AP® Calculus AB
2002 Sample Student Responses

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NO CALCULATOR ALLOWED

B1

x	−1.5	−1.0	−0.5	0	0.5	1.0	1.5
$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

Work for problem 6(a)

$$\int_0^{1.5} (3f'(x)+4)\,dy$$

$$\left[3F(x)+4x\right]_0^{1.5}$$

$$(3F(1.5)+4(1.5))-(3F(0)+4(0))$$

$$-3+6+21$$

$$\boxed{24}$$

Work for problem 6(b)

$f(1)=-4$

$f'(1)=5$

$y+4=5(x-1)$

$y=5x-9$

$y(1.2)=5(1.2)-9$

$y(1.2)=6-9$

$\boxed{y(1.2)=-3}$

The approximation is less than the actual value because on the interval, $f''(x)>0$, so $f(x)$ is therefore concave up. If this is true then tangent lines will fall below the curve, and all points on them will be below it as well.

6 6 6 6 6 6 6 6 6 6 B2

NO CALCULATOR ALLOWED

Work for problem 6(c)

f is differentiable for all real numbers, so f'(x) is differentiable for all real numbers

Mean Value Theorem, says that on the interval (a, b) there exists a f'(c) such that

$f'(c) = \frac{f(b) - f(a)}{b - a}$

It applies to 2nd derivative as well.

$f''(c) = \frac{f'(.5) - f'(0)}{.5 - 0} = r$

$f''(c) = \frac{3 - 0}{.5 - 0} = r$

$f''(c) = 6 = r$

$r = 6$

Work for problem 6(d)

x	.5	.1	-.5	0
g'(x)	-7			∅

$g'(x) = 4x - 1$

$g'(1.5) =$

$g'(0)^+ = 4x + 1 = 1$

$g'(0)^- = 4x - 1 = -1$

No because the function is not differentiable at x = 0. This violates the condition that f(x) is differentiable at all numbers, and in fact f'(0) = 0, because derivative doesn't exist at zero, function cannot be the same, 6 6 6 6 6 6 6 6 6 6

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$f(x)$	−1	−4	−6	−7	−6	−4	−1
$f'(x)$	−7	−5	−3	0	3	5	7

Work for problem 6(a)

$\int_0^{1.5} (3f'(x)+4)\,dx \qquad \int(3f'(x)+4)\,dx = 3f(x)+4x$

$[3f(x)+4x]\Big|_0^{1.5} = 3(-1)+6-((-7)(3))$

$= -3+6+21$

$= 3+21 = \boxed{24}$

Work for problem 6(b)

$f'(1) = 5 =$ m of tangent line

$y = 5x+b \qquad$ pt $(1,-4)$

$-4 = 5+b \qquad \boxed{y = 5x-9}$

$b = -9$

$f(1.2) \approx 5(1.2)-9 \approx 6-9 \approx \boxed{-3}$

this is an aproximation b/c x=1.2 is close enough to x=1 that one can use tan line for x=1.

NO CALCULATOR ALLOWED

D_2

Work for problem 6(c)

$f''(c) = r \quad r > 0$

$f''(x)$ = slope of $f'(x)$

for $f'(x) \rightarrow (0.5, 3) \quad (0, 0)$

slope $= f''(x) = \frac{3-0}{0.5} = 6$

Mean value and Rolle's theroren

Work for problem 6(d)

$g'(x) = 4x-1$ for $x < 0$

$= 4x+1$ for $x \geq 0$

compare to $f'(x)$ on table

no; because at $x=0$ $g'(x)=1$
and on table $f'(0) = 0$

$1 \neq 0$

so g and f can't be the same function