2023

AP® Statistics

Sample Student Responses and Scoring Commentary

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Free-Response Question 5

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Question 5: Multi-Focus — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each section of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	The scatterplot reveals a strong, positive, roughly linear association between the chest circumference and weight of tule elk. There are no points that seriously deviate from the straight-line pattern of the points in the plot.	**Essentially correct (E)** if the response provides a description that includes at least three of components 1-4 and component 5: 1. Direction of association (positive or increasing) 2. Strength of association (strong) 3. Form of association (linear or approximately linear) 4. Unusual features (no points with large discrepancies from the straight-line pattern exhibited by most of the points on the plot) 5. Context (association between chest circumference and weight of tule elk) **Partially correct (P)** if the response satisfies only one or two components out of components 1-4 and component 5 *OR* if the response satisfies at least two out of components 1-4 but does not satisfy component 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- To satisfy component 4 it is sufficient to simply indicate that there are no unusual features or no outliers.
- To satisfy component 5 it is minimally sufficient for the response to refer to the association or relationship between chest circumference or measurement and weight without explicitly mentioning tule elk.
- The quality of communication in part (a) should be considered if holistic scoring is required.

Model Solution	Scoring
(b) **(i)** The predicted weight of a male tule elk with a chest circumference of 145.9 cm is $-350.3 + 3.7455(145.9) \approx 196.17$ kg. **(ii)** The residual for a male tule elk with a chest circumference of 145.9 cm with an actual weight of 204.3 kg is $204.3 - 196.17 \approx 8.13$ kg.	**Essentially correct (E)** if the response satisfies the following two components: 1. Provides the correct value of 196.17 kg in part (b-i) with work shown 2. Provides the correct value of 8.13 kg in part (b-ii), or a value consistent with the predicted weight calculated for component 1, with work shown **Partially correct (P)** if the response satisfies only one of the two components required for an E *OR* the response gives both correct values with no work shown. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that includes an undefined or incorrect variable to identify work shown (e.g., x or $\hat{p}$) does not satisfy component 1.

	Model Solution	Scoring
(c)	The value of the slope of the least-squares regression line is 3.7455. This value indicates that the predicted weight of a tule elk increases by 3.7455 kilograms for each additional centimeter of chest circumference.	**Essentially correct (E)** if the response satisfies the following three components: 1. Identifies the value of the slope as 3.7455 2. Provides an interpretation that references an increase of a number of kilograms of weight for each one-centimeter increase in chest circumference 3. Indicates that the slope represents a change in a prediction using non-deterministic language such as "predicted," "estimated," "expected," or "average" **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

- The value of the slope, 3.7455, may be rounded to 3.746, 3.75, or 3.7, but not to 3 or 4, to satisfy the numerical requirement in component 1.
- A response that only contains 3.7455 in the interpretation satisfies component 1.
- A calculation of slope may satisfy component 1 provided that two points from the line are used in the calculation.
- Units of measurements must be correctly specified for both weight and length to satisfy component 2.
- It is not required to refer specifically to the "least-squares regression line."

Model Solution	Scoring
(d) **(i)** The degrees of freedom for the test of slope are $n - 2 = 30 - 2 = 28$. The t-table shows that for 28 degrees of freedom, the p-value for a one-sided test would be 0.001. Because this is a two-sided test, the p-value is $(2)(0.001) = 0.002$. **(ii)** Because the p-value $= 0.002$ is less than $\alpha = 0.05$, reject the null hypothesis. There is sufficient statistical evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different from 4.5 kg/cm.	**Essentially correct (E)** if the response satisfies the following three components: 1. Gives a correct p-value of 0.002, consistent with the two-sided alternative hypothesis 2. Provides correct comparison of the p-value to alpha (p-value is less than/greater than alpha) *AND* provides a correct decision about the null and/or alternative hypothesis 3. Provides a conclusion statement in context, consistent with, and in terms of, the alternative hypothesis using non-deterministic language **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes

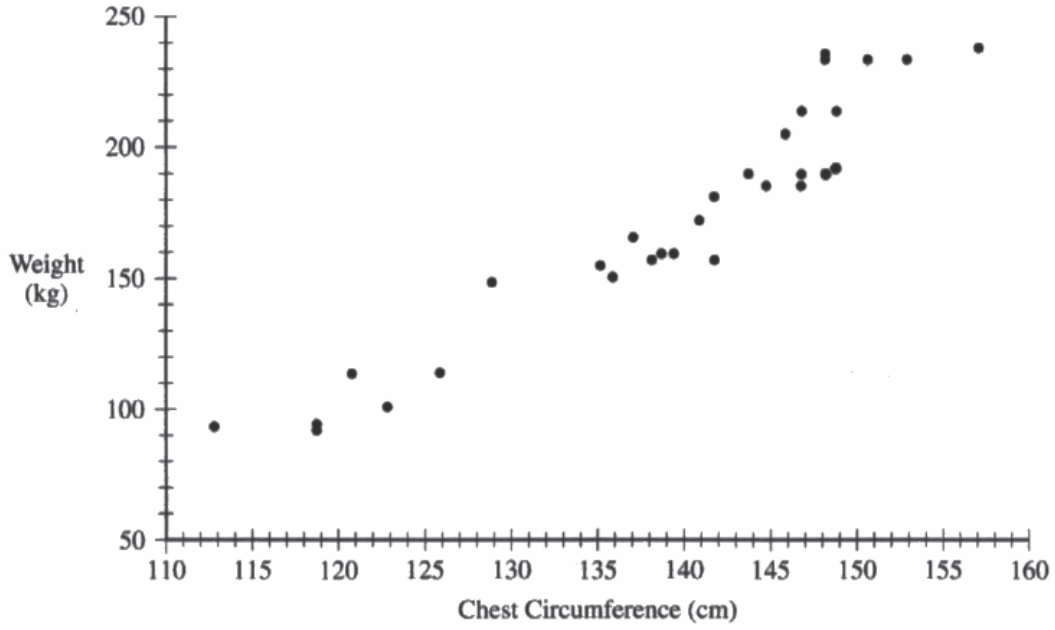
- The response need not make an explicit decision about the null hypothesis (reject H_o or fail to reject H_o).
- If the conclusion and justification are consistent with an incorrect p-value (or an incorrect value of the test statistic, or an incorrect confidence interval, or an incorrect critical value), the response may satisfy component 2.
- To satisfy the p-value comparison in component 2, the response can compare the value of the test statistic to the appropriate critical value (e.g., 2.048 or -2.048).
- If an explicit decision is stated and the conclusion statement is inconsistent with the decision, component 2 is not satisfied.
- The decision part of component 2 may be satisfied by implying the decision within the conclusion statement (sufficient evidence/insufficient evidence for the alternative hypothesis).
- If the response includes a statement that is equivalent to accepting the null hypothesis (e.g., "we conclude that the mean population slope is 4.5"), then component 2 is not satisfied.
- A student who doesn't give a p-value, but gives a correct conclusion based on a p-value $< \alpha$ with a consistent conclusion *OR* a p-value $> \alpha$ with a consistent conclusion may be scored P, e.g.,
  - If p were less than alpha, then reject H_0 and say there is sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
  - If p were greater than alpha, then fail to reject H_0 and say there is not sufficient evidence that the population slope for the linear regression of weight vs. chest circumference for male tule elk is different than 4.5 kg/cm.
- If the response includes an incorrect interpretation of the p-value (e.g., "this is the probability that the null hypothesis is true"), then component 2 is not satisfied.
- The quality of communication in part (d) should be considered if holistic scoring is required.

Scoring for Question 5 Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.	**Score**
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**
If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.	

Question 5

Begin your response to **QUESTION 5** on this page.

5. Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.



(a) Describe the relationship between chest circumference and weight of male tule elk in context.

The relationship between chest circumference and weight of male tule elk is positive, strong, and linear. There arent any outliers.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

Following is the equation of the least-squares regression line relating chest circumference and weight for male tule elk.

predicted weight = –350.3 + 3.7455(chest circumference)

(b) The weight of one male tule elk with a chest circumference of 145.9 cm is 204.3 kg.

(i) Using the equation of the least-squares regression line, calculate the predicted weight for this male tule elk. Show your work.

$\hat{\text{weight}} = -350.3 + 3.7455(145.9)$

$= 196.168$

The predicted weight for this male tule elk is 196.168kg.

(ii) Calculate the residual for this male tule elk. Show your work.

$\text{residual} = y - \hat{y}$

$= 204.3 - 196.168$

$= 8.13$

The predicted weight of the male tule elk was 8.13 kg less than the actual weight.

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

The equation of the least-squares regression line relating chest circumference and weight for male tule elk is repeated here.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

(c) Interpret the slope of the least-squares regression line in context.

For every 1 cm increase in chest circumference, the predicted weight increases by 3.7455 kg.

(d) The sambar, another species of deer, is similar in size to the tule elk. The slope of the population regression line relating chest circumference and weight for all male sambars is 4.5 kilograms per centimeter. A wildlife biologist wants to determine whether the slope of the population regression line for male tule elk is different than that for male sambars. Let β represent the slope of the population regression line for male tule elk. The wildlife biologist conducted a test of the following hypotheses using the sample of 30 tule elk.

$$H_0: \beta = 4.5$$
$$H_a: \beta \neq 4.5$$

The test statistic was calculated to be 3.408. Assume all conditions for inference were met.

(i) Determine the p-value of the test.

p-value = 2(tcdf(lower=3.408, upper=∞, df=28))

p-value = 0.002

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

(ii) At a significance level of $\alpha = 0.05$, what conclusion should the wildlife biologist make regarding the slope of the population regression line for male tule elk? Justify your response.

Because the p-value = 0.002 is less than $\alpha = 0.05$, the wildlife biologist should reject Ho. There is convincing evidence that the slope of the population regression line for male tule elk is different than that for male sambars.

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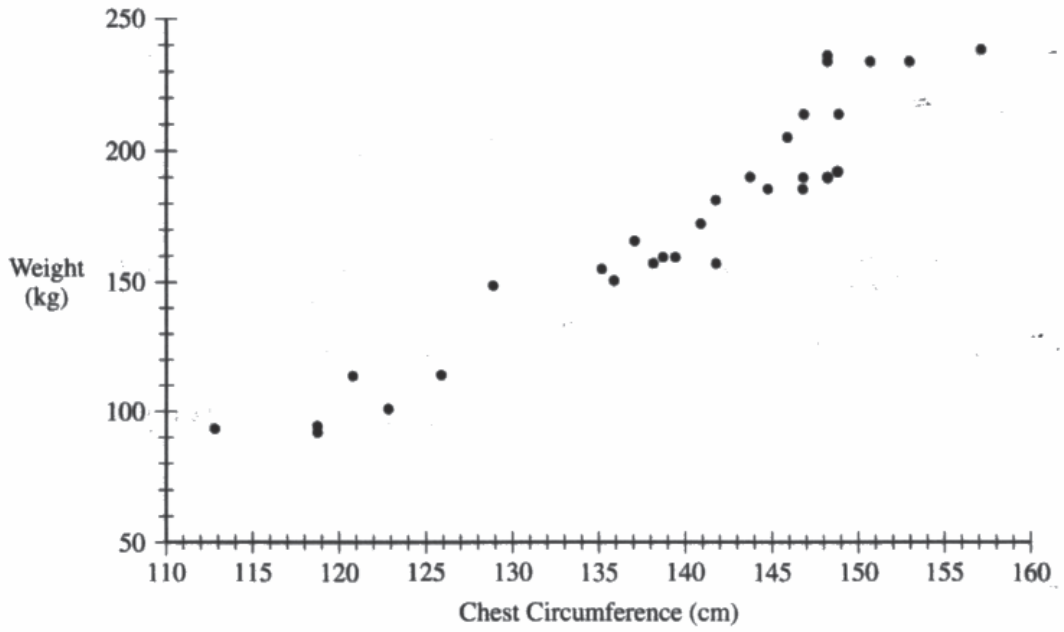
Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Sample 5B

Question 5

Begin your response to **QUESTION 5** on this page.

5. Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.



(a) Describe the relationship between chest circumference and weight of male tule elk in context.

linear

positive

moderate to strong correlation

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

Following is the equation of the least-squares regression line relating chest circumference and weight for male tule elk.

predicted weight = –350.3 + 3.7455(chest circumference)

(b) The weight of one male tule elk with a chest circumference of 145.9 cm is 204.3 kg.

(i) Using the equation of the least-squares regression line, calculate the predicted weight for this male tule elk. Show your work.

$-350.3 + 3.7455(145.9)$

$= 196.17$ kg

(ii) Calculate the residual for this male tule elk. Show your work.

$204.3 - 196.17 = 8.1$ kg

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

The equation of the least-squares regression line relating chest circumference and weight for male tule elk is repeated here.

predicted weight = −350.3 + 3.7455(chest circumference)

(c) Interpret the slope of the least-squares regression line in context.

for every 1 cm increase in chest circumference for male tule elk, the predicted increase in weight is 3.7455 kg

(d) The sambar, another species of deer, is similar in size to the tule elk. The slope of the population regression line relating chest circumference and weight for all male sambars is 4.5 kilograms per centimeter. A wildlife biologist wants to determine whether the slope of the population regression line for male tule elk is different than that for male sambars. Let β represent the slope of the population regression line for male tule elk. The wildlife biologist conducted a test of the following hypotheses using the sample of 30 tule elk.

$H_0: \beta = 4.5$

$H_a: \beta \neq 4.5$

The test statistic was calculated to be 3.408. Assume all conditions for inference were met.

(i) Determine the *p*-value of the test.

two-tailed test

1 − tcdf(lower: −3.408, upper: 3.408, df: 30−2)

= .002 p-value

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Sample 5B

Question 5

Continue your response to **QUESTION 5** on this page.

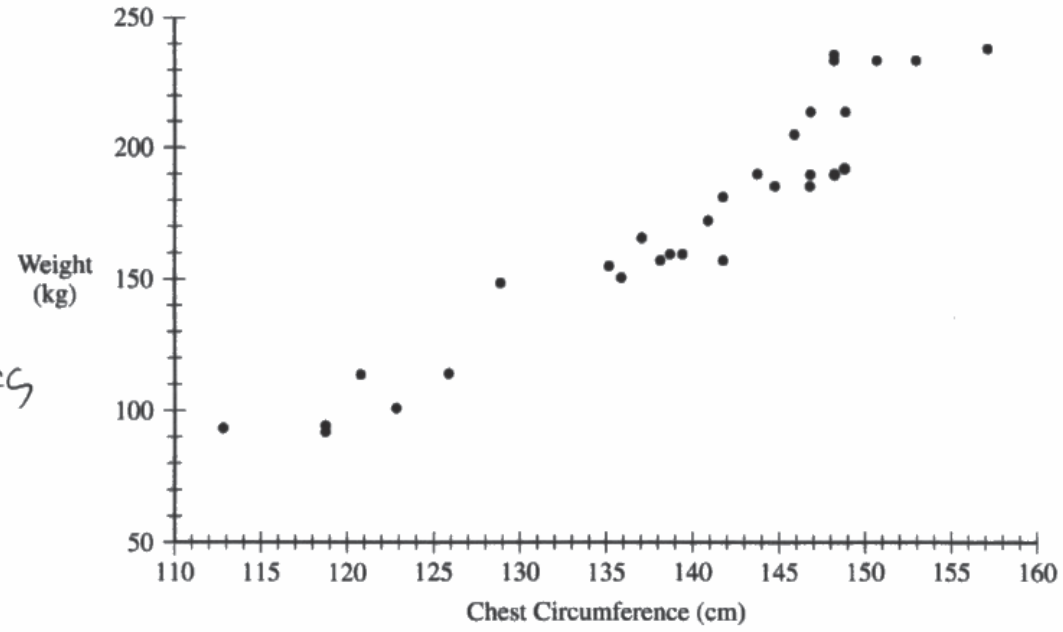
(ii) At a significance level of $\alpha = 0.05$, what conclusion should the wildlife biologist make regarding the slope of the population regression line for male tule elk? Justify your response.

because the p-value is less than the level of significance, there is statistically significant evidence to suggest that the slope of the population regression line for male tule elk is different than that for male sambars. reject H_0.

Question 5

Begin your response to **QUESTION 5** on this page.

5. Wildlife biologists are interested in the health of tule elk, a species of deer found in California. An important measurement of tule elk health is their weight. The weight of a tule elk is difficult to measure in the wild. However, chest circumference, which is believed to be related to the weight of a tule elk, can easily be measured from a safe distance using a harmless laser. A study was done to investigate whether chest circumference, in centimeters (cm), could be used to accurately estimate the weight, in kilograms (kg), of male tule elk. For the study, wildlife biologists captured 30 male tule elk, measured their chest circumference and weight, and then released the elk. The data for the 30 male tule elk are shown in the scatterplot.



DUFS

(a) Describe the relationship between chest circumference and weight of male tule elk in context.

The relationship between chest circumference and weight of male tule elk is strong, positive, and linear with no unusual features.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Sample 5C

Question 5

Continue your response to **QUESTION 5** on this page.

Following is the equation of the least-squares regression line relating chest circumference and weight for male tule elk.

predicted weight = –350.3 + 3.7455(chest circumference)

(b) The weight of one male tule elk with a chest circumference of 145.9 cm is 204.3 kg.

(i) Using the equation of the least-squares regression line, calculate the predicted weight for this male tule elk. Show your work.

predicted weight = $-350.3 + 3.7455(145.9)$

$= \boxed{196.2\text{kg}}$

(ii) Calculate the residual for this male tule elk. Show your work.

$\frac{\text{Predicted} - \text{actual}}{\text{predicted}} = \text{residual}$

$\frac{196.2 - 204.3}{196.2} = \boxed{-0.0413}$

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Continue your response to **QUESTION 5** on this page.

The equation of the least-squares regression line relating chest circumference and weight for male tule elk is repeated here.

$$\text{predicted weight} = -350.3 + 3.7455(\text{chest circumference})$$

(c) Interpret the slope of the least-squares regression line in context.

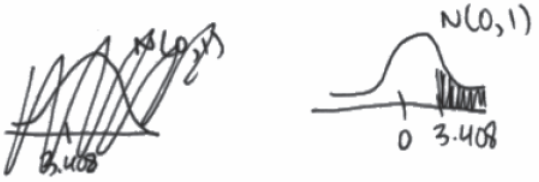
For every cm increase in chest circumference, the weight in kg increases by 3.7455.

(d) The sambar, another species of deer, is similar in size to the tule elk. The slope of the population regression line relating chest circumference and weight for all male sambars is 4.5 kilograms per centimeter. A wildlife biologist wants to determine whether the slope of the population regression line for male tule elk is different than that for male sambars. Let β represent the slope of the population regression line for male tule elk. The wildlife biologist conducted a test of the following hypotheses using the sample of 30 tule elk.

$$H_0: \beta = 4.5$$
$$H_a: \beta \neq 4.5$$

The test statistic was calculated to be 3.408. Assume all conditions for inference were met.

(i) Determine the p-value of the test.



normal cdf (lower = 3.408, upper = 999, $\mu = 0$, $\sigma = 1$)

p-value $\approx 3.27 \times 10^{-4}$

Sample 5C

Question 5

Continue your response to **QUESTION 5** on this page.

(ii) At a significance level of $\alpha = 0.05$, what conclusion should the wildlife biologist make regarding the slope of the population regression line for male tule elk? Justify your response.

Since the p-value $\approx 3.27 \times 10^{-4} < \alpha = 0.05$, reject H_0. We have convincing evidence that the slope of the population regression line for male tule elk is NOT different than that for male sambars.

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Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 5

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student's ability to (1) describe the relationship exhibited on a scatterplot between an independent variable and response variable in context; (2) calculate the predicted *y*-value for a specific observation, when provided a least-squares regression equation; (3) calculate the residual for an observation; (4) interpret the slope for a provided least-squares regression equation, in context; (5) determine the *p*-value for a two-sided test of slope, when the test statistic is provided; and (6) compare the *p*-value for a test of slope to a significance level, then reach a conclusion regarding the slope, in context.

This question primarily assesses skills in skill category 2: Data Analysis, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (2.A) Describe data presented numerically or graphically, (2.B) Construct numerical or graphical representations of distributions, (2.C) Calculate summary statistics, relative positions of points within a distribution, correlation, and predicted response, (3.E) Calculate a test statistic and find a *p*-value, provided conditions for inference are met, (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim, and (4.E) Justify a claim using a decision based on significance tests.

This question covers content from Unit 2: Exploring Two-Variable Data and Unit 9: Inference for Quantitative Data: Slopes of the course framework in the AP Statistics Course and Exam Description. Refer to topics 2.4, 2.6, 2.7, 2.8, and 9.5, and learning objectives DAT-1.A, DAT-1.D, DAT-1.E, DAT-1.H, DAT-3.N, and VAR-7.M.

Sample: 5A
Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a) the response correctly describes the association as positive, strong, and linear with sufficient context, satisfying component 1, component 2, component 3, and component 5. The response indicates "[t]here arent any outliers," satisfying component 4. Part (a) was scored essentially correct (E). In part (b) the response provides the correct predicted weight value with adequate work and the correct value of the residual with adequate work, satisfying both components. Part (b) was scored essentially correct (E). In part (c) the response identifies the value of the slope within the interpretation, satisfying component 1. The response provides an interpretation that references a one-centimeter increase in chest circumference resulting in a weight increase of 3.7455 kilograms, satisfying component 2. The response uses the non-deterministic language, "predicted," satisfying component 3. Part (c) was scored essentially correct (E). In part (d) the response gives a correct *p*-value, satisfying component 1. The response provides a correct comparison of the *p*-value to α and provides a correct decision about the null hypotheses, reject H_0. The response also includes the correct decision by stating "[t]here is convincing evidence" for the alternative hypothesis, satisfying component 2. The response provides a non-definitive conclusion statement in context in terms of the alternative hypothesis, satisfying component 3. Part (d) was scored essentially correct (E).

Sample: 5B
Score: 3

The response earned the following: Part (a) – P; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a) the response provides a minimal description that includes positive, strong, and linear, satisfying component 1, component 2, and component 3. The response does not include context and does not mention unusual features, so component 4 and component 5 are not satisfied. Part (a) was scored partially correct (P). In

Question 5 (continued)

part (b) the response provides the correct predicted weight value with adequate work and the correct value of the residual with adequate work, satisfying both components. Part (b) was scored essentially correct (E). In part (c) the response identifies the value of the slope within the interpretation, satisfying component 1. The response provides an interpretation that references a one-centimeter increase in chest circumference resulting in a weight increase of 3.7455 kilograms, satisfying component 2. The response uses the non-deterministic language, "predicted," satisfying component 3. Part (c) was scored essentially correct (E). In part (d) the response gives a correct *p*-value, satisfying component 1. The response provides a correct comparison of the *p*-value to α and provides a correct decision by stating "there is statistically significant evidence" for the alternative hypothesis, satisfying component 2. The response provides a non-definitive conclusion statement in context in terms of the alternative hypothesis, satisfying component 3. Part (d) was scored essentially correct (E). The score of 3.5 requires a holistic approach. Although the communication in part (d) is very good, part (a) contains minimal statements and does not include any context. The response was scored 3.

Sample: 5C
Score: 2

The response earned the following: Part (a) – E; Part (b) – P; Part (c) – P; Part (d) – I.

In part (a) the response correctly describes the association as positive, strong, and linear with sufficient context, satisfying component 1, component 2, component 3, and component 5. The response indicates "no unusual features," satisfying component 4. Part (a) was scored essentially correct (E). In part (b) the response provides the correct predicted weight value with adequate work shown, satisfying component 1. The response reverses the observed and predicted values resulting in a negative residual and continues by dividing by the predicted value. The response attempts to calculate a *z*-score or chi-squared test statistic. Component 2 is not satisfied. Part (b) was scored partially correct (P). In part (c) the response identifies the value of the slope within the interpretation, satisfying component 1. The response provides an interpretation that references a one-centimeter increase in chest circumference resulting in a weight increase of 3.7455 kilograms, satisfying component 2. The response does not use non-deterministic language such as "predicted" or "on average," so component 3 is not satisfied. Part (c) was scored partially correct (P). In part (d) the response gives an incorrect *p*-value based on the normal distribution, so component 1 is not satisfied. The response correctly links a *p*-value less than α to rejecting the null hypothesis. However, the decision is reversed by "having convincing evidence" in favor of the null hypothesis ("NOT different" being the equivalent of equal). Component 2 is not satisfied. The conclusion statement does not refer to the alternative hypothesis, so component 3 is not satisfied. Part (d) was scored incorrect (I).