AP® Calculus AB
2002 Scoring Commentary
Form B

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Question 1

This problem involved definite integration in the solution of standard questions about area and volume. Students were required to first find the point of intersection of two graphs to determine the proper interval of integration and were expected to use the numerical equation solver of a graphing calculator for this purpose. Once this interval was determined, each of the three parts of the problem required setting up and evaluating an appropriate definite integral to find (a) the area of the region, (b) the volume generated by revolving the same region about the x-axis, and (c) the volume of a solid described in terms of cross sections based in the region. Students were expected to use the numerical integration capabilities of a graphing calculator in each case.

The mean score was 5.90.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 1 point in part (a), 3 points in part (b), 2 points in part (c), and the universal region point. The student lost the answer points in parts (a) and (c) for incorrectly evaluating the volume integrals.

Question 2

This problem presented the rate of change of the amount of pollutant in a lake along with an initial condition and a safety level criterion. Part (a) asked for an interpretation, requiring the calculation of a derivative value with justification. Parts (b) and (c) asked students to find when the pollutant reached its minimum, and whether or not that minimum was below the given safety level. These answers required sufficient justification. Finding the time at which the minimum occurred required solving $P'(t) = 0$, and it was expected that students would use the numerical equation solver of a graphing calculator for this purpose. Justifying that an absolute minimum occurred at a specific time *t* required an explanation that suitably addressed all values $t \geq 0$. Such a justification could appeal to the sign of the derivative P' for these values. Determining whether the lake was safe, and supporting that determination, was most directly accomplished by evaluating the minimum value by using a definite integral. Part (d) asked for a tangent line approximation to $P(t)$ using the initial condition. While this approximation may have provided reasonable estimates of $P(t)$ for *t* near 0, in this case students were asked to use the linear model to predict the time at which the pollutant reached a safe level. This prediction was noticeably different from the answer found in part (b).

The mean score was 3.96.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 1 point in part (a), 2 points in part (b), 2 points in part (c), and 2 points in part (d). In part (b), the student lost the point awarded for justification. In part (c), the student reached an incorrect conclusion since the integral was not evaluated correctly.

Question 3

This problem presented students with the velocity and initial position of a particle moving along the *x*-axis. Part (a) asked students for a graph of this given velocity, which served as a visual reference throughout the problem. Part (b) required an interpretation of the sign of the velocity as it related to movement of the particle. Part (c) asked for the total distance traveled, a result found by calculating a definite integral of the speed over the appropriate time interval. To answer part (d), students needed to decide whether there was a time b for which the displacement of the particle, given by a definite integral of velocity, was zero. The sketch made in part (a) provided strong visual clues, but students were not permitted to appeal only to this sketch in supporting an answer. An adequate justification might have appealed to the periodicity of the velocity in establishing that the displacement was positive for all values of b in the given interval.

The mean score was 4.10.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 1 point in part (a), 3 points in part (b), 1 point in part (c), and 2 points in part (d). The student lost 2 points in part (c) since no turning point was identified.

Question 4

In this problem, students were given a graphical representation of a function f and a second function g that was defined in terms of a definite integral of f. The questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of f. Part (a) asked for calculations of $g(6)$, $g'(6)$, and $g''(6)$. Each value could be found directly using the graph of f. Using the fact that $f = g'$, part (b) required relating the sign of f (positive or negative) to the behavior of g (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of f to the concavity of the graph of g. Finally, part (d) asked for a trapezoidal approximation of a definite integral involving f. All necessary values for the calculation could be obtained from the graph of f and students could take advantage of the symmetry of the graph.

The mean score was 4.47.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 3 points in part (b), 2 points in part (c), and 0 points in part (d). The student miscalculated $g(6)$ in part (a), losing one point. In part (d), the student incorrectly used the absolute value of the first and last terms in the calculation.

Question 5

This problem presented a separable differential equation. Part (a) asked for the particular solution to the equation satisfying a horizontal tangent line condition. While no initial value condition was supplied explicitly, students could determine that the point of tangency was $(3, -2)$. Part (a) further asked students to classify the critical point at $x = 3$ as a local minimum, local maximum, or neither. The second derivative test was the most straightforward way to determine this, having used implicit differentiation with the original equation to find an expression for y'' in terms of x and y. Part (b) supplied an explicit initial condition and asked students to solve the separable differential equation. The solution was straightforward, but did require students to choose the appropriately signed square root of a quadratic expression.

The mean score was 3.19.

Sample A (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: 1 point in part (a) and 6 points in part (b). The student did not use the first derivative test correctly and arrived at an incorrect conclusion.

Question 6

This problem presented a common related rates setting with two ships moving along perpendicular paths. The variables were related by the Pythagorean Theorem and the trigonometry of a right triangle. Part (a) asked students to calculate the distance between the ships at a specific instant. The purpose was to prompt students to establish the proper Pythagorean relationship between x, y, and the distance between the ships. Part (b) then asked students to relate the rate of change of this distance to the two given rates of change of x and y. This required differentiation of the relationship in part (a) with respect to t, followed by the appropriate substitutions. Part (c) shifted attention to an angle in the right triangle determined by the positions of the two ships and a lighthouse. A trigonometric ratio was the relationship required for finding the desired rate of change of this angle.

The mean score was 4.56.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 1 point in part (a), 3 points in part (b), and 3 points in part (c). The student lost evaluation points in part (c) and in part (d).