AP

AP® Statistics
Sample Student Responses and Scoring Commentary

Inside:

Free-Response Question 3

☑ **Scoring Guidelines**

☑ **Student Samples**

☑ **Scoring Commentary**

Question 3: Focus on Probability and Sampling Distributions | 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

Model Solution	Scoring
(a) The random variable X is the dollar value of the cash prize in a bath fizzy. **(i)** The proportion of bath fizzies containing \$1 is equal to the $P(X = \$1)$ and $P(X = \$1)$ $= 1 - (0.2 + 0.05 + 0.05 + 0.01 + 0.01)$ $= 0.68.$ **(ii)** The proportion of bath fizzies that contain at least \$10 is equal to the $P(X \geq \$10)$ and $P(X \geq \$10)$ $= 0.05 + 0.05 + 0.01 + 0.01$ $= 0.12.$	**Essentially correct (E)** if the response satisfies at least three of the following four components: 1. Correctly calculates the proportion of bath fizzies containing \$1 2. Provides correct supporting work for the value calculated in component 1 3. Correctly calculates the proportion of bath fizzies containing at least \$10 4. Provides correct supporting work for the value calculated in component 3 **Partially correct (P)** if the response satisfies only two of the four components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:
- A response that provides a correct percentage, instead of a proportion, may satisfy components 1 and 3, such as 68 percent or 12 percent.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(b) Given a bath fizzy contains at least \$10, then the probability that it contains \$100 is $P(X = \$100 \mid X \geq \$10)$ $= \dfrac{0.01}{0.12}$ $\approx 0.0833.$	**Essentially correct (E)** if the response satisfies the following two components: 1. Correctly calculates the requested probability 2. Shows work consistent with their response to part (a-ii) **Partially correct (P)** if the response satisfies only one of the two components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- A specific conditional probability statement is not required, but if correctly given should be considered a positive in holistic scoring.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(c) The expected value of the distribution of X is $E(X) = 1(0.68) + 5(0.2) + 10(0.05)$ $\qquad + 20(0.05) + 50(0.01) + 100(0.01)$ $= \$4.68.$ The expected value is the mean of the cash prizes that result from the long run of many, many trials of randomly selecting bath fizzies and determining the amount each contains.	**Essentially correct (E)** if the response satisfies both components 1 and 2 *AND* at least two of components 3–5: 1. States the correct expected value 2. Shows appropriate work to calculate the expected value 3. Interpretation includes the concept of repeating the selection process over a long period of time 4. Interpretation includes the concept of an average or mean 5. Interpretation includes the context of receiving a cash prize **Partially correct (P)** if the response does not meet the criteria for E but satisfies two or three of components 1–4. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:
- Supporting work for finding the expected value must include at least two of the terms in the equation to show the pattern, such as $1(0.68) + 5(0.2) + \ldots$.
- Calculator notation does not satisfy component 2, such as $1 - \text{VAR STATS}(L1, L2)$.
- Responses that satisfy only components 1 and 5 or only components 2 and 5 receive a score of I but should be considered a positive in holistic scoring.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Model Solution	Scoring
(d) The expected value of the distribution of X in euros is $4.68(0.89) \approx 4.17$ euros.	**Essentially correct (E)** if the response correctly calculates the number of euros for the expected value showing work and including units.
	Partially correct (P) if the response correctly calculates the number of euros for the expected value but is missing either work or units.
	Incorrect (I) if the response does not meet the criteria for E or P.

Additional Notes:
- A response that does not have at least one decimal place in the final response (e.g., rounded to an integer) should be scored no more than P.
- The response can either use the expected value calculated in part (c) or first convert all the values in the probability distribution to euros to find the new expected value.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Scoring for Question 3
Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.

	Score
Complete Response	4
Substantial Response	3
Developing Response	2
Minimal Response	1

If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.

Question 3

Begin your response to **QUESTION 3** on this page.

3. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable, X, represent the dollar value of the cash prize in a bath fizzy. The probability distribution of X is shown in the table.

Cash prize, x	$1	$5	$10	$20	$50	$100
Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(handwritten column labels above table: / B C D Z F)

(a) Based on the probability distribution of X, answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain $1.

$$P(x=1) = 1 - (P(x=1))^c$$

68% of bath fizzies contain 1 dollar.

$$P(x=1)^c = 0.2 + 0.05 + 0.05 + 0.01 + 0.01 \qquad 1 - 0.32 = 0.68$$

(ii) Calculate the proportion of bath fizzies that contain at least $10.

$$P(C \cup D \cup E \cup F) = 0.05 + 0.05 + 0.01 + 0.01 = 0.12$$

12% of bath fizzies contain $10 or more.

(b) Based on the probability distribution of X, calculate the probability that a randomly selected bath fizzy contains $100, given that it contains at least $10. Show your work.

$$P(A|B) = \frac{P(A \cap A)}{P(B)} = \frac{0.01}{0.12} = 0.0833$$

$P(A) =$ Bath fizzy contains $100
$P(B) =$ Bath fizzy contains at least $10

There is an 8.3% chance that a bath fizzy contains $100 if it is given that it contains at least $10.

    Page 6   **GO ON TO THE NEXT PAGE.**

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Q5214/6

Question 3

Continue your response to **QUESTION 3** on this page.

(c) Based on the probability distribution of X, calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work.

$$E(x) = x_1 P(x_1) + x_2 P(x_2) + \ldots$$

$$E(x) = 1(0.68) + 5(0.2) + 10(0.05) + 20(0.05) + 50(0.01) + 100(0.01)$$

$$E(x) = 4.68$$

On average, the cash prize is expected to be $4.68 in the bath fizzie.

(d) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

$$N_{(0.89x)} = 4.68(0.89) = 4.17$$

On average in France, the cash prize in euros is expected to be 4.17 in the bath fizzy.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/7

Question 3

Begin your response to **QUESTION 3** on this page.

3. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable, X, represent the dollar value of the cash prize in a bath fizzy. The probability distribution of X is shown in the table.

Cash prize, x	$1	$5	$10	$20	$50	$100
Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(a) Based on the probability distribution of X, answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain $1. There is a ~~80~~ 68% chance of a $1 cash prize.

.2 + .05 + 0.5 + .01 + .01 = 32
1 - .32 = .68

(ii) Calculate the proportion of bath fizzies that contain at least $10. There is a 12% chance of the cash prize being at least $10.

(b) Based on the probability distribution of X, calculate the probability that a randomly selected bath fizzy contains $100, given that it contains at least $10. Show your work. There is a 8.33% chance of getting a $100 prize if you are getting at least $10

Page 6   **GO ON TO THE NEXT PAGE.**

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Q5214/6

Question 3

Continue your response to **QUESTION 3** on this page.

(c) Based on the probability distribution of X, calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work. The expected value is 4.68,

$$1(.68) + 5(.2) + 10(.05) + 20(.05) + 50(.01) + 100(.01) = 4.68$$

(d) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

The expected value in euros is 4.1652 euros.

$$= 4.68 \times .89 = 4.1652$$

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Q5214/7

Question 3

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3. Bath fizzies are mineral tablets that dissolve and create bubbles when added to bathwater. In order to increase sales, the Fizzy Bath Company has produced a new line of bath fizzies that have a cash prize in every bath fizzy. Let the random variable, X, represent the dollar value of the cash prize in a bath fizzy. The probability distribution of X is shown in the table.

Cash prize, x	$1	$5	$10	$20	$50	$100
Probability of cash prize, $P(X = x)$	$P(X = \$1)$	0.2	0.05	0.05	0.01	0.01

(a) Based on the probability distribution of X, answer the following. Show your work.

(i) Calculate the proportion of bath fizzies that contain $1.

$$P(X = \$1) = 1 - P(X \neq \$1) = 1 - (0.2 + 0.05 + 0.05 + 0.01 + 0.01)$$
$$= \underline{0.68}$$

(ii) Calculate the proportion of bath fizzies that contain at least $10.

$$P(X \geq \$10) = 0.05 + 0.05 + 0.01 + 0.01 = \underline{0.12}$$

(b) Based on the probability distribution of X, calculate the probability that a randomly selected bath fizzy contains $100, given that it contains at least $10. Show your work.

$$P(\text{contains } \$100 \mid \text{contains at least } \$10) = \frac{P(\text{contain } \$100 \cap \text{contain at least } \$10)}{P(\text{contain at least } \$10)}$$

$$= \frac{0.01}{0.12} = 0.083$$

    Page 6   **GO ON TO THE NEXT PAGE.**

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Q5214/6

Question 3

Continue your response to **QUESTION 3** on this page.

(c) Based on the probability distribution of X, calculate and interpret the expected value of the distribution of the cash prize in the bath fizzies. Show your work.

The expected proportion of both fizzies that have $1 is 0.68. The ex. prop. for those who have $5 is 0.2. The ex. prop. for those who have $10 is 0.05. The ex. prop. for those who have $20 is 0.05. The ex. prop for those who have $50 is 0.01. The ex. prop. for those who have $100 is 0.01.

(d) The Fizzy Bath Company would like to sell the bath fizzies in France, where the currency is euros. Suppose the conversion rate for dollars to euros is 1 dollar = 0.89 euros. Using your expected value from part (c), calculate the expected value, in euros, of the distribution of the cash prize in the bath fizzies. Show your work.

$$\$1(0.89) = 0.89 \text{ euros}$$
$$\$5(0.89) = 4.45 \text{ euros}$$
$$\$10(0.89) = 8.9 \text{ euros}$$
$$\$20(0.89) = 17.8 \text{ euros}$$
$$\$50(0.89) = 44.5 \text{ euros}$$
$$\$100(0.89) = 89 \text{ euros}$$

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/7

Question 3

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student's ability to (1) calculate single and compound discrete proportions when provided the probability distribution; (2) calculate a conditional probability for a discrete distribution; (3) calculate and interpret the expected value for a discrete distribution; and (4) convert an expected value to a different unit of measurement.

This question primarily assesses skills in skill category 3: Using Probability and Simulation. Skills required for responding to this question include (3.A) Determine relative frequencies, proportion, or probabilities using simulation or calculations, (3.B) Determine parameters for probability distributions, and (3.C) Describe probability distributions.

This question covers content from Unit 4: Probability, Random Variables, and Probability Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 4.3, 4.5. 4.8, and 4.9 with the following learning objectives VAR-4.A, VAR-4.D, VAR-5.C, and VAR-5.F.

Sample: 3A
Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a) the response satisfies at least three of the four components by correctly calculating the requested proportions (components 1 and 3) and providing supporting work for each (components 2 and 4). The response includes these values as percentages, which was not required but is acceptable. Part (a) was scored essentially correct (E). In part (b) the response correctly calculates the conditional probability (component 1) and provides supporting work (component 2) consistent with the work in part (a-ii). Part (b) was scored essentially correct (E). In part (c) the response satisfies both components 1 and 2 by calculating the expected value and showing appropriate work. Next, the response satisfies at least two of components 3–5. The response provides an interpretation of expected value but does not satisfy component 3 as it does not refer to selection over the long run. The interpretation includes the concept of average (component 4) and includes context (component 5). Part (c) was scored essentially correct (E). In part (d) the response correctly calculates the expected value, provides work, and includes the units of euros. Part (d) was scored essentially correct (E).

Sample: 3B
Score: 3

The response earned the following: Part (a) – E; Part (b) – P; Part (c) – P; Part (d) – E.

In part (a) the response satisfies at least three of the four components by correctly calculating the requested proportions (components 1 and 3) and providing supporting work for only one proportion (component 2). The response includes these values as percentages, which was not required but is acceptable. Part (a) was scored essentially correct (E). In part (b) the response correctly calculates the conditional probability (component 1) but does not provide supporting work (component 2). Part (b) was scored partially correct (P). In part (c) the response satisfies both components 1 and 2 by calculating the expected value and showing appropriate work. However, the response does not satisfy at least two of components 3–5 as no interpretation is provided. Part (c) was scored partially correct (P). In part (d) the response correctly calculates the expected value, provides work, and includes the units of euros. Part (d) was scored essentially correct (E).

Question 3 (continued)

Sample: 3C
Score: 2

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – I; Part (d) – I.

In part (a) the response satisfies at least three of the four components by correctly calculating the requested proportions (components 1 and 3) and providing supporting work (components 2 and 4) for each. Part (a) was scored essentially correct (E). In part (b) the response correctly calculates the conditional probability (component 1) and provides supporting work (component 2). Part (b) was scored essentially correct (E). In part (c) the response only provides a list of each of the possible prize amounts with associated probabilities so does not satisfy any of components 1–5. Part (c) was scored incorrect (I). In part (d) the response only provides a list of each of the possible prize amounts converted to euros and does not use this information to calculate the expected value in euros. Part (d) was scored incorrect (I).