AP Calculus AB
2000 Student Samples

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Work for problem 5(a)

(a) $xy^2 - x^3y = 6$

$$x(2y)\left(\frac{dy}{dx}\right) + y^2 - 3x^2y - x^3\left(\frac{dy}{dx}\right) = 0$$

$$\left(\frac{dy}{dx}\right)2yx - \left(\frac{dy}{dx}\right)x^3 = 3x^2y - y^2$$

$$\frac{dy}{dx}(2yx - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2yx - x^3}$$

Work for problem 5(b)

$(1)y^2 - (1)y = 6$

$y^2 - y - 6 = 0$

$(y+2)(y-3) = 0$ $(1,-2)$ and $(1,3)$

$y = -2$ or 3

at $(1,-2)$

$$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{3(1)(-2) - 4}{2(-2)(1) - (1)} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$$

$y = 2(x-1) - 2$

$\boxed{y = 2x - 4}$

at $(1,3)$

$$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{3(3) - (3)^2}{2(3) - 1} = \frac{9-9}{6-1} = 0$$

$\boxed{y = 3}$

Continue problem 5 on page 13.

Work for problem 5(c)

(c) $\frac{dy}{dx}$ does not exist, y does exist

& $\frac{dy}{dx}$ does not change sign on either side of pt.

$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ which D.N.E.

when $2xy - x^3 = 0$

$x(2y - x^2) = 0$

$x = 0$ or $2y - x^2 = 0$

$2y = x^2$

$y = \frac{x^2}{2}$

y does exist when $xy^2 - x^3y = 6$

$(0)y - (0)y = 6$

$0 \neq 6$

$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$

$\frac{x^5}{4} - \frac{x^5}{2} = 6$

$\frac{x^5}{4} - \frac{2x^5}{4} = 6$

$-\frac{x^5}{4} = 6$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

Work for problem 5(a)

$$xy^2 - x^3y = 6$$

$$(x \cdot 2yy') + (y^2) - (x^3y') + (3x^2y) = 0$$

$$2xyy' + y^2 - x^3y' - 3x^2y = 0$$

$$2xyy' - x^3y' = 3x^2y - y^2$$

$$y'(2xy - x^3) = 3x^2y - y^2$$

$$y' = \frac{3x^2y - y^2}{2xy - x^3}$$

Work for problem 5(b)

$x = 1$

$$xy^2 - x^3y = 6$$

$$(1)y^2 - (1)^3y = 6$$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2)$$

$$y = 3, -2$$

So pts are (1,3) and (1,-2)

$$m = \frac{3x^2y - y^2}{2xy - x^3} \quad \text{for } (1,3)$$

$$m = 0$$

$$m = \frac{3(1)^2(3) - (3)^2}{2(1)(3) - (1)^3}$$

$$m = \frac{9-9}{6-1} = 0$$

- so eq^n for (1,3) ; $m = 0$; $y - 3 = 0(x-1)$

$$y = 3$$

$$m = \frac{3x^2y - y^2}{2xy - x^3} \quad \text{at } (1,-2)$$

$$m = \frac{3(1)^2(-2) - (-2)^2}{2(1)(-2) - (1)^3}$$

$$= \frac{-6-4}{-4-1} = \frac{-16}{-5} =$$

- eq^n for (1,-2) ; $m = 2$; $y + 2 = 2(x-1)$

Continue problem 5 on page 13.

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Work for problem 5(c)

tangent line = vertical when denominator of $\frac{dx}{dy}$ is 0

so $0 = 2xy - x^3$

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Work for problem 5(a)

$$xy^2 - x^3y = 6$$

$$x \cdot 2y\frac{dy}{dx} + y^2 \cdot 1 - x^3\frac{dy}{dx} + y(-3x^2) = 0$$

$$2xy\frac{dy}{dx} - x^3\frac{dy}{dx} = 3x^2y - y^2$$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$$

Work for problem 5(b)

$$(1)y^2 - (1)^3y = 6$$

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$\frac{1 \pm \sqrt{-1^2 - 4(1)(-6)}}{2(1)}$$

$$\frac{1 \pm \sqrt{25}}{1}$$

$$\frac{1+5}{1} \text{ or } \frac{1-5}{1}$$

$$6 \text{ or } -4$$

$$\boxed{(1,6) \text{ and } (1,-4)}$$

$$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$$

$$\frac{3(1)^2(6) - 6^2}{2(1)(6) - (1)^3} = \frac{18-36}{12-1} = \frac{-18}{11} = \frac{dy}{dx}$$

$$y - 6 = -\frac{18}{11}(x-1)$$

$$\boxed{y = -\frac{18}{11}(x-1) + 6}$$

$$\frac{3x^2y - y^2}{2xy - x^3} = \frac{dy}{dx}$$

$$\frac{3(1)^2(4) - 4^2}{2(1)(4) - 1^3} = \frac{12-16}{8-1} = -\frac{4}{7}$$

$$y - 4 = -\frac{4}{7}(x-1)$$

$$\boxed{y = -\frac{4}{7}(x-1) + 4}$$

Continue problem 5 on page 13.

Work for problem 5(c)

$$2xy - x^3 = 0$$

$$2x\left(-\tfrac{4}{7}(x-1)+4\right) - x^3 = 0$$

$$2x\left(-\tfrac{4}{7}x + \tfrac{4}{7} + 4\right)$$

$$-\tfrac{8}{7}x^2 + \tfrac{8}{7}x + 8x = 0$$

$$-\tfrac{8}{7}x^2 + 64\tfrac{4}{7}x = 0$$

$$\tfrac{8}{7}x(x+8) = 0$$

$$x = 0$$
$$x = -8$$