



## AP<sup>®</sup> Calculus AB 2002 Sample Student Responses Form B

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Work for problem 2(a)

$$a) P'(9) = 1 - 3(e)^{-0.2\sqrt{9}}$$

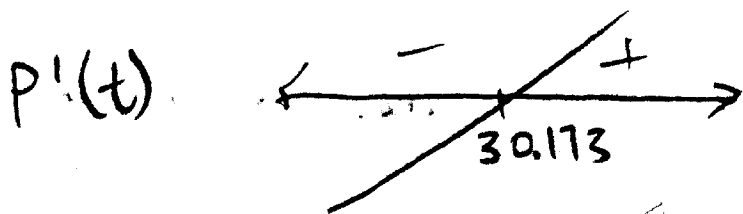
$$= -0.646 \text{ gallons/day.}$$

No.  $P'(9)$  is negative, so the amount of pollutant is decreasing.

Work for problem 2(b)

$$b) P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$$

$$t = 30.173$$



minimum at  $t = 30.173$

Work for problem 2(c)

$$50 + \int_0^{30.173} P'(t) dt$$

$$= 50 - 14.895$$

$$= 35.104 \text{ gallons}$$

At day 30, there will be 35.104 gallons of pollutant left, and  $35.104 < 40$ ,  
 $\therefore$  it will be safe.

Work for problem 2(d)

$$P'(0) = 1 - 3e^{-0.2 \cdot 0}$$

$$= 1 - 3e^0$$

$$= 1 - 3$$

$$= -2$$

$$P(0) = 50$$

$$y - 50 = -2(x)$$

$$y = -2x + 50$$

$$y \leq 40$$

$$-2x + 50 \leq 40$$

$$-2x \leq -10$$

$$x \geq 5$$

It predicts that at  
 $t = 5$  the  
 lake will become safe.

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Work for problem 2(a)

$$P'(a) = 1 - 3e^{-0.2\sqrt{a}} = -0.646 \text{ gallons/day}$$

The level of pollutant is decreasing because the rate is negative, as it is decreasing.

Work for problem 2(b)

gallons of pollutant at a min when  $P'(t) = 0$ 

$$1 - 3e^{-0.2\sqrt{t}} = 0$$

$$3e^{-0.2\sqrt{t}} = 1$$

$$e^{-0.2\sqrt{t}} = 1/3$$

$$-0.2\sqrt{t} = \ln(1/3)$$

$$\sqrt{t} = \frac{\ln(1/3)}{-0.2}$$

$$t = \left( \frac{\ln(1/3)}{-0.2} \right)^2$$

$$= 30.174$$

$$\approx 30 \text{ days}$$

C<sub>2</sub>

Work for problem 2(c) no. of gallons present at the lake

$$= 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}}) dt$$

$$= 50.000 \text{ gallons.}$$

the lake is not safe because the no. of gallons is above 40 gallons.

Work for problem 2(d)

slope of tangent =  $1 - 3e^{-0.2\sqrt{t}}$ 

$$\text{at } t=0; \text{ gallons} = 50 \Rightarrow m_T|_{(0,50)} = 1 - 3e^{-0.2\sqrt{0}} = \textcircled{-2}$$

$$\text{equation of tangent: } y = -2x + 50$$

$$40 \text{ lake is safe} \Rightarrow 40 = -2x + 50$$

$$\Rightarrow -2x = -10$$

$$\Rightarrow x = 5$$

after 5 days.

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