



## AP Calculus AB 2000 Scoring Guidelines

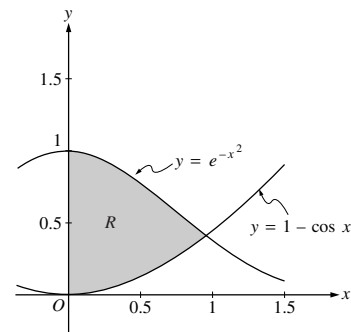
**The materials included in these files are intended for non-commercial use by AP teachers for course and exam preparation; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.**

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT<sup>®</sup>, the PSAT/NMSQT<sup>™</sup>, the Advanced Placement Program<sup>®</sup> (AP<sup>®</sup>), and Pacesetter<sup>®</sup>. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2001 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, and the acorn logo are registered trademarks of the College Entrance Examination Board.

Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $y = e^{-x^2}$ ,  $y = 1 - \cos x$ , and the  $y$ -axis, as shown in the figure above.



- (a) Find the area of the region  $R$ .
- (b) Find the volume of the solid generated when the region  $R$  is revolved about the  $x$ -axis.
- (c) The region  $R$  is the base of a solid. For this solid, each cross section perpendicular to the  $x$ -axis is a square. Find the volume of this solid.

Region  $R$

$$e^{-x^2} = 1 - \cos x \text{ at } x = 0.941944 = A$$

(a) Area =  $\int_0^A (e^{-x^2} - (1 - \cos x)) dx$   
 = 0.590 or 0.591

(b) Volume =  $\pi \int_0^A ((e^{-x^2})^2 - (1 - \cos x)^2) dx$   
 =  $0.55596\pi = 1.746$  or 1.747

(c) Volume =  $\int_0^A (e^{-x^2} - (1 - \cos x))^2 dx$   
 = 0.461

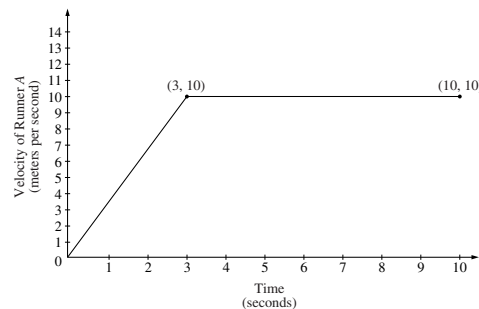
1 : Correct limits in an integral in (a), (b), or (c).

2 { 1 : integrand  
 1 : answer

3 { 2 : integrand and constant  
 < - 1 > each error  
 1 : answer

3 { 2 : integrand  
 < - 1 > each error  
 Note: 0/2 if not of the form  
 $k \int_c^d (f(x) - g(x))^2 dx$   
 1 : answer

Two runners,  $A$  and  $B$ , run on a straight racetrack for  $0 \leq t \leq 10$  seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner  $A$ . The velocity, in meters per second, of Runner  $B$  is given by the function  $v$  defined by  $v(t) = \frac{24t}{2t + 3}$ .



- (a) Find the velocity of Runner  $A$  and the velocity of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- (b) Find the acceleration of Runner  $A$  and the acceleration of Runner  $B$  at time  $t = 2$  seconds. Indicate units of measure.
- (c) Find the total distance run by Runner  $A$  and the total distance run by Runner  $B$  over the time interval  $0 \leq t \leq 10$  seconds. Indicate units of measure.

(a) Runner  $A$ : velocity  $= \frac{10}{3} \cdot 2 = \frac{20}{3}$   
 $= 6.666$  or  $6.667$  meters/sec

Runner  $B$ :  $v(2) = \frac{48}{7} = 6.857$  meters/sec

(b) Runner  $A$ : acceleration  $= \frac{10}{3} = 3.333$  meters/sec<sup>2</sup>

Runner  $B$ :  $a(2) = v'(2) = \frac{72}{(2t + 3)^2} \Big|_{t=2}$   
 $= \frac{72}{49} = 1.469$  meters/sec<sup>2</sup>

(c) Runner  $A$ : distance  $= \frac{1}{2}(3)(10) + 7(10) = 85$  meters

Runner  $B$ : distance  $= \int_0^{10} \frac{24t}{2t + 3} dt = 83.336$  meters

2 { 1 : velocity for Runner  $A$   
 1 : velocity for Runner  $B$

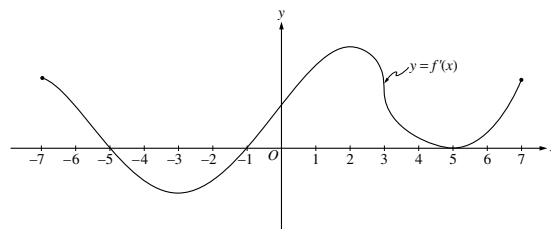
2 { 1 : acceleration for Runner  $A$   
 1 : acceleration for Runner  $B$

4 { 2 : distance for Runner  $A$   
 1 : method  
 1 : answer  
 2 : distance for Runner  $B$   
 1 : integral  
 1 : answer

(units) meters/sec in part (a), meters/sec<sup>2</sup> in part (b), and meters in part (c), or equivalent.

1: units

The figure above shows the graph of  $f'$ , the derivative of the function  $f$ , for  $-7 \leq x \leq 7$ . The graph of  $f'$  has horizontal tangent lines at  $x = -3$ ,  $x = 2$ , and  $x = 5$ , and a vertical tangent line at  $x = 3$ .



- (a) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative minimum. Justify your answer.
- (b) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f$  attains a relative maximum. Justify your answer.
- (c) Find all values of  $x$ , for  $-7 < x < 7$ , at which  $f''(x) < 0$ .
- (d) At what value of  $x$ , for  $-7 \leq x \leq 7$ , does  $f$  attain its absolute maximum? Justify your answer.

(a)  $x = -1$

$f'(x)$  changes from negative to positive at  $x = -1$

2 { 1 : answer  
1 : justification

(b)  $x = -5$

$f'(x)$  changes from positive to negative at  $x = -5$

2 { 1 : answer  
1 : justification

(c)  $f''(x)$  exists and  $f'$  is decreasing on the intervals  $(-7, -3)$ ,  $(2, 3)$ , and  $(3, 5)$

2 { 1 :  $(-7, -3)$   
1 :  $(2, 3) \cup (3, 5)$

(d)  $x = 7$

The absolute maximum must occur at  $x = -5$  or at an endpoint.

$f(-5) > f(-7)$  because  $f$  is increasing on  $(-7, -5)$

The graph of  $f'$  shows that the magnitude of the negative change in  $f$  from  $x = -5$  to  $x = -1$  is smaller than the positive change in  $f$  from  $x = -1$  to  $x = 7$ .

Therefore the net change in  $f$  is positive from  $x = -5$  to  $x = 7$ , and  $f(7) > f(-5)$ . So  $f(7)$  is the absolute maximum.

3 { 1 : answer  
1 : identifies  $x = -5$  and  $x = 7$  as candidates  
— or —  
indicates that the graph of  $f$  increases, decreases, then increases  
1 : justifies  $f(7) > f(-5)$

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of  $\sqrt{t+1}$  gallons per minute, for  $0 \leq t \leq 120$  minutes. At time  $t = 0$ , the tank contains 30 gallons of water.

- (a) How many gallons of water leak out of the tank from time  $t = 0$  to  $t = 3$  minutes?
- (b) How many gallons of water are in the tank at time  $t = 3$  minutes?
- (c) Write an expression for  $A(t)$ , the total number of gallons of water in the tank at time  $t$ .
- (d) At what time  $t$ , for  $0 \leq t \leq 120$ , is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1:  $\int_0^3 \sqrt{t+1} dt = \frac{2}{3}(t+1)^{3/2} \Big|_0^3 = \frac{14}{3}$

– or –

Method 2:  $L(t)$  = gallons leaked in first  $t$  minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

(b)  $30 + 8 \cdot 3 - \frac{14}{3} = \frac{148}{3}$

(c) Method 1:

$$\begin{aligned} A(t) &= 30 + \int_0^t (8 - \sqrt{x+1}) dx \\ &= 30 + 8t - \int_0^t \sqrt{x+1} dx \end{aligned}$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

- (d)  $A'(t) = 8 - \sqrt{t+1} = 0$  when  $t = 63$   
 $A'(t)$  is positive for  $0 < t < 63$  and negative for  $63 < t < 120$ . Therefore there is a maximum at  $t = 63$ .

Method 1:

$$3 \begin{cases} 2 : \text{definite integral} \\ 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

– or –

Method 2:

$$3 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{solves for } C \text{ using } L(0) = 0 \\ 1 : \text{answer} \end{cases}$$

1 : answer

Method 1:

$$2 \begin{cases} 1 : 30 + 8t \\ 1 : -\int_0^t \sqrt{x+1} dx \end{cases}$$

– or –

Method 2:

$$2 \begin{cases} 1 : \text{antiderivative with } C \\ 1 : \text{answer} \end{cases}$$

$$3 \begin{cases} 1 : \text{sets } A'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$$

Consider the curve given by  $xy^2 - x^3y = 6$ .

- (a) Show that  $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$ .
- (b) Find all points on the curve whose  $x$ -coordinate is 1, and write an equation for the tangent line at each of these points.
- (c) Find the  $x$ -coordinate of each point on the curve where the tangent line is vertical.

(a)  $y^2 + 2xy \frac{dy}{dx} - 3x^2y - x^3 \frac{dy}{dx} = 0$

$$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

(b) When  $x = 1$ ,  $y^2 - y = 6$   
 $y^2 - y - 6 = 0$   
 $(y - 3)(y + 2) = 0$   
 $y = 3, y = -2$

At  $(1, 3)$ ,  $\frac{dy}{dx} = \frac{9 - 9}{6 - 1} = 0$

Tangent line equation is  $y = 3$

At  $(1, -2)$ ,  $\frac{dy}{dx} = \frac{-6 - 4}{-4 - 1} = \frac{-10}{-5} = 2$

Tangent line equation is  $y + 2 = 2(x - 1)$

(c) Tangent line is vertical when  $2xy - x^3 = 0$

$$x(2y - x^2) = 0 \text{ gives } x = 0 \text{ or } y = \frac{1}{2}x^2$$

There is no point on the curve with  $x$ -coordinate 0.

When  $y = \frac{1}{2}x^2$ ,  $\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$$2 \left\{ \begin{array}{l} 1 : \text{implicit differentiation} \\ 1 : \text{verifies expression for } \frac{dy}{dx} \end{array} \right.$$

$$4 \left\{ \begin{array}{l} 1 : y^2 - y = 6 \\ 1 : \text{solves for } y \\ 2 : \text{tangent lines} \end{array} \right.$$

Note: 0/4 if not solving an equation of the form  $y^2 - y = k$

$$3 \left\{ \begin{array}{l} 1 : \text{sets denominator of } \frac{dy}{dx} \text{ equal to } 0 \\ 1 : \text{substitutes } y = \frac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \text{into the equation for the curve} \\ 1 : \text{solves for } x\text{-coordinate} \end{array} \right.$$

Consider the differential equation  $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$ .

- (a) Find a solution  $y = f(x)$  to the differential equation satisfying  $f(0) = \frac{1}{2}$ .  
 (b) Find the domain and range of the function  $f$  found in part (a).

(a)  $e^{2y} dy = 3x^2 dx$

$$\frac{1}{2}e^{2y} = x^3 + C_1$$

$$e^{2y} = 2x^3 + C$$

$$y = \frac{1}{2} \ln(2x^3 + C)$$

$$\frac{1}{2} = \frac{1}{2} \ln(0 + C); \quad C = e$$

$$y = \frac{1}{2} \ln(2x^3 + e)$$

- 6 {
- 1 : separates variables
  - 1 : antiderivative of  $dy$  term
  - 1 : antiderivative of  $dx$  term
  - 1 : constant of integration
  - 1 : uses initial condition  $f(0) = \frac{1}{2}$
  - 1 : solves for  $y$
- Note: 0/1 if  $y$  is not a logarithmic function of  $x$

Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

(b) Domain:  $2x^3 + e > 0$

$$x^3 > -\frac{1}{2}e$$

$$x > \left(-\frac{1}{2}e\right)^{1/3} = -\left(\frac{1}{2}e\right)^{1/3}$$

Range:  $-\infty < y < \infty$

- 3 {
- 1 :  $2x^3 + e > 0$
  - 1 : domain
  - Note: 0/1 if 0 is not in the domain
  - 1 : range

Note: 0/3 if  $y$  is not a logarithmic function of  $x$