



AP Calculus AB 2001 Free-Response Questions

The materials included in these files are intended for use by AP teachers for course and exam preparation in the classroom; permission for any other use must be sought from the Advanced Placement Program. Teachers may reproduce them, in whole or in part, in limited quantities, for face-to-face teaching purposes but may not mass distribute the materials, electronically or otherwise. These materials and any copies made of them may not be resold, and the copyright notices must be retained as they appear here. This permission does not apply to any third-party copyrights contained herein.

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT[®], the PSAT/NMSQT[™], the Advanced Placement Program[®] (AP[®]), and Pacesetter[®]. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

Copyright © 2001 by College Entrance Examination Board. All rights reserved. College Board, Advanced Placement Program, AP, and the acorn logo are registered trademarks of the College Entrance Examination Board.

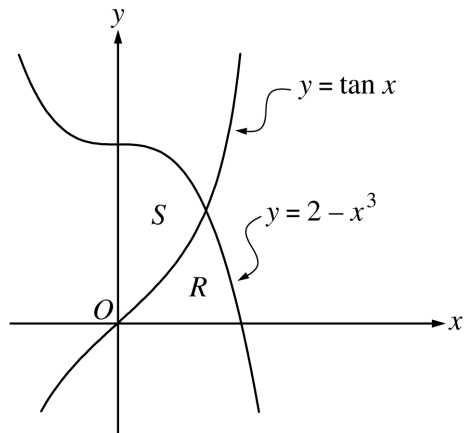
2001 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



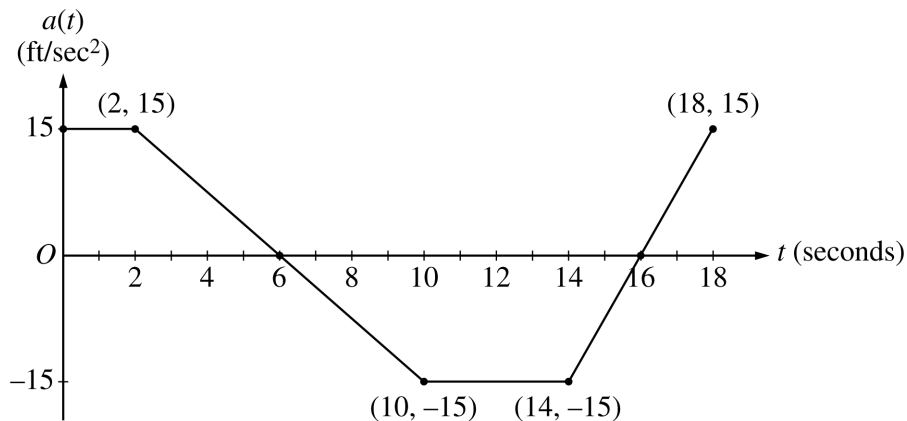
1. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$. The region S is bounded by the y -axis and the graphs of $y = 2 - x^3$ and $y = \tan x$.
- Find the area of R .
 - Find the area of S .
 - Find the volume of the solid generated when S is revolved about the x -axis.
-

2001 AP® CALCULUS AB FREE-RESPONSE QUESTIONS

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

2. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t . The table above shows the water temperature as recorded every 3 days over a 15-day period.
- (a) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.
- (b) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
- (c) A student proposes the function P , given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t , where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.
- (d) Use the function P defined in part (c) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \leq t \leq 15$ days.
-

2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS



3. A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \leq t \leq 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2 , is the piecewise linear function defined by the graph above.
- Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?
 - At what time in the interval $0 \leq t \leq 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?
 - On the time interval $0 \leq t \leq 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
 - At what times in the interval $0 \leq t \leq 18$, if any, is the car's velocity equal to zero? Justify your answer.
-

END OF PART A OF SECTION II

2001 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

-
4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$$h'(x) = \frac{x^2 - 2}{x} \text{ for all } x \neq 0.$$

- (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
- (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
- (c) Write an equation for the line tangent to the graph of h at $x = 4$.
- (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

-
5. A cubic polynomial function f is defined by

$$f(x) = 4x^3 + ax^2 + bx + k$$

where a , b , and k are constants. The function f has a local minimum at $x = -1$, and the graph of f has a point of inflection at $x = -2$.

- (a) Find the values of a and b .
- (b) If $\int_0^1 f(x) dx = 32$, what is the value of k ?

-
6. The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

- (a) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.
- (b) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

END OF EXAMINATION