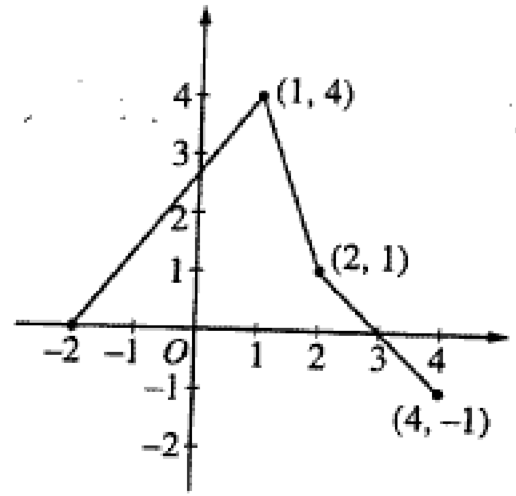
AP Calculus AB
1999 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

B



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$$g(4) = \int_1^4 f(t)dt = \int_1^2 f(t)\,dt + \int_2^4 f(t)\,dt$$
$$= \frac{5}{2} + 0 = \frac{5}{2}$$

$$g(-2) = \int_1^{-2} f(t)dt = -\int_{-2}^{1} f(t)\,dt = -\left(\tfrac{1}{2}(3)(4)\right) = -6$$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$$g'(x) = f(x)$$
$$g'(1) = f(1) = 4$$

Continue problem 5 on page 13.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

absolute minimum would occur at endpoint or when $g'(x) = 0$

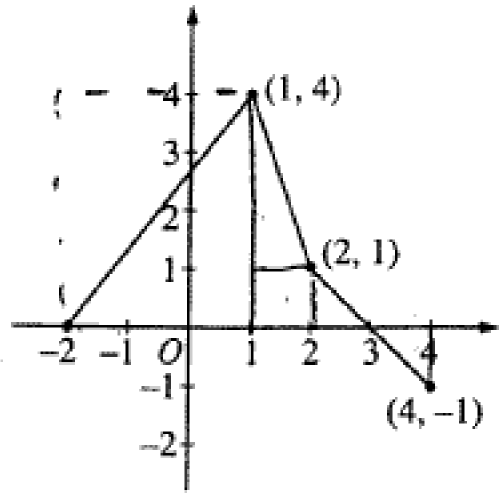
	x	$g(x)$
endpt / $g'(x)=0$	-2	-6
$g'(x)=0$	3	3
endpt	4	$\frac{5}{2}$

since $g(-2) < g(3)$ and $g(-2) < g(4)$
the absolute minimum occurs at -2
and is $g(-2) = -6$.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

$x=1$ is an inflection point because $g''(x) > 0$ for $x < 1$ and $g''(x) < 0$ for $x > 1$. $x = 2$ is not an inflection point because $g''(x) < 0$ for $x < 2$ and $x > 2$.

D



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$g(x) = \int_1^x f(t)\,dt$

$g(4) = \int_1^4 f(t)\,dt = 1 + \frac{3}{2} + \frac{1}{2} - \frac{1}{2} \Rightarrow$

$\boxed{g(4) = \frac{5}{2}}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^1 f(t)\,dt = -\left[\frac{3 \times 4}{2}\right] = -6$

$\boxed{g(-2) = -6}$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$\frac{dg}{dt} = \frac{d}{dt}\int_1^x f(t)\,dt$

$\frac{dg}{dt} = f(t)\Big|_x$

$\boxed{\frac{d}{dt}g(1) = 4}$

Continue problem 5 on page 13.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

$g' = f(t) = 0$
$t = 3$

f: + + + + 0 − − − (3), pos | neg

$t > 3$, g increasing
$t < 3$, g decreasing
∴ $t = 3$ is rel max

⇨ absolute min must be one of the endpoints

OR $x = 4$, $x = -2$

$$-\int_{-2}^{1} f(t)\,dt < \int_{1}^{4} f(t)\,dt$$

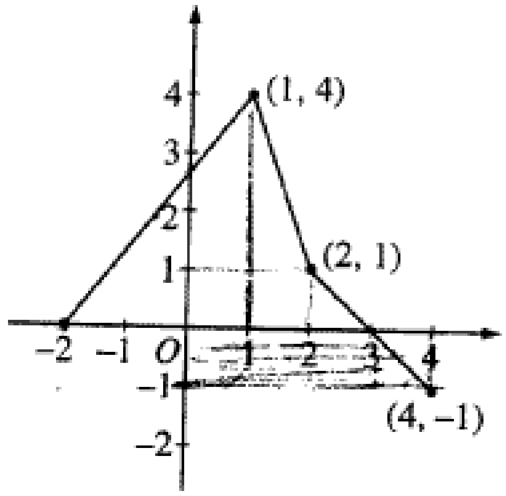
$x = -2$ is absolute minimum because the area between f(t) and the x axis in the interval [-2, 1] by -1 is clearly less than the area between f(t) and the x axis on the interval [1, 4].

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

$x = 1$ is a point of inflection because the slope of f(t) [which equals $g''(x)$] changes from positive to negative at $x = 1$. At $x = 2$, the slope of f(t) stays positive for $1 < x < 2$ and $2 < x < 4$.

GO ON TO THE NEXT PAGE

E



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

(a) Compute $g(4)$ and $g(-2)$.

$g(4) = \int_1^4 f(t)\,dt$

~~$A = \frac{1}{2}(b_1 + b_2)h$~~

~~$A = \frac{1}{2}($~~

~~$g(1) = \int_0^1 (4/3 x$~~

$A_1 = \frac{1}{2}(b_1 + b_2)h$

$A_1 = \frac{1}{2}(6.5) \times 1$

$A_1 = 3.25$

$A_2 = \frac{1}{2}(4+1)(1)$

$A_2 = 2.5$

$g(4) = 3.25 + 2.5 + 1$

$\boxed{g(4) = 6.75}$

$g(-2) = \int_1^{-2} f(t)\,dt$

$A_\Delta = \frac{1}{2}bh$

$A_\Delta = \frac{1}{2}(2 \cdot 2.5)$

$A_\Delta = 5$

$\boxed{g(-2) = 5}$

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

$g'(x) = f(x)$

$g'(1) = f(1)$

$f(1) = 4$

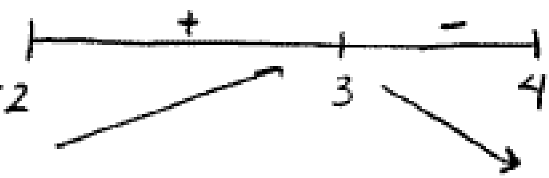
$\boxed{\text{Instantaneous rate of change} = 4}$

Continue problem 5 on page 13.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

$g'(x) = f(t)$

$f(t)$



$f(-2) = 0$

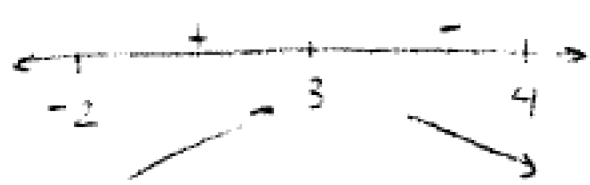
$f(4) = -1$

$\boxed{\therefore \text{ There is an absolute minimum for } g(x) \text{ over } [-2,4] \text{ at } x = 4}$

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

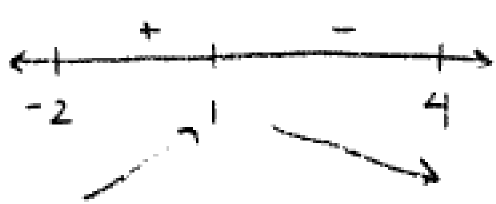
$g'(x) = f(x)$

$f(x)$



$f'(x) = g''(x)$

$f'(x)$



$\boxed{x = 1 \text{ is a pt of inflection for } g(x)}$

GO ON TO THE NEXT PAGE