AP® Calculus AB
2002 Scoring Commentary

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Question 1

This problem presented a region bounded between two graphs and two vertical lines. Students were asked to use both integration and differentiation to answer some straightforward questions about this region. Part (a) required the use of a definite integral to find the area of the region. In part (b), the region is revolved about a horizontal line, resulting in a solid with cross sections in the shape of "washers," and the volume of the region was asked for, requiring another use of a definite integral. Students were expected to use the numerical integration capabilities of a graphing calculator to evaluate these definite integrals. Part (c) required the use of differentiation to find the absolute minimum height of the region.

The mean score was 4.08.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 4 points in part (b), and 1 point in part (c). In part (c), the student set the derivative of $h(x)$ equal to 0, but did not find a critical point in the interval $\frac{1}{2} \le x \le 1$. This made the student ineligible for the answer point.

Question 2

This problem involved a "real-life" model of people entering and leaving an amusement park, and students were expected to interpret the meanings of their calculations in that context. In part (a), students needed to recognize that the function E represented the rate of accumulation of people entering the park, and hence that the total accumulation over a time interval could be obtained by a definite integral. Part (b) took the same idea a step further, asking students to calculate ticket revenues based on different pricing over two time intervals. Part (c) presented a function H defined in terms of a definite integral. The value of $H'(17)$ was easily computed, provided students recognized that the Fundamental Theorem of Calculus could be applied. More importantly, students were expected to interpret, in words, the meanings of both $H(17)$ and $H'(17)$. These interpretations played a role in part (d), which essentially asked students to find when the maximum value of the function H is achieved.

The mean score was 3.13.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 3 points in part (a), 0 points in part (b), 2 points in part (c), and 2 points in part (d). The student lost 1 point in part (b) since the function L, rather than the function E, was used in the second integral. In part (c), 1 point was lost since an incorrect value for $H'(17)$ was given.

Question 3

This problem presented the velocity and initial position of an object moving along the x-axis. Part (a) required a knowledge of the relationship between velocity and its derivative, acceleration. Part (b) addressed the distinction between velocity and speed by presenting students with two statements that could appear contradictory, but in fact, are both true. The words "velocity" and "speed" are often used interchangeably in everyday language, but the technical distinction between the two is highlighted in calculus. Parts (c) and (d) also focused on this distinction. The total distance traveled by the object was calculated with a definite integral of speed (absolute value of velocity). Finding the position of the object at time $t = 4$ involved displacement, calculated with a definite integral of velocity.

The mean score was 3.12.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 1 point in part (a), 1 point in part (b), 3 points in part (c), and 2 points in part (d). In part (b), the student earned only the point for noting that Statement II was correct, failing to give a correct reason. The student did note that velocity was decreasing, but gave an incorrect reason, and did not earn the point for Statement I.

Question 4

In this problem, students were given a graphical representation of a function *f*, and another function *g* that was defined in terms of a definite integral of *f*. While it was possible to find piecewise algebraic definitions for *f* and *g*, the questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of *f*. Part (a) asked for calculations of $g(-1)$, $g'(-1)$, and $g''(-1)$. These values could be found using, respectively, an area, ordinate, and slope related to the graph of *f*. Using the fact that $f = g'$, part (b) required relating the sign of *f* (positive or negative) to the behavior of *g* (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of *f* to the concavity of the graph of *g*. Part (d) asked for a sketch of the graph of *g*. Utilizing previous parts of the problem helped in determining characteristics of the graph.

The mean score was 3.51.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). The student had the incorrect sign in the calculation of $g(-1)$ in part (a), and the graph in part (d) was translated vertically.

Question 5

This problem presented a common related rates setting with several variables (radius, depth, volume) related by geometry to the water evaporating in a conical container. Part (a) asked students to calculate the volume of water when its depth was $h = 5$ cm. The purpose of this part was to prompt students to establish the relationships among the radius, depth, and volume variables. Part (b) then asked students to relate the rate of change of the volume to the given rate of change of the depth of water. Part (c) introduced another related quantity, the exposed surface area, and asked students to verify a direct proportionality relationship between the rate of change of the volume and the exposed surface area of the water. The constant of proportionality in this case was precisely the given constant rate of change of the water's depth.

The mean score was 2.29.

Sample A (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: 1 point in part (a), 5 points in part (b), 0 points in part (c), and the global units point. In part (c), the student found the derivative of the area function, rather than the derivative of the volume function.

Question 6

This problem presented data in tabular form for a function f and its derivative f', along with information about the sign of the second derivative. Part (a) required students to use the Fundamental Theorem of Calculus and some basic properties of integrals to calculate a specific definite integral. Part (b) asked for the calculation of a linear approximation using appropriate data from the table. Students needed to interpret the sign of the second derivative in terms of concavity and relate this information to the tangent line. Part (c) required students to recognize that the Mean Value Theorem could be applied to f' to show the existence of a real number c such that $f''(c) = 6$, the average rate of change of f' over the interval $[0, 0.5]$. Part (d) presented students with a piecewise algebraic expression for a function g that fit all of the given points on the graph of f. To determine that $f \neq g$, students needed to appeal to some inconsistency with the first or second derivatives of the respective functions.

The mean score was 2.33.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (b), the student lost the reason point by not referring to the given information that the graph of f was concave up on the interval $1 < x < 2$. In part (d), the student made an incorrect statement in the justification by stating that $g'(0) = 1$, when in fact $g'(0)$ does not exist.