



AP Calculus AB 2000 Student Samples

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Work for problem 5(a)

$$(a) \quad xy^2 - x^3y = 6$$

$$x(2y)\left(\frac{dy}{dx}\right) + y^2 - 3x^2y - x^3\left(\frac{dy}{dx}\right) = 0$$

$$\left(\frac{dy}{dx}\right)2yx - \left(\frac{dy}{dx}\right)x^3 = 3x^2y - y^2$$

$$\frac{dy}{dx}(2yx - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2yx - x^3}$$

Work for problem 5(b)

$$(1)y^2 - (1)y = 6$$

$$y^2 - y - 6 = 0$$

$$(y+2)(y-3) = 0 \quad (1, -2) \text{ and } (1, 3)$$

$$\text{at } (1, -2) \quad y = -2 \text{ or } 3$$

$$\frac{dy}{dx} \Big|_{(1, -2)} = \frac{3(1)(-2) - 4}{2(-2)(1) - (1)} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$$

$$y = 2(x-1) - 2$$

$$y = 2x - 4$$

$$\text{at } (1, 3) \quad \frac{dy}{dx} \Big|_{(1, 3)} = \frac{3(3) - (3)^2}{2(3) - 1} = \frac{9 - 9}{6 - 1} = 0$$

$$y = 3$$

Continue problem 5 on page 13.

Work for problem 5(c)

(c) $\frac{dy}{dx}$ does not exist, y does exist.
 $\frac{dy}{dx}$ does not change sign on either side of pt.

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3} \text{ which D.N.E.}$$

when $2xy - x^3 = 0$

$$x(2y - x^2) = 0$$

$$x = 0 \text{ or } 2y - x^2 = 0$$

$$2y = x^2$$

$$y = \frac{x^2}{2}$$

vals

$$(0)y - (0)y = 6$$

$$0 \neq 6$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$\frac{x^5}{4} - \frac{x^5}{2} = 6$$

$$\frac{x^5}{4} - \frac{2x^5}{4} = 6$$

$$-\frac{x^5}{4} = 6$$

$$x^5 = -24$$

$$x = \sqrt[5]{-24}$$

y does exist when
 $xy^2 - x^3y = 6$

Work for problem 5(a)

$$xy^2 - (x^3y) = 6$$

$$(x \cdot 2yy') + (y^2) - (x^3y') + (3x^2y \cdot y') = 0$$

$$2xyy' + y^2 - x^3y' - 3x^2y \cdot y' = 0$$

$$2xyy' - x^3y' = 3x^2y - y^2$$

$$y'(2xy - x^3) = 3x^2y - y^2$$

$$y' = \frac{3x^2y - y^2}{2xy - x^3}$$

Work for problem 5(b)

$$x = 1$$

$$m = \frac{3x^2y - y^2}{2xy - x^3} \quad \text{for } (1, 3) \quad m = 0$$

$$m = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$$

$$m = \frac{9 - 9}{6 - 1} = 0$$

$$xy^2 - x^3y = 6$$

$$(1)y^2 - (1)^3y = 6$$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y - 3)(y + 2)$$

$$y = 3, -2$$

So p+s are (1, 3) and (1, -2)

$$m = \frac{3x^2y - y^2}{2xy - x^3} \quad \text{at } (1, -2)$$

$$m = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$$

$$= \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$$

- so eqⁿ for (1, 3) ; $y - 3 = 0(x - 1)$
 $y = 3$

- eqⁿ for (1, -2) ; $y + 2 = 2(x - 1)$
 $m = 2$

Continue problem 5 on page 13.

Work for problem 5(c)

tangent line = vertical when denominator of $\frac{dx}{dy}$
is 0

$$\text{so } 0 = 2xy - x^3$$

Work for problem 5(a)

$$xy^2 - x^3y = 6$$

$$x \cdot 2y \frac{dy}{dx} + y^2 \cdot 1 - x^3 \frac{dy}{dx} + y(-3x^2) = 0$$

$$2xy \frac{dy}{dx} - x^3 \frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx} (2xy - x^3) = 3x^2y - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$$

Work for problem 5(b)

$$(1)y^2 - (1)^3y = 6$$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$\boxed{(1, 6) \text{ and } (1, -4)}$$

$$\frac{1 \pm \sqrt{-1^2 - 4(1)(-6)}}{2(1)}$$

$$\frac{1 \pm \sqrt{25}}{1}$$

$$\frac{1+5}{1} \text{ or } \frac{1-5}{1}$$

$$6 \text{ or } -4$$

$$\rightarrow y - 6 = \frac{-18}{11}(x - 1)$$

$$\boxed{y = \frac{-18}{11}(x - 1) + 6}$$

$$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$$

$$\frac{3(1)^2(-4) - (-4)^2}{2(1)(-4) - 1^3} = \frac{12 - 16}{8 - 1} = \frac{-4}{7}$$

$$y - 4 = \frac{-4}{7}(x - 1)$$

$$\boxed{y = \frac{-4}{7}(x - 1) + 4}$$

$$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$$

$$\frac{3(1)^2(6) - 6^2}{2(1)(6) - (1)^3} = \frac{18 - 36}{12 - 1} = \frac{-18}{11} = \frac{dy}{dx}$$

Continue problem 5 on page 13.

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Work for problem 5(c)

$$2xy - x^3 = 0$$

$$2x \left(-\frac{4}{7}(x-1) + 4 \right) - x^3 = 0$$

$$2x \left(-\frac{4}{7}x + \frac{4}{7} + 4 \right)$$

$$-\frac{8}{7}x^2 + \frac{8}{7}x + \frac{8}{1}x = 0$$

$$-\frac{8}{7}x^2 + \frac{64}{7}x = 0$$

$$\frac{8}{7}x(x + 8) = 0$$

$$x = 0$$

$$x = -8$$