



AP[®] Calculus AB 2002 Sample Student Responses

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NO CALCULATOR ALLOWED

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

B₁

Work for problem 6(a)

$$\int_0^{1.5} (3f'(x) + 4) dx$$

$$[3F(x) + 4x]_0^{1.5}$$

$$(3F(1.5) + 4(1.5)) - (3F(0) + 4(0))$$

$$-3 + 6 + 21$$

$$\textcircled{24}$$

Work for problem 6(b)

$$F(1) = -4$$

$$F'(1) = 5$$

$$y + 4 = 5(x - 1)$$

$$y = 5x - 9$$

$$y(1.2) = 5(1.2) - 9$$

$$y(1.2) = 6 - 9$$

$$\boxed{y(1.2) = -3}$$

The approximation is less than the actual value because on the interval, $F''(x) > 0$, so $f(x)$ is therefore concave up. If this is true then tangent lines will fall below the curve, and all points on them will be below it as well.

NO CALCULATOR ALLOWED

f is differentiable for all real numbers, so $f'(x)$ is differentiable for all real numbers

Work for problem 6(c)

Mean Value Theorem says that on the interval (a, b) there exists a $f'(c)$ such

$$f''(c) = \frac{f'(1.5) - f'(0)}{1.5 - 0} = r \quad \text{that} \quad f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f''(c) = \frac{3 - 0}{1.5 - 0} = r$$

$$f''(c) = 6 = r$$

$$r = 6$$

It applies to 2nd derivative as well.

Work for problem 6(d)

x	1.5	.1	-.5	0
$g'(x)$	-7			\emptyset

$$g'(x) = 4x - 1$$

$$g'(1.5) =$$

$$g'(0)^+ = 4x + 1 = 1$$

$$g'(0)^- = 4x - 1 = -1$$

No because the function is not differentiable at $x = 0$. This violates the condition that

$f(x)$ is differentiable at all numbers, and in fact

$$f'(0) = 0,$$

because derivative doesn't exist at zero, function cannot be the same.

NO CALCULATOR ALLOWED

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Work for problem 6(a)

$$\int_0^{1.5} (3f'(x) + 4) dx = \int_0^{1.5} (3f'(x) + 4) dx = 3f(x) + 4x$$

$$= 3(-1) + 6 - ((-7)(3))$$

$$= -3 + 6 + 21$$

$$= 3 + 21 = \boxed{24}$$

Work for problem 6(b)

$$f'(1) = 5 = m \text{ of tangent line}$$

$$y = 5x + b$$

$$pt (1, -4)$$

$$-4 = 5 + b$$

$$b = -9$$

$$\boxed{y = 5x - 9}$$

$$f(1.2) \approx 5(1.2) - 9 \approx 6 - 9 = \boxed{-3}$$

this is an approximation $\forall c$ $x=1.2$ is close enough to $x=1$ that one can use tan line for $x=1$.

Work for problem 6(c)

$$f''(0) = 5 \quad r > 0$$

$$\text{for } f'(x) \rightarrow (0.5, 3) \quad (0, 0)$$

$$f''(x) = \text{slope of } f'(x)$$

$$\rightarrow \text{slope} (= f''(x)) = \frac{3-0}{0.5} = \boxed{6}$$

Mean value and Rolle's theorem

Work for problem 6(d)

$$g'(x) = \begin{cases} 4x-1 & \text{for } x < 0 \\ 4x+1 & \text{for } x \geq 0 \end{cases}$$

$$= 4x+1 \quad \text{for } x \geq 0$$

compare to $f'(x)$ on table

no; because at $x=0$ $g'(x) = 1$
and on table $f'(0) = 0$

$$1 \neq 0$$

so g and f can't be the same function