



AP[®] Calculus AB 2002 Scoring Guidelines Form B

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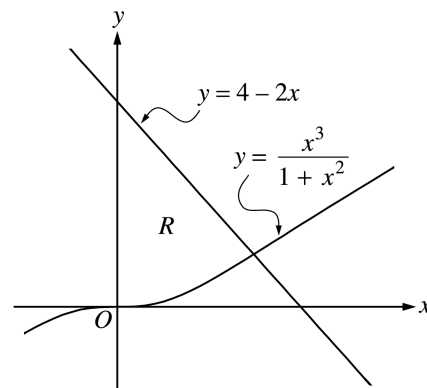
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2002 SCORING GUIDELINES (Form B)

Question 1

Let R be the region bounded by the y -axis and the graphs of

$y = \frac{x^3}{1+x^2}$ and $y = 4 - 2x$, as shown in the figure above.

- (a) Find the area of R .
 (b) Find the volume of the solid generated when R is revolved about the x -axis.
 (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x -axis is a square. Find the volume of this solid.



Region R

$$\frac{x^3}{1+x^2} = 4 - 2x \text{ at } x = 1.487664 = A$$

(a) Area = $\int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right) dx$
 = 3.214 or 3.215

(b) Volume
 = $\pi \int_0^A \left((4 - 2x)^2 - \left(\frac{x^3}{1+x^2} \right)^2 \right) dx$
 = 31.884 or 31.885 or 10.149π

(c) Volume = $\int_0^A \left(4 - 2x - \frac{x^3}{1+x^2} \right)^2 dx$
 = 8.997

1 : Correct limits in an integral in (a), (b), or (c).

2 { 1 : integrand
 1 : answer

3 { 2 : integrand and constant
 < -1 > each error
 1 : answer

3 { 2 : integrand
 < -1 > each error
 note: 0/2 if not of the form
 $k \int_c^d (f(x) - g(x))^2 dx$
 1 : answer

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Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

- (a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?
- (b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
- (c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
- (d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
 so the amount is not increasing at this time.

1 : answer with reason

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
 $t = (5 \ln 3)^2 = 30.174$
 $P'(t)$ is negative for $0 < t < (5 \ln 3)^2$ and positive for $t > (5 \ln 3)^2$. Therefore there is a minimum at $t = (5 \ln 3)^2$.

3 { 1 : sets $P'(t) = 0$
 1 : solves for t
 1 : justification

(c) $P(30.174) = 50 + \int_0^{30.174} (1 - 3e^{-0.2\sqrt{t}}) dt$
 $= 35.104 < 40$, so the lake is safe.

3 { 1 : integrand
 1 : limits
 1 : conclusion with reason
 based on integral of $P'(t)$

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

2 { 1 : slope of tangent line
 1 : answer

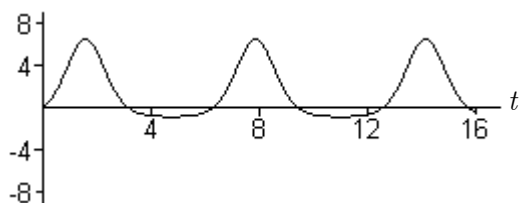
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Question 3

A particle moves along the x -axis so that its velocity v at any time t , for $0 \leq t \leq 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

- (a) On the axes provided, sketch the graph of $v(t)$ for $0 \leq t \leq 16$.
- (b) During what intervals of time is the particle moving to the left? Give a reason for your answer.
- (c) Find the total distance traveled by the particle from $t = 0$ to $t = 4$.
- (d) Is there any time t , $0 < t \leq 16$, at which the particle returns to the origin? Justify your answer.

(a) $v(t)$



1 : graph

three "humps"
 periodic behavior
 starts at origin
 reasonable relative max and min values

(b) Particle is moving to the left when

$$v(t) < 0, \text{ i.e. } e^{2\sin t} < 1.$$

$$(\pi, 2\pi), (3\pi, 4\pi) \text{ and } (5\pi, 6\pi]$$

3 { 2 : intervals
 < -1 > each missing or incorrect interval
 1 : reason

(c) $\int_0^4 |v(t)| dt = 10.542$

or

$$v(t) = e^{2\sin t} - 1 = 0$$

$$t = 0 \text{ or } t = \pi$$

$$x(\pi) = \int_0^\pi v(t) dt = 10.10656$$

$$x(4) = \int_0^4 v(t) dt = 9.67066$$

$$|x(\pi) - x(0)| + |x(4) - x(\pi)| \\ = 10.542$$

3 { 1 : limits of 0 and 4 on an integral of
 $v(t)$ or $|v(t)|$
 or
 uses $x(0)$ and $x(4)$ to compute distance
 1 : handles change of direction at student's
 turning point
 1 : answer
 note: 0/1 if incorrect turning point

(d) There is no such time because

$$\int_0^T v(t) dt > 0 \text{ for all } T > 0.$$

2 { 1 : no such time
 1 : reason

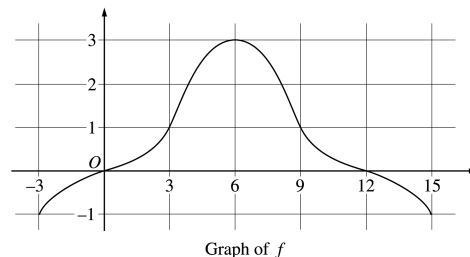
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Question 4

The graph of a differentiable function f on the closed interval $[-3,15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let

$$g(x) = 5 + \int_6^x f(t) dt \text{ for } -3 \leq x \leq 15.$$

- (a) Find $g(6)$, $g'(6)$, and $g''(6)$.
 (b) On what intervals is g decreasing? Justify your answer.
 (c) On what intervals is the graph of g concave down? Justify your answer.
 (d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t) dt$ using six subintervals of length $\Delta t = 3$.



(a) $g(6) = 5 + \int_6^6 f(t) dt = 5$
 $g'(6) = f(6) = 3$
 $g''(6) = f'(6) = 0$

$$3 \left\{ \begin{array}{l} 1 : g(6) \\ 1 : g'(6) \\ 1 : g''(6) \end{array} \right.$$

(b) g is decreasing on $[-3,0]$ and $[12,15]$ since
 $g'(x) = f(x) < 0$ for $x < 0$ and $x > 12$.

$$3 \left\{ \begin{array}{l} 1 : [-3,0] \\ 1 : [12,15] \\ 1 : \text{justification} \end{array} \right.$$

(c) The graph of g is concave down on $(6,15)$ since
 $g' = f$ is decreasing on this interval.

$$2 \left\{ \begin{array}{l} 1 : \text{interval} \\ 1 : \text{justification} \end{array} \right.$$

(d) $\frac{3}{2}(-1 + 2(0 + 1 + 3 + 1 + 0) - 1)$
 $= 12$

1 : trapezoidal method

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Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

- (a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f . Find the x -coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.
- (b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.

(a) $\frac{dy}{dx} = 0$ when $x = 3$

$$\frac{d^2y}{dx^2} \Big|_{(3,-2)} = \frac{-y - y'(3-x)}{y^2} \Big|_{(3,-2)} = \frac{1}{2},$$

so f has a local minimum at this point.

or

Because f is continuous for $1 < x < 5$, there is an interval containing $x = 3$ on which

$y < 0$. On this interval, $\frac{dy}{dx}$ is negative to the left of $x = 3$ and $\frac{dy}{dx}$ is positive to the

right of $x = 3$. Therefore f has a local minimum at $x = 3$.

(b) $y \, dy = (3-x) \, dx$

$$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C$$

$$8 = 18 - 18 + C; C = 8$$

$$y^2 = 6x - x^2 + 16$$

$$y = -\sqrt{6x - x^2 + 16}$$

$$3 \left\{ \begin{array}{l} 1 : x = 3 \\ 1 : \text{local minimum} \\ 1 : \text{justification} \end{array} \right.$$

$$6 \left\{ \begin{array}{l} 1 : \text{separates variables} \\ 1 : \text{antiderivative of } dy \text{ term} \\ 1 : \text{antiderivative of } dx \text{ term} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition } g(6) = -4 \\ 1 : \text{solves for } y \end{array} \right.$$

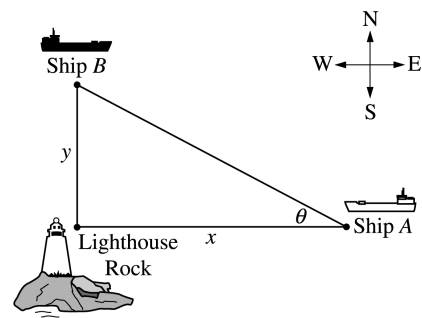
Note: max 3/6 [1-1-1-0-0-0] if no constant of integration

Note: 0/6 if no separation of variables

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Question 6

Ship *A* is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour (km/hr). Ship *B* is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship *A* and Lighthouse Rock at time t , and let y be the distance between Ship *B* and Lighthouse Rock at time t , as shown in the figure above.



- (a) Find the distance, in kilometers, between Ship *A* and Ship *B* when $x = 4$ km and $y = 3$ km.
- (b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 4$ km and $y = 3$ km.
- (c) Let θ be the angle shown in the figure. Find the rate of change of θ , in radians per hour, when $x = 4$ km and $y = 3$ km.

(a) Distance = $\sqrt{3^2 + 4^2} = 5$ km

(b) $r^2 = x^2 + y^2$

$$2r \frac{dr}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

or explicitly:

$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dt} = \frac{1}{2\sqrt{x^2 + y^2}} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} \right)$$

At $x = 4$, $y = 3$,

$$\frac{dr}{dt} = \frac{4(-15) + 3(10)}{5} = -6 \text{ km/hr}$$

(c) $\tan \theta = \frac{y}{x}$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{\frac{dy}{dt} x - \frac{dx}{dt} y}{x^2}$$

At $x = 4$ and $y = 3$, $\sec \theta = \frac{5}{4}$

$$\frac{d\theta}{dt} = \frac{16}{25} \left(\frac{10(4) - (-15)(3)}{16} \right)$$

$$= \frac{85}{25} = \frac{17}{5} \text{ radians/hr}$$

1 : answer

4 { 1 : expression for distance
 2 : differentiation with respect to t
 < -2 > chain rule error
 1 : evaluation

4 { 1 : expression for θ in terms of x and y
 2 : differentiation with respect to t
 < -2 > chain rule, quotient rule, or
 transcendental function error
 note: 0/2 if no trig or inverse trig
 function
 1 : evaluation