AP® Calculus AB
2002 Sample Student Responses
Form B

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A1

Work for problem 2(a)

a) $P'(9) = 1 - 3(e)^{-0.2\sqrt{9}}$

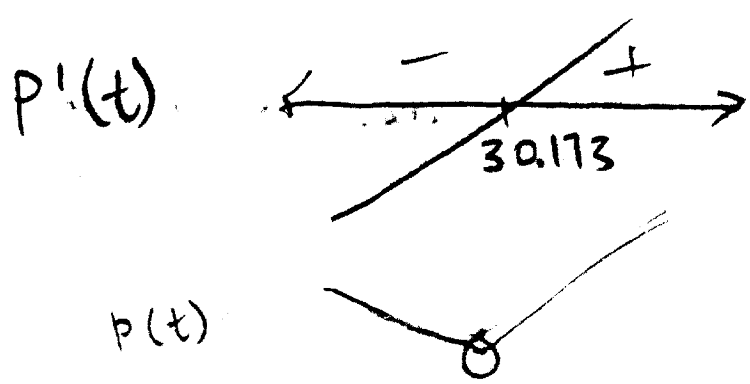
$= -0.646$ gallons/day.

No. $P'(9)$ is negative, so the amount of pollutant is decreasing.

Work for problem 2(b)

b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$

$t = 30.173$



minimum at $t = 30.173$

A2

Work for problem 2(c)

$$50 + \int_0^{30.173} p'(t)\, dt$$

$$= 50 - 14.895$$

$$= 35.104 \text{ gallons}$$

At day 30, there will be 35.104 gallons of pollutant left, and 35.104 < 40,

$\therefore$ it will be safe.

Work for problem 2(d)

$p'(0) = 1 - 3e^{-0.2 \cdot 0}$

$= 1 - 3e^{0}$

$= 1 - 3$

$= -2$

$P(0) = 50$

$y - 50 = -2(x)$

$y = -2x + 50$

$y \le 40$

$-2x + 50 \le 40$

$-2x \le -10$

$x \ge 5$

It predicts that at $t = 5$ the lake will become safe.

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C1

Work for problem 2(a)

$P'(9) = 1 - 3e^{-0.2\sqrt{9}} = -0.646$ gallons/day

the level of pollutant is decreasing because the rate is negative, as it is decreasing.

Work for problem 2(b) gallons of pollutant at a min when $P'(t) = 0$

$1 - 3e^{-0.2\sqrt{t}} = 0$

$3e^{-0.2\sqrt{t}} = 1$

$e^{-0.2\sqrt{t}} = 1/3$

$-0.2\sqrt{t} = \ln 1/3$

$\sqrt{t} = \dfrac{\ln 1/3}{-0.2}$

$t = \left(\dfrac{\ln 1/3}{-0.2}\right)^2$

$= 30.174$

≈ 30 days

C_2

Work for problem 2(c)

no of gallons present at the lake

$= 50 + \int_0^{30.174} (1-3e^{-0.2\sqrt{t}})\,dt$

$= 50.000$ gallons.

the lake is not safe because the no. of gallons is above 40 gallons.

Work for problem 2(d)

slope of tangent $= 1-3e^{-0.2\sqrt{t}}$

at $t=0$; gallons $=50 \Rightarrow m_T\Big|_{(0,50)} = 1-3e^{-0.2\sqrt{0}} = -2$

equation of tangent: $y = -2x+50$

40 lake is safe $\Rightarrow 40 = -2x+50$

$\Rightarrow -2x = -10$

$\Rightarrow x = 5$

after 5 days.

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