



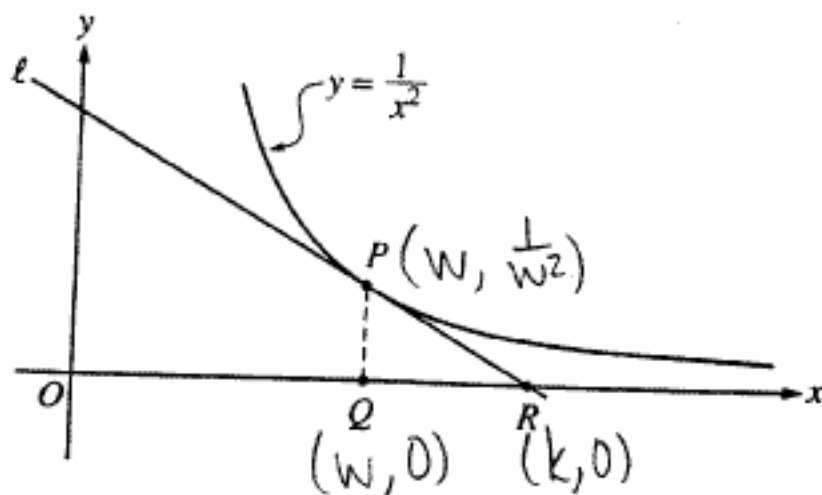
AP Calculus AB 1999 Sample Student Responses

The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[™], the Advanced Placement Program[®] (AP[®]), and Pacesetter[®]. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2001 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, and the acorn logo are registered trademarks of the College Entrance Examination Board.



6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P , with coordinates $(w, \frac{1}{w^2})$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x -axis at point R , with coordinates $(k, 0)$.

- (a) Find the value of k when $w = 3$.

$$y = \frac{1}{x^2} = (x)^{-2}$$

$$y' = -2(x)^{-3} = -\frac{2}{x^3}$$

$$w = 3$$

$$y' = -\frac{2}{3^3} = -\frac{2}{27}$$

$$b = \frac{1}{9} = \frac{1}{3}$$

$$y = -\frac{2}{27}x + \frac{1}{3}$$

$$0 = -\frac{2}{27}x + \frac{1}{3}$$

$$\frac{1}{3} = \frac{2}{27}x$$

$$x = \boxed{k = 4.5}$$

- (b) For all $w > 0$, find k in terms of w .

$$y = mx + b$$

$$m = -\frac{2}{w^3}$$

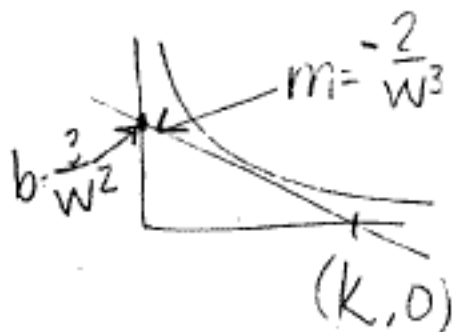
$$b = \frac{3}{w^2}$$

$$y = -\frac{2}{w^3}x + \frac{3}{w^2}$$

point $(k, 0)$

$$0 = -\frac{2}{w^3} \cdot k + \frac{3}{w^2}$$

$$k = \frac{+\frac{3}{w^2}}{+\frac{2}{w^3}} = \boxed{k = \frac{3}{2}w}$$



- (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

$$k = \frac{3}{2}w$$

$$\frac{dk}{dt} = \frac{3}{2} \frac{dw}{dt}$$

$$\frac{dk}{dt} = \frac{3}{2} \cdot 7 = \boxed{\frac{21}{2} \frac{\text{units}}{\text{sec}}}$$

- (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

$$A_{\triangle PQR} = \frac{1}{2}bh$$

$$h = \frac{1}{w^2}$$

$$b = k - w$$

$$k = \frac{3}{2}w$$

$$b = \frac{3}{2}w - w$$

$$b = \frac{1}{2}w$$

$$\text{Area} = \left(\frac{1}{2}\right) \left(\frac{1}{w^2}\right) \left(\frac{w}{2}\right)$$

$$\text{Area} = \frac{1}{4w} \cdot \frac{1}{4}(w)^{-1}$$

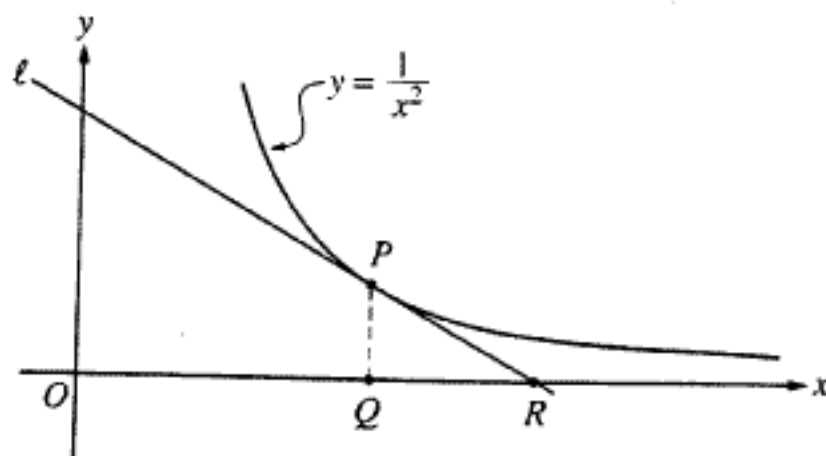
$$\frac{dA}{dt} = -\frac{1}{4w^2} \cdot \frac{dw}{dt} = \left(-\frac{1}{4}\right) \left(\frac{1}{25}\right) (7)$$

$$\frac{dA}{dt} = \frac{-7 \text{ unit}^2}{100 \text{ sec}} = -0.07 \frac{\text{unit}^2}{\text{sec}}$$

decreasing

END OF EXAMINATION

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.



6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P , with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x -axis at point R , with coordinates $(k, 0)$.

- (a) Find the value of k when $w = 3$.



slope

$$y = \frac{1}{x^2} = x^{-2}$$

$$P = \left(3, \frac{1}{9}\right)$$

$$y' = -2x^{-3}$$

$$Q = (3, 0)$$

$$y' = -.07407$$

$$(y - \frac{1}{9}) = -.07407(x - 3)$$

$$y = -.07407x + .3333$$

$$0 = -.07407x + .3333$$

$$k = x = \boxed{4.500}$$

- (b) For all $w > 0$, find k in terms of w .

$$y - \frac{1}{w^2} = (-2w^{-3})(x - w)$$

$$y = 0 \quad x = k$$

$$-\frac{1}{w^2} = (-2w^{-3})(k - w)$$

$$\frac{w^3}{2w^2} = k - w$$

$$\frac{1}{2}w = k - w$$

$$k = \boxed{1.5w}$$

- (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

$$K = 1.5w \quad dw = 7$$

$$\frac{dK}{dt} = 1.5dw$$

$$\frac{dK}{dt} = (1.5)(7)$$

$$\frac{dK}{dt} = \boxed{10.5 \text{ units per second}}$$

- (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

$$A = \frac{1}{2} \left(\frac{1}{w^2} \right) (K - w) \quad dw = 7 \quad w = 5$$

$$dK = 10.5 \quad K = 4.5$$

$$A = \frac{1}{2} \cdot w^{-2} \cdot (K - w)$$

$$A = \frac{w^{-2}}{2} (K - w)$$

$$\frac{dA}{dt} = \frac{w^{-2}}{2} (0 \cdot K - dw) + (K - w)(-w^{-3})$$

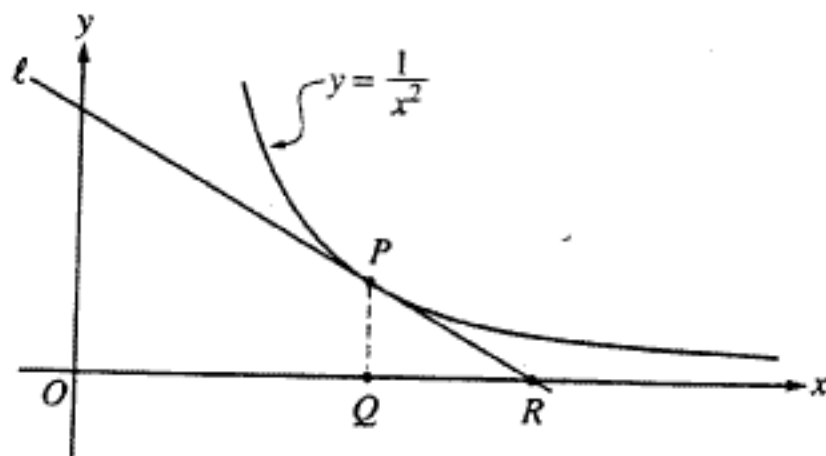
$$= (.02)(3.5) + (-.5)(-.008)$$

$$= .07 + .004$$

$$\frac{dA}{dt} = \boxed{.074 \text{ units}^2 \text{ per second}}$$

Increasing

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.



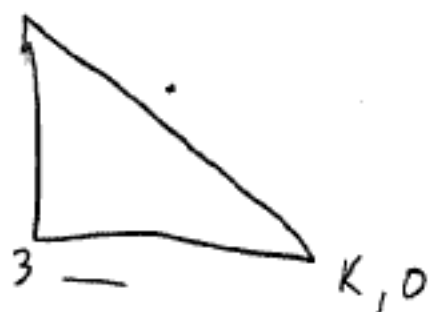
6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P , with coordinates $(w, \frac{1}{w^2})$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x -axis at point R , with coordinates $(k, 0)$.

(a) Find the value of k when $w = 3$.

$$y = x^{-2} \quad y' = -\frac{2}{w^3} \quad \text{at point } p \text{ if } w=3 \text{ then } y(3) = \frac{1}{3^2} = \frac{1}{9}$$

ℓ 's slope $m = -\frac{2}{27}$

$3, \frac{1}{9}$



$$\frac{0 - \frac{1}{9}}{k - 3} = -\frac{2}{27}$$

$$k = 4.5$$

(b) For all $w > 0$, find k in terms of w .

$$y' = -\frac{2}{w^3}$$

$$-\frac{2}{w^3} = \frac{\frac{1}{w^2} - 0}{w - k}$$

$$-2(w - k) = w$$

$$-2w + 2k = w$$

$$2k = w + 2w$$

$$k = \frac{3w}{2}$$

- (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

$$\frac{dw}{dt} = 7$$

$$w = 7t + C$$

-
- (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

$$\text{Area of } \triangle = \frac{1}{2} (k-w) \frac{1}{w^2}$$

END OF EXAMINATION

-
- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK COVER OF THIS BOOKLET.
 - CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX ON THE BACK COVER.
 - MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMINATIONS YOU HAVE TAKEN THIS YEAR.