



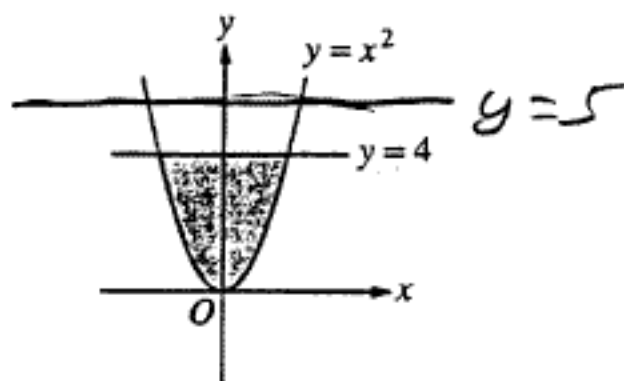
## AP Calculus AB 1999 Sample Student Responses

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2. The shaded region,  $R$ , is bounded by the graph of  $y = x^2$  and the line  $y = 4$ , as shown in the figure above.

(a) Find the area of  $R$ .

$$\int_{-2}^2 4 - x^2 dx = \frac{32}{3}$$

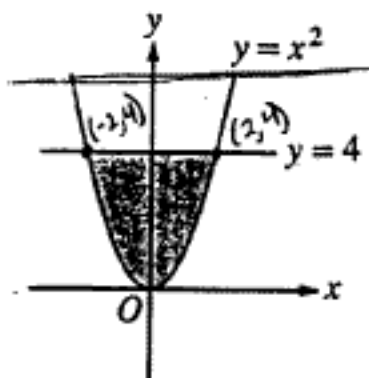
(b) Find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.

$$\pi \int_{-2}^2 4^2 - x^4 dx = \pi \left[ 16x - \frac{1}{5}x^5 \right]_{-2}^2} = 51.2\pi$$

- (c) There exists a number  $k$ ,  $k > 4$ , such that when  $R$  is revolved about the line  $y = k$ , the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .

$$\pi \int_{-2}^2 (k-x^2)^2 - (k-4)^2 dx = 51.2\pi$$

GO ON TO THE NEXT PAGE 



2. The shaded region,  $R$ , is bounded by the graph of  $y = x^2$  and the line  $y = 4$ , as shown in the figure above.

(a) Find the area of  $R$ .

$$A = \int_{-2}^2 (4 - x^2) dx = 2 \int_0^2 (4 - x^2) dx = 2 \left[ 4x - \frac{1}{3} x^3 \right]_0^2$$

$$A = 2 \left( 4(2) - \frac{1}{3}(2)^3 - 0 - 0 \right) = 2 \left( 8 - \frac{8}{3} \right) = \frac{32}{3}$$

$$\boxed{A_R = \frac{32}{3}}$$

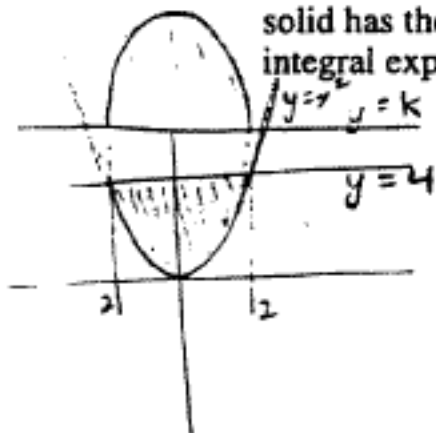
(b) Find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.

$$V = \pi \int_{-2}^2 (4^2 - (x^2)^2) dx = 2\pi \int_0^2 (16 - x^4) dx = 2\pi \left[ 16x - \frac{1}{5} x^5 \right]_0^2$$

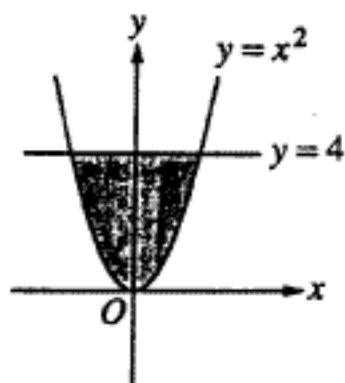
$$V = 2\pi \left[ 16(2) - \frac{1}{5}(2)^5 - 0 - 0 \right] = 2\pi \left( 32 - \frac{32}{5} \right) = \frac{256\pi}{5}$$

$$\boxed{V = \frac{256\pi}{5} = 160.850}$$

- (c) There exists a number  $k$ ,  $k > 4$ , such that when  $R$  is revolved about the line  $y = k$ , the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .



$$V = \pi \int_{-2}^2 ((k-4)^2 - (k-x^2)^2) dx$$



2. The shaded region,  $R$ , is bounded by the graph of  $y = x^2$  and the line  $y = 4$ , as shown in the figure above.

(a) Find the area of  $R$ .

$$\int_{-2}^2 x^2 - 4 = \int_{-2}^0 \frac{x^3}{3} - 4x + \int_0^2 \frac{x^3}{3} - 4x = \boxed{\frac{32}{3}}$$

(b) Find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.

$$V = \pi \int_{-2}^2 (x^2 - 4)^2 dx = \pi \int_{-2}^2 x^4 - 16x dx$$

$$V = \pi \left[ \frac{x^5}{5} - 16x \right]_{-2}^2 \Rightarrow \boxed{V \approx 59.733\pi}$$

- (c) There exists a number  $k$ ,  $k > 4$ , such that when  $R$  is revolved about the line  $y = k$ , the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .

$$59.733\pi = \pi \int_{-2}^2 (x^2 - k)^2 dx$$

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