



AP Calculus AB 1999 Sample Student Responses

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t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
→ 6	10.8
9	11.2
→ 12	11.4
15	11.3
→ 18	10.7
21	10.2
→ 24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$. Using correct units, explain the meaning of your answer in terms of water flow.

$$\sum_{i=1}^4 R(c_i) \Delta t = 10.4 \cdot 6 + 11.2 \cdot 6 + 11.3 \cdot 6 + 10.2 \cdot 6$$

where $c_i =$ midpoint of interval
($t = 3, 9, 15, 21$)

$= 258.6$ gallons

$=$ # of gallons of water to flow out of a pipe from $t = 0$ to $t = 24$

- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Yes $\rightarrow \frac{R(24) - R(0)}{24 - 0} = 0$, therefore, by the Mean Value Theorem, there is some t in $(0, 24)$ such that $R'(t) = 0$

- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.

$$\frac{1}{79} \int_0^{24} (768 + 23t - t^2) dt$$

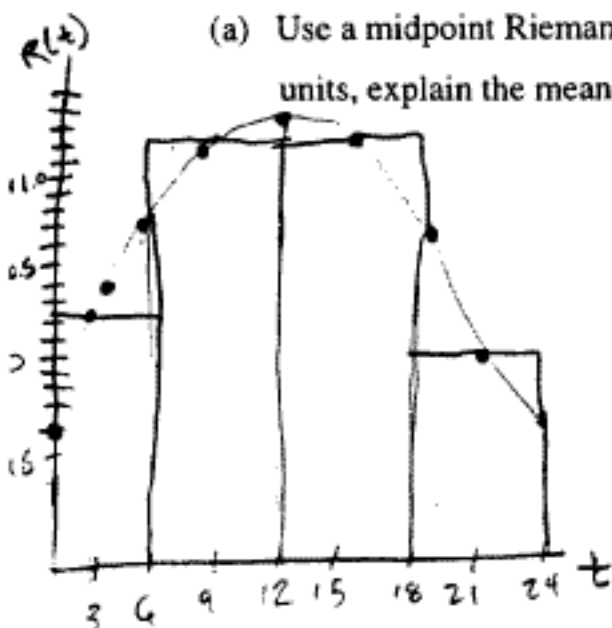
24

$$\approx 10.7848 \text{ gallons / hour}$$

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$. Using correct units, explain the meaning of your answer in terms of water flow.



$$\text{b.h.} \\ (2 \cdot 10.4) + (2 \cdot 11.2) + (2 \cdot 11.3) + (2 \cdot 10.2)$$

$$\int_0^{24} R(t) dt \approx 86.2 \text{ gallons}$$

It means that 86.2 gallons of water flowed out of the pipe for that 24 hour period.

- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

Yes there is. At approx. $t \approx 12$ the slope of a tangent line to that point is 0. $\therefore R'(t) = 0$ at that point.

- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79} (768 + 23t - t^2)$.
Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
Indicate units of measure.

$$\text{Total Flow} = \int_0^{24} Q(t) dt$$

$$= 258.83544 \text{ gallons}$$

$$\text{average rate} = \frac{258.83544 \text{ gallons}}{24 \text{ hours}}$$

$$= 10.785 \text{ gallons/hour}$$

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t . The table above shows the rate as measured every 3 hours for a 24-hour period.

- (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t) dt$. Using correct units, explain the meaning of your answer in terms of water flow.

$$RS = \frac{b-a}{n} [f(x_1) + f(x_2) + f(x_3) + \dots + f(x_n)]$$

$$RS = \frac{24}{4} [f(3) + f(9) + f(15) + f(21)]$$

$$RS = 6 [10.4 + 11.2 + 11.3 + 10.2]$$

$$RS = 258.600 \text{ gallons}$$

after 24 hours 258,600 gallons of water have flowed from the pipe

- (b) Is there some time t , $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

$R'(t)$ is the slope of $R(t)$
 - if $R(t)$ is a velocity then $R'(t)$
 is the acceleration or change in velocity
 between time $t=12$ and time $t=15$ the
 change in velocity changes from positive
 to negative so $R'(t)$ must = 0
 at some time t $12 \leq t \leq 15$ which is
 between 0 and 24

- (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$.
 Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period.
 Indicate units of measure.

$$\text{Avg Rate} = \frac{Q(24) - Q(0)}{24 - 0}$$

$$Q(24) = \frac{1}{79}(768 + 23(24) - (24)^2)$$

$$Q(24) = 9.418$$

$$Q(0) = \frac{1}{79}(768 + 23(0) - (0)^2)$$

$$Q(0) = 9.722$$

$$\text{Avg Rate} = \frac{9.418 - 9.722}{24}$$

$$\text{Avg Rate} = -0.013 \text{ gallons per hour}$$