2023

AP

AP® Statistics

Sample Student Responses and Scoring Commentary

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Free-Response Question 6

☑ Scoring Guidelines

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Question 6: Investigative Task — 4 points

General Scoring Notes

- Each part of the question (indicated by a letter) is initially scored by determining if it meets the criteria for essentially correct (E), partially correct (P), or incorrect (I). The response is then categorized based on the scores assigned to each letter part and awarded an integer score between 0 and 4 (see the table at the end of the question).
- The model solution represents an ideal response to each part of the question, and the scoring criteria identify the specific components of the model solution that are used to determine the score.

	Model Solution	Scoring
(a)	Let X represent the amount of gold applied to a necklace randomly selected from necklaces produced with this machine. The random variable X has an approximately normal distribution with mean 300 mg and standard deviation 5 mg. Then, $P(296 < X < 304)$ $= P(X < 304) - P(X \le 296)$ $= P\left(Z < \frac{304-300}{5}\right) - P\left(Z \le \frac{296-300}{5}\right)$ $= P(Z < 0.8) - P(Z \le -0.8)$ $\approx 0.7881 - 0.2119 \approx 0.5763.$	**Essentially correct (E)** if the response satisfies the following three components: 1. Indicates the use of a normal distribution with mean 300 and standard deviation 5 2. Specifies the correct event (boundary value and direction) or an event consistent with values reported in component 1 3. Provides the correct probability of 0.5763 or a probability consistent with components 1 and 2 **Partially correct (P)** if the response satisfies only two of the three components. **Incorrect (I)** if the response does not satisfy the criteria for E or P.

Additional Notes:

Component 1

- A response may satisfy component 1 by any of the following or a combination of the following:
  - Graphical: Displaying a graph of a normal density function with the appropriate scale on the horizontal axis showing the mean and the standard deviation for the distribution of the amount of gold applied to the necklaces.
  - Calculator function syntax: Labeling correct values of the mean and standard deviation in a "normalcdf" statement, such as:
    normalcdf(lower = 296, upper = 304, mean = 300, standard deviation = 5).
    Correct specification of the lower and upper bounds is not required to satisfy component 1.
  - Words: Using a statement such as "normal distribution with mean 300 and standard deviation 5."
  - Standard Notation: Using standard notation such as $N(300,5)$ or $N(300,(5)^2)$.
  - Z-score: Displaying the correct mean and standard deviation in a z-score calculation that includes "z," such as $z = \frac{304-300}{5}$.

Component 2

- A response may satisfy component 2 by any of the following or a combination of the following:
  - Graphical: Displaying a graph of a normal density function with the region of interest ($296 < \text{amount of gold} < 304$ or $-0.8 < Z < 0.8$) clearly identified. The shaded area does not need to be proportional, but the boundaries should be on the proper side of the mean and the shading should be in the proper direction.
  - Calculator function syntax: Labeling the lower and upper bounds of the region of interest in a "normalcdf" statement, such as:
    - normalcdf(lower = 296, upper = 304, mean = 300, standard deviation = 5).
    - normalcdf(lower = –0.8, upper = 0.8, $\mu = 0$, $\sigma = 1$).
  - Words: Specifying the correct event in words with correct numerical value for the boundary value and correct direction, such as "the probability that the amount of gold applied to a randomly selected necklace is between 296 and 304 mg" or $P(296 < \text{amount of gold} < 304)$.
  - Standard Notation: Using standard notation such as $P(296 < X < 304)$ or $P\left(\frac{296-300}{5} < Z < \frac{304-300}{5}\right)$ or $P(-0.8 < Z < 0.8)$.

General

- Minor errors in statistical notation may be ignored. However, this is considered poor communication if holistic scoring is required.
- If the only error in the response to part (a) is the reversal of the numerator for the *z*-score, $(300-296)$ or $(300-304)$, the response is scored P.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.
- A response that satisfies components 1 and 2 by including more than calculator syntax (e.g., standard random variable notation and probability notation) is considered good communication and may be considered if holistic scoring is required.

Model Solution	Scoring
(b) **(i)** If the machine is working properly, and the sample mean amount of gold, $\bar{X}$, has a sampling distribution that follows a normal distribution with mean 300 mg and standard deviation $\frac{5}{\sqrt{2}} \approx 3.5355$ mg, then $P(\bar{X} > 303) = P\left(Z > \frac{303 - 300}{\frac{5}{\sqrt{2}}}\right) \approx P(Z > 0.8485) \approx 0.198.$ **(ii)** Observing a sample mean amount of 303 mg would not provide convincing evidence that the population mean amount of gold being applied by the machine is something other than 300 mg because the probability of observing a sample mean that differs from 300 mg by 3 mg or more is large, around $0.198(2) = 0.396$.	**Essentially correct (E)** if the response satisfies components 1, 4, and 5 *AND* at least one of components 2 or 3: 1. In part (b-i), indicates the use of a normal (or an approximately normal) distribution and identifies the correct parameter values (mean 300 mg and standard deviation $\frac{5}{\sqrt{2}} \approx 3.5355$ mg) with work shown or a correct formula 2. In part (b-i), specifies the correct event (boundary value and direction) 3. In part (b-i), provides the correct probability of 0.198 or the probability consistent with components 1 and 2 4. In part (b-ii), indicates whether observing a sample mean of 303 mg would provide convincing evidence that the population mean is not 300 mg, consistent with the response to part (b-i) 5. In part (b-ii), provides justification based on the probability obtained in part (b-i) or a correctly computed probability in part (b-ii) **Partially correct (P)** if the response does not meet the requirements for E and satisfies at least three of the five components *OR* the response satisfies only components 4 and 5, consistent with the response to part (b-i) *OR* the response satisfies only component 1 and one of the other four components. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

Component 1

- A response may satisfy component 1 by any of the following or a combination of the following:
  - Graphical: Displaying a graph of a normal density function with the appropriate scale on the horizontal axis showing mean and standard deviation for the distribution of mean amount of gold.
  - Calculator function syntax: Labeling correct values of the mean and standard deviation in a "normalcdf" statement, such as "normalcdf" or "ncdf", e.g.,
    - $\text{normalcdf}\left(\text{lower} = 303, \text{ upper} = \infty, \text{ mean} = 300, \text{ standard deviation} = \frac{5}{\sqrt{2}}\right).$

    Correct specification of the upper and lower bounds is not required to satisfy component 1.
  - Words**:** Using a statement such as "approximately normal distribution with mean 300 and standard deviation $\frac{5}{\sqrt{2}}$."
  - Standard Notation: Using standard notation such as $N\left(300, \frac{5}{\sqrt{2}}\right)$ or $N\left(300, \left(\frac{5}{\sqrt{2}}\right)^2\right)$.
  - *Z*-score: Displaying the correct mean and standard deviation in a *z*-score calculation, which includes "*z*," such as $z = \frac{303 - 300}{\frac{5}{\sqrt{2}}}$.
  - To satisfy the supporting work criterion of component 1 the response must clearly indicate that the standard deviation for $\overline{X}$ is computed by dividing 5 by the square root of 2. This may be indicated with $\frac{5}{\sqrt{2}}$, words, or standard notation such as $\frac{\sigma}{\sqrt{n}}$.

Component 2

- A response may satisfy component 2 by any of the following or a combination of the following:
  - Graphical: Displaying a graph of a normal density function with the region of interest ($\overline{X} > 303$ or $Z > 0.8485$) clearly identified. The shaded area does not need to be proportional, but the boundary should be on the proper side of the mean and the shading should be in the proper direction.
  - Calculator function syntax: Labeling the lower and upper bounds of the region of interest in a "normalcdf" statement, such as:
    - $\text{normalcdf}\left(\text{lower} = 303, \text{ upper} = \infty, \text{ mean} = 300, \text{ standard deviation} = \frac{5}{\sqrt{2}}\right).$
    - $\text{normalcdf}\left(\text{lower} = 0.8485, \text{ upper} = \infty, \mu = 0, \sigma = 1\right).$
  - Words: Specifying the correct event in words with correct numerical value for the boundary and correct direction, such as the "probability that the mean amount of gold applied to two randomly selected necklaces is greater than 303 mg" or $P(\text{mean amount of gold} > 303)$.
  - Standard Notation: Using standard notation such as or $P\left(\overline{X} > 303\right)$ or $P\left(Z > \frac{303 - 300}{3.5355}\right)$ or $P(Z > 0.8485)$.

Component 3

- A response may satisfy component 3 if the reported probability is consistent with components 1 and 2. If a normal distribution with a mean of 300 mg and standard deviation 5 mg is used, the probability reported should be 0.274.

General

- Minor errors in statistical notation may be ignored. However, this is considered poor communication and may be considered if holistic scoring is required.
- If the only error in part (b) is the reversal of the numerator for the *z*-score $(300 - 303)$, the response is scored P.
- An arithmetic or transcription error in a response can be ignored if correct work is shown.

Alternate Solution:

- A response that calculates the probability that the *total* amount of gold on the two randomly selected necklaces is greater than 606 mg $(2 \times 303\,\text{mg})$ can be scored essentially correct if the response demonstrates:
  - The use of a normal (or approximately normal) distribution and the correct mean and standard deviation for the distribution of the total amount of gold applied to two randomly selected necklaces, (e.g., $N(600, 7.071)$, or mean $= 2(300) = 600$ mg and standard deviation $= \sqrt{2(5^2)} \approx 7.071$ mg, or $z = \frac{606 - 600}{7.071} \approx 0.8485$), satisfying component 1.
  - Specifies the correct event, including the correct boundary value of $x_{total} = 2(303) = 606$ or $z \approx 0.8485$, and the correct direction, satisfying component 2.
  - The probability is correctly computed using the mean and the standard deviation of the total amount of gold applied to two randomly selected necklaces. An arithmetic error can be ignored if correct work is shown and the result is between 0 and 0.5.
- While the probability of a difference in part (b-ii) is twice the probability found in part (b-i), it is not necessary for a response to double the probability in part (b-i). However, if the probability in part (b-i) is doubled, that should be considered if holistic scoring is required.

Model Solution	Scoring
(c) **(i)** The sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$ is skewed to the right. Almost all values of the simulated ranges are between 0 mg and about 25 mg and the center of the distribution is about 6 mg. **(ii)** As the value of the population standard deviation increases, the variation (spread) in the distribution of the sample range increases and the mean of the distribution of the sample range also increases.	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. In part (c-i), describes the shape as positively skewed (or skewed to the right) 2. In part (c-i), describes the center of the distribution as about 6 mg 3. In part (c-i), describes the spread of the simulated sample ranges as having most values between 0 mg and 30 mg 4. In part (c-ii), indicates that the mean (or center) of the distribution of the sample range increases as the population standard deviation increases 5. In part (c-ii), indicates that the variation (or spread) of the distribution of the sample range increases as the population standard deviation increases **Partially correct (P)** if the response satisfies three of the five components *OR* the response satisfies only components 4 and 5 *OR* the response only satisfies components 2 and 4 *OR* the response satisfies only components 3 and 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- The centrality component of the response to part (c-i) may be satisfied with any reasonable description of center, such as a comment on the approximate value of mean or median of the distribution or indication that the center is somewhere between 4 and 10 mg.
- The spread component of the response to part (c-i) may be satisfied with a reasonable statement about the inter-quartile range. Any value between 5 and 7, inclusive, is considered reasonable.
- Although the actual range of the sample distribution of the sample range is infinite, a comment that the range is any value between 20 and 30 may be scored as satisfying the spread component of the response to part (c-i).
- The response to part (c-ii) need not comment on the skewness of the distributions. Any comment on the skewness of the distributions should be considered as extraneous in scoring the response.

Model Solution	Scoring
(d) **(i)** No, a sample range of 10 mg is not unusual if the machine is working properly with a standard deviation of 5 mg. Although observing a sample range around 10 mg or greater is much more likely if the population standard deviation is 8 mg or 12 mg than when the population standard deviation is 5 mg, the graph of the sampling distribution of the sample range for samples of size 2 from a normal distribution with $\sigma = 5$ mg indicates that 10 mg is not an unusual value for the range when $\sigma = 5$ mg. There is about a 20% chance that a random sample of two necklaces would yield a range of 10 mg or more when the machine is working properly. **(ii)** No, Cleo's sample mean of 303 mg and range of 10 mg do not indicate that the machine is not working properly. As noted in part (b-i), the probability that the sample mean would be equal to or greater than 303 mg when the machine is working properly is almost 20% so having a sample mean of 303 mg is not unusual. Furthermore, it is less than one standard deviation, $\frac{5}{\sqrt{2}} = 3.5355$ mg, away from 300 mg. As indicated in part (d-i), the probability of a range of 10 mg or greater when the population standard deviation is 5 mg is also about 20%, so not unusual. There is not statistically significant evidence to show the machine is not working properly.	**Essentially correct (E)** if the response satisfies four or five of the following five components: 1. In part (d-i), indicates a sample range of 10 mg is not unusual 2. In part (d-i), justifies the response to component 1 by indicating that the graph of the sampling distribution of the sample range when $\sigma = 5$ mg shows that values of 10 mg or greater occur often 3. In part (d-ii), indicates there is not convincing evidence that the machine is not working properly 4. In part (d-ii), justifies the response by indicating that a sample mean of 303 mg would not be unusual when the machine is working properly with one of the following • 303 mg is less than one standard deviation away from 300 mg • The probability that the mean is greater than or equal to 303 mg was shown in (b) to have probability of 0.198, which is not unusual • Some other reasonable justification 5. In part (d-ii), justifies the response by indicating that a sample range of 10 mg is not unusual when the machine is working properly as discussed in part (d-i) **Partially correct (P)** if the response does not meet the requirements for E and satisfies at least three of the five components *OR* the response satisfies components 1 and 2 *OR* the response satisfies component 3 and either component 4 or 5. **Incorrect (I)** if the response does not meet the criteria for E or P.

Additional Notes:

- An argument that a sample range of 10 mg or more is more likely to occur (or closer to the center of the sampling distribution of the range) when $\sigma = 8$ mg or $\sigma = 12$ mg is not required for component 2, but by itself does not satisfy component 2. Inclusion of this argument should be considered if holistic scoring is required.

- Other reasonable justifications to satisfy component 4 include arguments based on a z-value close to zero, a one-sample z-test or one-sample t-test for a population mean, or a one sample z-interval or t-interval for a population mean.

Alternate Solutions:

- A response that uses the intersection (or union) of the event 'the sample mean is greater than or equal to 303 mg' and the event 'the sample range is greater than or equal to 10 mg' can satisfy components 3, 4, and 5.
  - Components 3, 4, and 5 are satisfied if the response shows, for example:
    - Let $P(M) = P(\text{mean} \geq 303) = 0.198$ (from part b-ii).
    - Let $P(R) = P(\text{range} \geq 10 \text{ if } \sigma = 5) \approx 0.2$ (from part d-i).
    - For Intersection**:** $P(M \text{ and } R) = P(M)P(R) = (0.198)(0.2) = 0.0396,$ because this probability is small, this is evidence that the machine is not working properly.
    - For Union**:** $P(M \text{ or } R) = P(M) + P(R) - P(M)P(R) = 0.198 + 0.2 - (0.198)(0.2) = 0.3584,$ because this probability is large, this is not evidence that the machine is not working properly.

General Notes:

- Context (amount of gold in necklaces) is not required in parts (a), (b), (c), or (d) of the response. However, including context is considered good communication and may be considered if holistic scoring is required.

Scoring for Question 6 Each essentially correct (E) part counts as 1 point, and each partially correct (P) part counts as ½ point.	**Score**
Complete Response	**4**
Substantial Response	**3**
Developing Response	**2**
Minimal Response	**1**
If a response is between two scores (for example, 2 ½ points), use a holistic approach to decide whether to score up or down, depending on the strength of the response and quality of the communication.	

Question 6

Begin your response to **QUESTION 6** on this page.

STATISTICS

SECTION II, Part B

Suggested Time—25 minutes

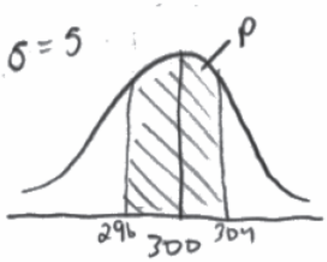
1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

   (a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

$\sigma = 5$



Lower bound: 296
upper bound: 304
μ: 300
σ: 5

probability $(296 < x < 304) = P(x < 304) - P(x < 296)$

$= P(z < 0.8) - P(z < -0.8)$

$= 0.788 - 0.212 = \boxed{0.576}$

GO ON TO THE NEXT PAGE.

Question 6

Continue your response to QUESTION 6 on this page.

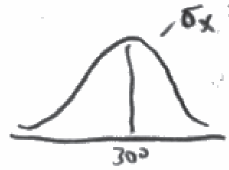
The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

(b) Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.

(i) Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.

i)



$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{2}} = 3.54$

$N(300, 3.54)$

$\bar{x} > 303$:

$z\text{-score} = \frac{303-300}{3.54} = 0.847$

$P(\bar{x} > 303) = P(z > 0.847)$

$= \boxed{0.198}$

(ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

NO. The probability of the sample mean amount of gold being 303 mg. or higher assuming that the true mean amount of gold is 300 mg is 0.198, which means it is not unlikely to get that extreme of an observation.

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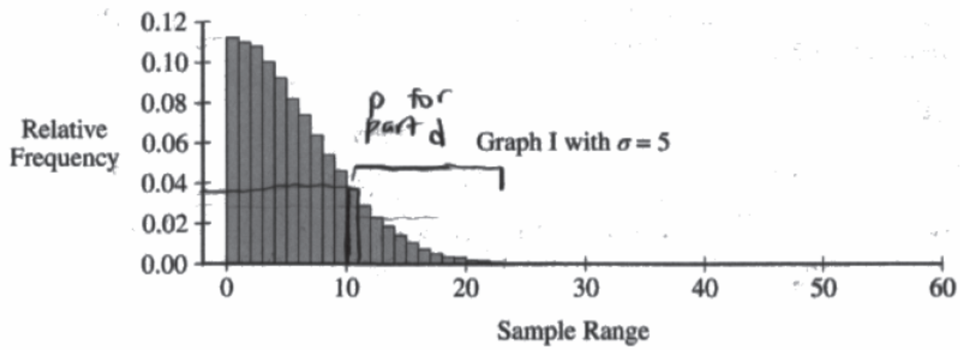
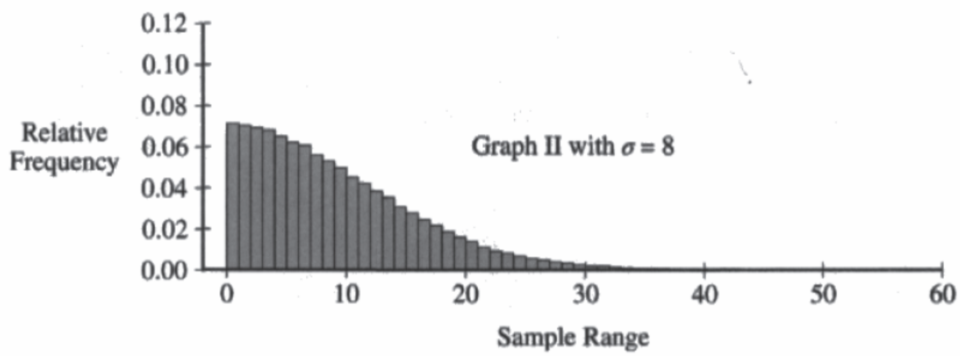
Question 6

Continue your response to QUESTION 6 on this page.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

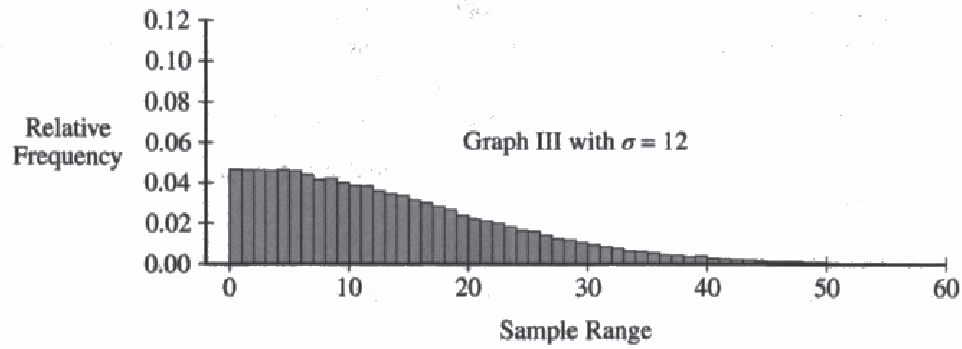
Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.





Question 6

Continue your response to **QUESTION 6** on this page.



(c) Use the information in the graphs to complete the following.

(i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

The distribution is skewed to the right and the median sample range is around 4-5 mg. The ranges have a range of about 23 mg.

(ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

The sampling distributions become less skewed to the right, and the range and median of the sample ranges increase.

Question 6

Continue your response to QUESTION 6 on this page.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg.

(d) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

(i) Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

using the sampling distributions for samples of size 2 and σ of 5 mg, ~~as a population distribution~~, the probability of getting a sample range of 10 mg or higher (the area under Graph I for sample ranges of 10 mg or higher) is approximately $0.025(12) = 0.30$. Since this prob is greater than the significance level of 0.05, the sample range is $\boxed{\text{NOT}}$ unusual

(ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

$\boxed{\text{NO.}}$ Both the sample mean of 303 mg and sample range of 10 mg were deemed to not be unusual.

Question 6

Begin your response to **QUESTION 6** on this page.

STATISTICS

SECTION II, Part B

Suggested Time—25 minutes

1 Question

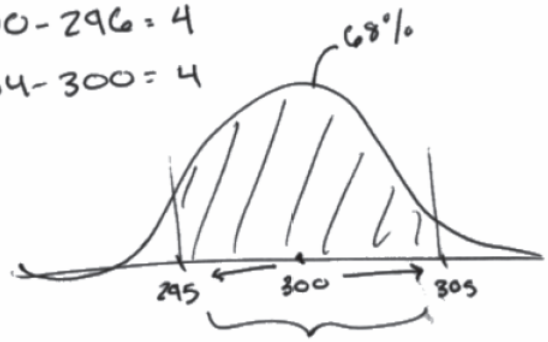
Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

   (a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.

300 - 296 = 4

304 - 300 = 4



within 1 standard deviation

therefore < .68

normCdf(296, 304, 300, 5) = $\boxed{.576}$

Question 6

Continue your response to **QUESTION 6** on this page.

The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

(b) Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.

(i) Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.

Sample $\sigma = \frac{5}{\sqrt{2}} = 3.54$

normCdf$(303, \infty, 300, 3.54) = \boxed{.198}$

(ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

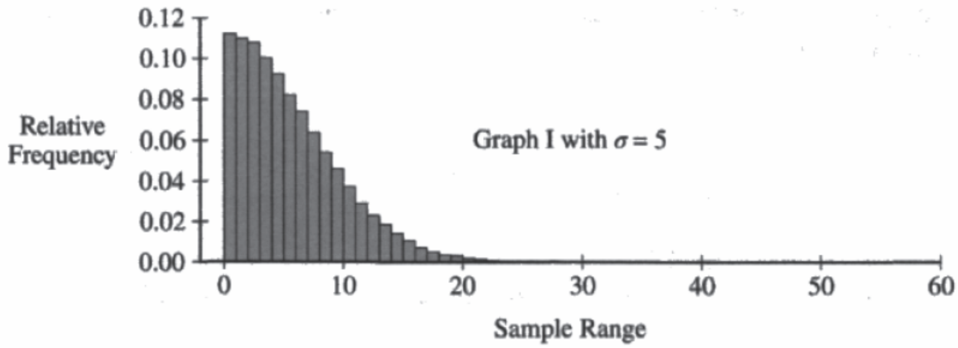
No, because a sample mean of 303mg is within 1 standard deviation of the sample. Due to the very small sample size of n=2, it is very likely the 3mg difference is due to random variation, and if the sample size was increased the sample mean would come much closer to the population mean.

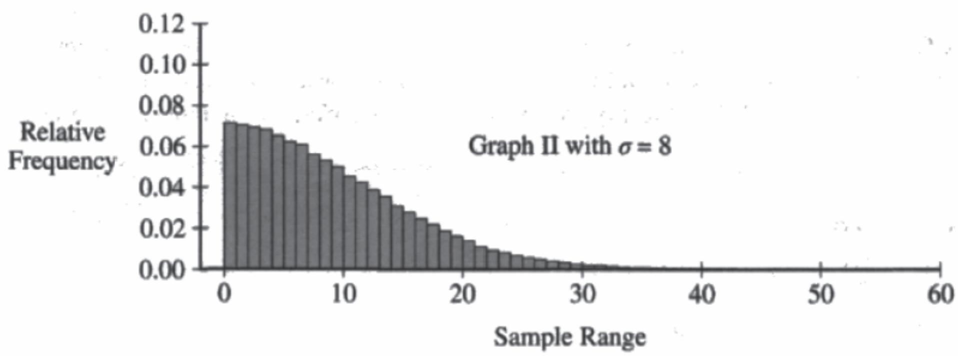
Question 6

Continue your response to **QUESTION 6** on this page.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.





Question 6

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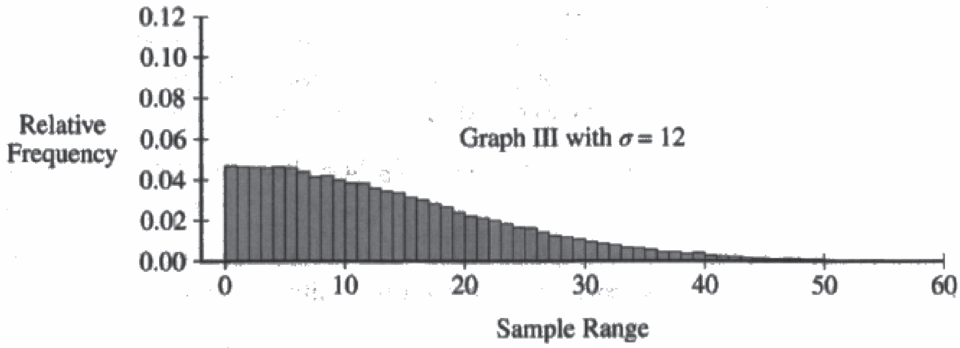


(c) Use the information in the graphs to complete the following.

(i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

Graph ~~one~~ I is strongly skewed right with a center range of about 5. There is minimal spread in this distribution with most sample means falling near the center. The distribution is unimodal with no apparent outliers.

(ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

As the standard deviation increases, the sampling distribution increases in spread. While they are all still unimodal with no clear outliers, the distribution flattens out a lot, increasing the range. It goes from around 20-30-50 with every 3 increase in standard deviation. This means the centers are also moving to the right and getting higher, even if the distributions are still skewed right.

GO ON TO THE NEXT PAGE.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Question 6

Continue your response to **QUESTION 6** on this page.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg.

(d) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

(i) Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

Yes, normally with a sampling distribution with a $\sigma = 5$, the range should be about 20mg. 10 mg is 10 less than the predicted range ~~with~~ which is 2 standard deviations away from the predicted range. That means this range should be considered unusual because there is only about a 3% chance it would fall there.

(ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

Yes, because the 10mg range is two standard deviations away from the predicted range at $\sigma = 5$, we can conclude the machine is not working properly. That is because there is only a ~~3%~~ 1.5% chance the range would fall this high above the predicted due to chance. So it is highly unlikely the machine is working right.

Sample 6C

Question 6

Begin your response to **QUESTION 6** on this page.

STATISTICS

SECTION II, Part B

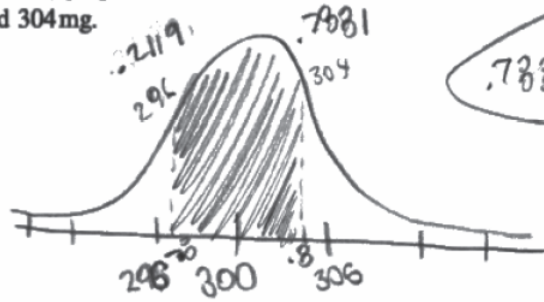
Suggested Time—25 minutes

1 Question

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A jewelry company uses a machine to apply a coating of gold on a certain style of necklace. The amount of gold applied to a necklace is approximately normally distributed. When the machine is working properly, the amount of gold applied to a necklace has a mean of 300 milligrams (mg) and standard deviation of 5 mg.

   (a) A necklace is randomly selected from the necklaces produced by the machine. Assuming that the machine is working properly, calculate the probability that the amount of gold applied to the necklace is between 296 mg and 304 mg.



.7881 − .2119 = .5762

304 − 300 = 4/5 = .8

296 − 300 = −4/5 = −.8

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Q5214/14

Question 6

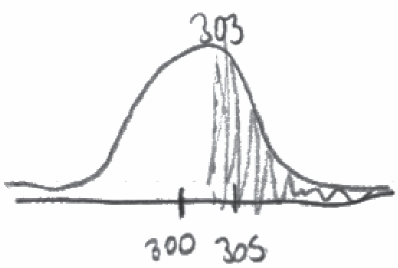
Continue your response to QUESTION 6 on this page.

The jewelry company wants to make sure the machine is working properly. Each day, Cleo, a statistician at the jewelry company, will take a random sample of the necklaces produced that day. Each selected necklace will be melted down and the amount of the gold applied to that necklace will be determined. Because a necklace must be destroyed to determine the amount of gold that was applied, Cleo will use random samples of size $n = 2$ necklaces.

Cleo starts by considering the mean amount of gold being applied to the necklaces. After Cleo takes a random sample of $n = 2$ necklaces, she computes the sample mean amount of gold applied to the two necklaces.

(b) Suppose the machine is working properly with a population mean amount of gold being applied of 300 mg and a population standard deviation of 5 mg.

(i) Calculate the probability that the sample mean amount of gold applied to a random sample of $n = 2$ necklaces will be greater than 303 mg.



$\frac{303-300}{5} = .6 = .7257$

$1 - .7257 = .2743$

(ii) Suppose Cleo took a random sample of $n = 2$ necklaces that resulted in a sample mean amount of gold applied of 303 mg. Would that result indicate that the population mean amount of gold being applied by the machine is different from 300 mg? Justify your answer without performing an inference procedure.

No, because it does not pass the central limit theorem.

CLT: $n > 30$

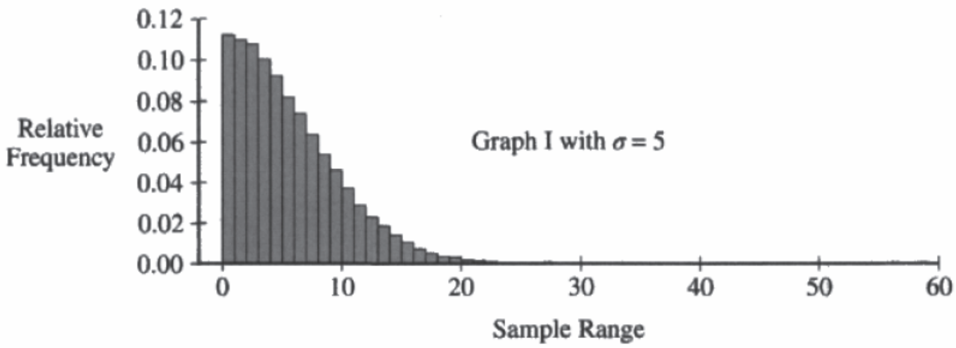
2 is not greater than 30

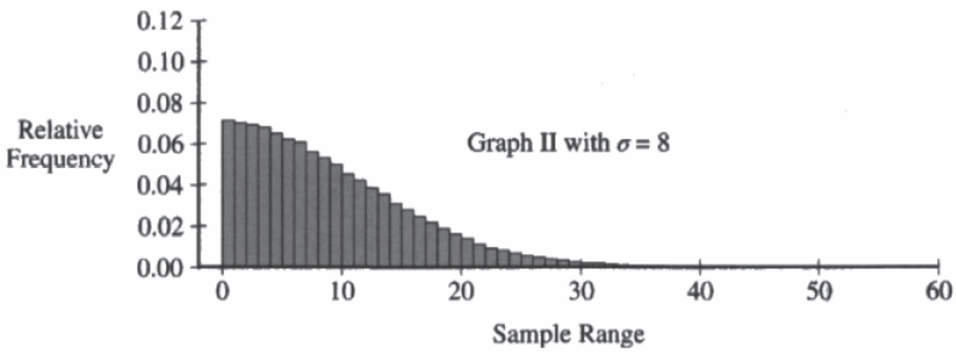
Question 6

Continue your response to **QUESTION 6** on this page.

Now, Cleo will consider the variation in the amount of gold the machine applies to the necklaces. Because of the small sample size, $n = 2$, Cleo will use the sample range of the data for the two randomly selected necklaces, rather than the sample standard deviation.

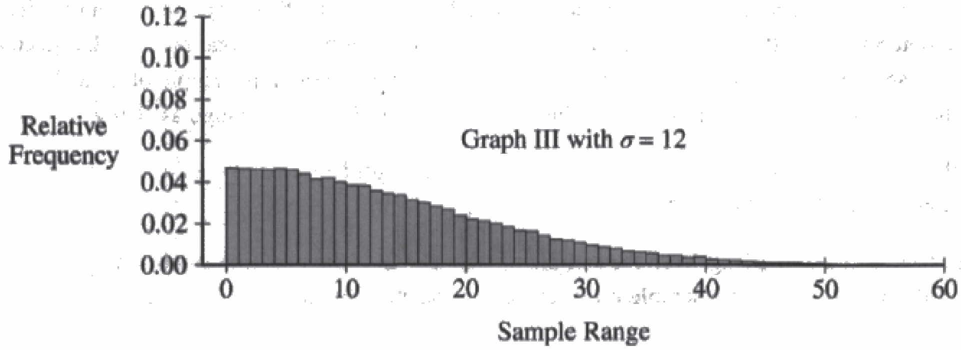
Cleo will investigate the behavior of the range for samples of size $n = 2$. She will simulate the sampling distribution of the range of the amount of gold applied to two randomly sampled necklaces. Cleo generates 100,000 random samples of size $n = 2$ independent values from a normal distribution with mean $\mu = 300$ and standard deviation $\sigma = 5$. The range is calculated for the two observations in each sample. The simulated sampling distribution of the range is shown in Graph I. This process is repeated using $\sigma = 8$, as shown in Graph II, and again using $\sigma = 12$, as shown in Graph III.





Question 6

Continue your response to QUESTION 6 on this page.



(c) Use the information in the graphs to complete the following.

(i) Describe the sampling distribution of the sample range for random samples of size $n = 2$ from a normal distribution with standard deviation $\sigma = 5$, as shown in Graph I.

The sampling distribution is strongly skewed to the right.

(ii) Describe how the sampling distribution of the sample range for samples of size $n = 2$ changes as the value of the population standard deviation σ increases.

As the s.d. increases, the sample distribution ~~range~~ sample ranges also increases.

Question 6

Continue your response to QUESTION 6 on this page.

Recall that Cleo needs to consider both the mean and standard deviation of the amount of gold applied to necklaces to determine whether the machine is working properly. Suppose that one month later, Cleo is again checking the machine to make sure it is working properly. Cleo takes a random sample of 2 necklaces and calculates the sample mean amount of gold applied as 303 mg and the sample range as 10 mg.

(d) Recall that the machine is working properly if the amount of gold applied to the necklaces has a mean of 300 mg and standard deviation of 5 mg.

(i) Consider Cleo's range of 10 mg from the sample of size $n = 2$. If the machine is working properly with a standard deviation of 5 mg, is a sample range of 10 mg unusual? Justify your answer.

Yes, because the sample range of the machine working properly is 5mg, which is smaller than 10mg, which also means that the s.d. is larger than 5mg.

(ii) Do Cleo's sample mean of 303 mg and range of 10 mg indicate that the machine is not working properly? Explain your answer.

Yes, because ~~a~~ range of 10mg contains data outside of the correct range of 5mg.

Question 6

Note: Student samples are quoted verbatim and may contain spelling and grammatical errors.

Overview

The primary goals of the question were to assess a student's ability to (1) calculate the probability a normally distributed random variable is between two values; (2) calculate the probability a sample mean is greater than a value using the distribution of the sample mean; (3) interpret a probability to determine whether it implies the population mean is a different value than what was assumed; (4) describe the sampling distribution of the sample range, when provided a graph of the results of a simulation; (5) describe how the sampling distribution of the sample range changes as the population standard deviation increases, based on simulated graphs; (6) determine if a value for the sample range is unlikely based on the results of a simulation; and (7) determine whether a machine is working properly based on the previously calculated probability for a mean and the simulated sampling distribution for the sample range.

This question primarily assesses skills in skill category 2: Data Analysis, skill category 3: Using Probability and Simulation, and skill category 4: Statistical Argumentation. Skills required for responding to this question include (2.A) Describe data presented numerically or graphically, (2.D) Compare distributions or relative positions of points within a distribution, (3.A) Determine relative frequencies, proportions, or probabilities using simulation or calculations, and (4.B) Interpret statistical calculations and findings to assign meaning or assess a claim.

This question covers content from Unit 1: Exploring One-Variable Data, Unit 4: Probability, Random Variables, and Probability Distributions, and Unit 5: Sampling Distributions of the course framework in the AP Statistics Course and Exam Description. Refer to topics 1.8, 1.9, 4.7 and 5.2, and learning objectives UNC-1.M, UNC-1.N, VAR-5.B, and VAR-6.A.

Sample: 6A
Score: 4

The response earned the following: Part (a) – E; Part (b) – E; Part (c) – E; Part (d) – E.

In part (a) the response shows a sketch of a normal curve. To the right of the sketch, the mean and standard deviation are satisfied. Therefore, the response satisfies component 1. The boundary values are stated, and standard probability notation is used, satisfying component 2. The response reports the correct probability, satisfying component 3. Part (a) was scored essentially correct (E). In part (b-i) the response shows correct work in computing the standard deviation, shows a sketch, and uses standard random variable notation, satisfying component 1. The response indicates the correct boundary with a probability statement, satisfying component 2. The response reports the correct probability, satisfying component 3. Although there is a small notational error, this was ignored in scoring the response. In part (b-ii) the response states, "No," satisfying component 4. The response interprets the probability from part (b-i) correctly, by stating, "it is not unlikely to get that extreme of an observation," satisfying component 5. The response satisfies all five components. Part (b) was scored essentially correct (E). In part (c-i) the response states, "The distribution is skewed to the right," satisfying component 1. The response indicates the median sample range is around 4–5. Because 5 mg is within the range of acceptable values, component 2 is satisfied. Finally, the response reports that the sample ranges have a range of about 23, satisfying component 3. In part (c-ii) the response states, "The sampling distributions become less skewed to the right," which is extraneous. The response goes on to indicate that the range and median of the sample ranges increase, satisfying components 4 and 5. Because all five components are satisfied, part (c) was scored essentially correct (E). In part (d-i) the response concludes the sample range is not unusual at the end of the response, satisfying component 1. The response indicates using the area under Graph I for ranges of 10 or greater. The response attempts to estimate the average height of 12 bars at 10 or greater to estimate the simulated *p*-value. Finally, the response uses this approximate probability to justify the response, satisfying component 2.

Question 6 (continued)

In part (d-ii) the response states, "No," satisfying component 3. The response then references the sample mean and range having already been deemed not to be unusual. Because the response made correct numerical arguments based on the probabilities in parts (b-i) and (d-i), the response satisfies components 4 and 5. Part (d) was scored essentially correct (E).

Sample: 6B
Score: 2

The response earned the following: Part (a) – P; Part (b) – P; Part (c) – E; Part (d) – I.

In part (a) the response uses a sketch of a normal distribution with the mean and one standard deviation, noted under the graph, satisfying component 1. The response does not label the graph, or use labeled calculator syntax, standard notation, or a statement in words to establish correct boundaries, and, therefore, does not satisfy component 2. Finally, the response reports a correct probability, satisfying component 3. Part (a) was scored partially correct (P). In part (b-i) the response indicates normality by calculator syntax and reports the correct standard error. However, the response does not identify the mean of the sampling distribution. Therefore component 1 is not satisfied. The unlabeled calculator boundaries do not satisfy component 2. The response goes on to calculate a correct probability, satisfying component 3. In part (b-ii) the response states, "No," satisfying component 4. The response justifies the answer by indicating that 303 is within one standard deviation of the mean, satisfying component 5. Part (b) was scored partially correct (P). In part (c-i) the response states, "Graph I is strongly skewed right," satisfying component 1. The response also reports a center of about 5, satisfying component 2. There is no acceptable value given for spread, failing to satisfy component 3. In part (c-i) the last two sentences of the response are considered extraneous. In part (c-ii) the response goes on to state, "the sampling distribution increases in spread," and "the centers are also moving to the right and getting higher," satisfying components 4 and 5. Because the response satisfies four of the five components, part (c) was scored essentially correct (E). In part (d-i) the response includes the incorrect answer of "Yes," failing to satisfy component 1, with incorrect justification. In part (d-ii) the response reiterates an incorrect conclusion and fails to correctly address the mean or the range. Part (d) was scored incorrect (I).

Sample: 6C
Score: 1

The response earned the following: Part (a) – E; Part (b) – P; Part (c) – I; Part (d) – I.

In part (a) the response uses a sketch of a normal curve, with mean, standard deviation, and boundary values labeled. The response also includes a correct probability. Part (a) was scored essentially correct (E). In part (b-i) the response uses the incorrect standard deviation, failing to satisfy component 1. The sketch has the correct boundary and direction, satisfying component 2. The response indicates a probability consistent with components 1 and 2, satisfying component 3. In part (b-ii) the response states, "No," satisfying component 4. However, the justification is incorrect, failing to satisfy component 5. Three of the five components are satisfied. Part (b) was scored partially correct (P). In part (c-i) the response indicates skewness to the right, satisfying component 1. The response fails to address components 2 or 3. In part (c-ii) the response states, "As the s.d. increases, the sample distribution for sample ranges also increases." However, the response does not indicate what it is about the sampling distribution that increases. Therefore, neither components 4 nor 5 are satisfied. Part (c) was scored incorrect (I). In part (d-i) the response includes the incorrect answer of "Yes," failing to satisfy component 1 with incorrect justification. In part (d-ii) the response reiterates an incorrect conclusion and fails to correctly address the mean or the range. Part (d) was scored incorrect (I). Because the response earned 1.5 points, holistic scoring must be used. In part (a), standard notation is not used, and the response contains an incorrect string of equalities. In part (b-i) the response includes an incorrect string of equalities. In part (b-ii) the response satisfies component 4 by stating, "No." However, the response is based on incorrect justification. In parts (c) and (d), the communication is weak. The response was scored 1.