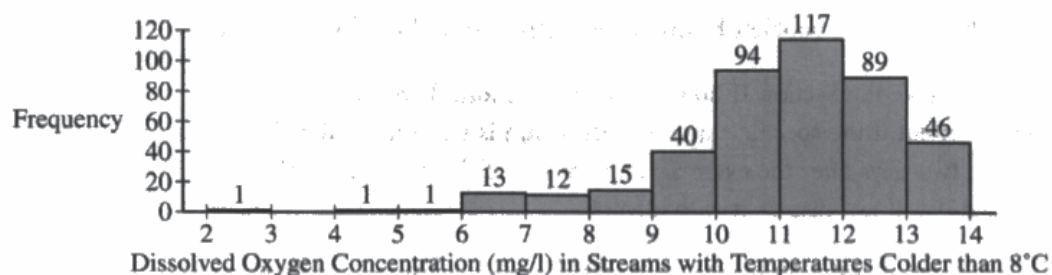**6 Questions**

Part A

Suggested Time—1 hour and 5 minutes

5 Questions

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. As part of a study on the chemistry of Alaskan streams, researchers took water samples from many streams with temperatures colder than 8°C and from many streams with temperatures warmer than 8°C. For each sample, the researchers measured the dissolved oxygen concentration, in milligrams per liter (mg/l).

Dissolved Oxygen Concentration (mg/l) in Streams with Temperatures Colder than 8°C

(a) The researchers constructed the histogram shown for the dissolved oxygen concentration in streams from the sample with water temperatures <u>colder</u> than 8°C. Based on the histogram, describe the distribution of dissolved oxygen concentration in streams with water temperatures <u>colder</u> than 8°C.
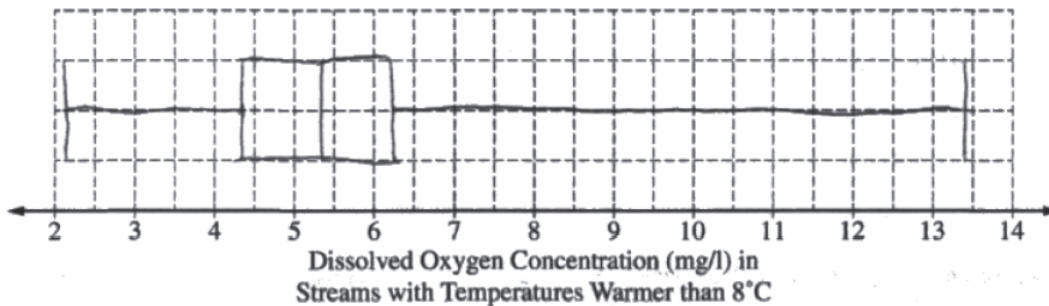
The distribution of Dissolved oxygen Concentration in streams colder than 8°c has a median between 11 and 12 mg/l. The spread is from 2 mg/l to 14 mg/l. The shape is unimodal skewed left. Unusual features include an outlier at 2-3 mg/l with a gap between 3-4 mg/l.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/2

Question 1

Continue your response to **QUESTION 1** on this page.

Min	Q1	Median	Q3	Max	Mean	Std. Dev.
2.10	4.39	5.43	6.12	13.45	5.54	1.64

(b) The researchers computed the summary statistics shown in the table for the dissolved oxygen concentration in streams from the sample with water temperatures warmer than 8°C. Use the summary statistics to construct a box plot for the dissolved oxygen concentration in streams with water temperatures warmer than 8°C. Do not indicate outliers.

Dissolved Oxygen Concentration (mg/l) in
Streams with Temperatures Warmer than 8°C

(c) The researchers believe that streams with higher dissolved oxygen concentration are generally healthier for wildlife. Which streams are generally healthier for wildlife, those with water temperature colder than 8°C or those with water temperature warmer than 8°C? Using characteristics of the distribution of dissolved oxygen concentration for temperatures colder than 8°C and characteristics of the distribution of dissolved oxygen concentration for temperatures warmer than 8°C, justify your answer.

Streams with temperature colder than 8°c are generally healthier. In the histogram of streams colder than 8°c, the median of dissolved oxygen levels was between 11 mg/l and 12 mg/l. This is around twice as much as the median for dissolved oxygen levels in warmer than 8°c streams. This is indicated in the histogram that displays a center line around 5.43 mg/l. This means that streams cooler than 8°c have a median that is higher than streams warmer than 8°c, making them generally healthier.

Page 3   **GO ON TO THE NEXT PAGE.**

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Q5214/3

Question 1

Begin your response to **QUESTION 1** on this page.

STATISTICS

SECTION II

Total Time—1 hour and 30 minutes
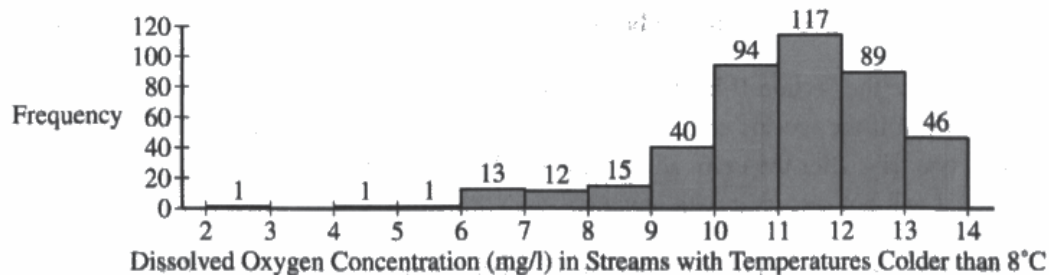
6 Questions

Part A

Suggested Time—1 hour and 5 minutes

5 Questions

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. As part of a study on the chemistry of Alaskan streams, researchers took water samples from many streams with temperatures colder than 8°C and from many streams with temperatures warmer than 8°C. For each sample, the researchers measured the dissolved oxygen concentration, in milligrams per liter (mg/l).

Dissolved Oxygen Concentration (mg/l) in Streams with Temperatures Colder than 8°C

(a) The researchers constructed the histogram shown for the dissolved oxygen concentration in streams from the sample with water temperatures <u>colder</u> than 8°C. Based on the histogram, describe the distribution of dissolved oxygen concentration in streams with water temperatures <u>colder</u> than 8°C.
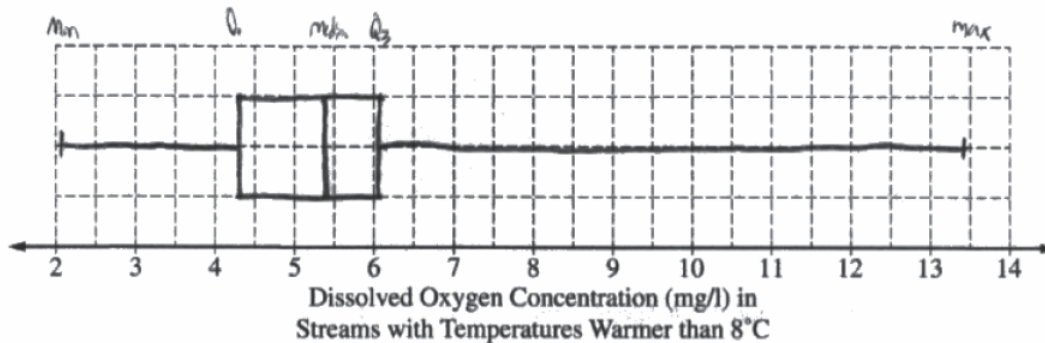
The distribution of dissolved oxygen concentration in streams with water temperatures colder than 8°c is left skewed and has one peak.

Page 2   **GO ON TO THE NEXT PAGE.**

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/2

Question 1

Continue your response to **QUESTION 1** on this page.

Min	Q1	Median	Q3	Max	Mean	Std. Dev.
2.10	4.39	5.43	6.12	13.45	5.54	1.64

(b) The researchers computed the summary statistics shown in the table for the dissolved oxygen concentration in streams from the sample with water temperatures warmer than 8°C. Use the summary statistics to construct a box plot for the dissolved oxygen concentration in streams with water temperatures warmer than 8°C. Do not indicate outliers.

Dissolved Oxygen Concentration (mg/l) in
Streams with Temperatures Warmer than 8°C

(c) The researchers believe that streams with higher dissolved oxygen concentration are generally healthier for wildlife. Which streams are generally healthier for wildlife, those with water temperature colder than 8°C or those with water temperature warmer than 8°C? Using characteristics of the distribution of dissolved oxygen concentration for temperatures colder than 8°C and characteristics of the distribution of dissolved oxygen concentration for temperatures warmer than 8°C, justify your answer.

The histogram given for temperatures colder than 8°C is left skewed meaning there is a higher frequency of Dissolved Oxygen Concentration on the right side of the graph meaning a higher concentration. The box plot made in part b has a long tail on the right meaning skewed right so there are more Dissolved Oxygen Concentration that are lower for temperatures warmer than 8°C. Because of these graph characteristics the colder water temperature than 8°C is generally healthier for wildlife.

Page 3   **GO ON TO THE NEXT PAGE.**

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/3

Question 1

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of this question were to assess a student's ability to (1) use data presented on a histogram to describe a variable within the context of a study; (2) create a box plot when provided summary statistics for a variable; and (3) determine which of two data sets match a description, when provided graphs and characteristics, and then provide a justification based on a comparison of the characteristics.

This question primarily assesses skills in skill category 2: Data Analysis. Skills required for responding to this question include (2.A) Describe data presented numerically or graphically, (2.B) Construct numerical or graphical representations of distributions, and (2.D) Compare distributions or relative positions of points within a distribution.

This question covers content from Unit 1: Exploring One-Variable Data of the course framework in the AP Statistics Course and Exam Description. Refer to topics, 1.6, 1.8, and 1.9, and learning objectives UNC-1.H, UNC-1.L, UNC-1.M, and UNC-1.N.

Sample: 1A
Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E.

In part (a) the response includes context, satisfying component 1. The response indicates that the distribution is "skewed left," satisfying component 2. The response provides the correct center "between 11 mg/l and 12 mg/l," satisfying component 3. The response notes a "range of 12" satisfying component 4. The response identifies a "possible" outlier, satisfying component 5. Part (a) was scored essentially correct (E). In part (b) the response correctly includes the beginning and end of the box, median line, and extends to the minimum and maximum values, satisfying all 5 components. Part (b) was scored essentially correct (E). In part (c) the response states colder streams are healthier, directly compares the medians of the two distributions, and discusses the shape of each distribution, satisfying components 1, 2, and 3. The response does not satisfy component 4. Part (c) was scored essentially correct (E).

Sample: 1B
Score: 3

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – P.

In part (a) the response includes context, shape, center, and spread, satisfying components 1–4. The response uses definitive language to identify "an outlier at $2 - 3$ mg/l," which is parallel to the discussion of a "gap between $3 - 4$ mg/l." Component 5 is not satisfied. Part (a) was scored essentially correct (E). In part (b) the response correctly includes the beginning and end of the box, median line, and extends to the minimum and maximum values, satisfying all five components. Part (b) was scored essentially correct (E). In part (c) the response includes that colder streams are healthier and compares centers, satisfying components 1 and 2. The response does not discuss skewness or spread, which does not satisfy components 3 and 4. Part (c) was scored partially correct (P).

Question 1 (continued)

Sample: 1C
Score: 2

The response earned the following: Part (a) – I; Part (b) – E; Part (c) – P.

In part (a) the response includes context and shape, satisfying component 1 and only one of the components 2–5. Part (a) was scored incorrect (I). In part (b) the response correctly includes the beginning and end of the box, median line, and extends to the minimum and maximum values, satisfying all five components. Part (b) was scored essentially correct (E). In part (c) the response states that colder streams are healthier for wildlife and indicates warmer streams are skewed right and colder streams are skewed left, satisfying components 1 and 3. The response does not satisfy components 2 or 4. Part (c) was scored partially correct (P).