AP® Calculus BC
2004 Sample Student Responses
Form B

1 1 1 1 1 1 1 1 1 1 A.

CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

speed $= \sqrt{(x'(0))^2 + (y'(0))^2}$

$\frac{dx}{dt}(0) = \sqrt{0^4+9} = \sqrt{9} = 3$ $\quad \frac{dy}{dt}(0) = 2e^0 + 5e^0 = 2+5 = 7$

speed $= \sqrt{3^2+7^2} = \sqrt{58} = 7.616$ (3d.p)

acceleration: $\frac{d^2x}{dt^2} = \frac{1}{2\sqrt{t^4+9}}(4t^3) = \frac{2t^3}{\sqrt{t^4+9}}$.

$\frac{d^2y}{dt^2} = 2e^t - 5e^{-t}$.

$\left(\frac{2t^3}{\sqrt{t^4+9}}, 2e^t - 5e^{-t}\right) \Rightarrow$ acceleration at $t=0$ $(0, -3)$

Work for problem 1(b)

m_T = slope $= \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2e^t + 5e^{-t}}{\sqrt{t^4+9}}$ at $t=0$ $\boxed{\frac{dy}{dx} = \frac{7}{3}}$

$y - y_0 = m_T(x - x_0) \Rightarrow y - 1 = \frac{7}{3}(x-4) \Rightarrow y - 1 = \frac{7}{3}x - \frac{28}{3}$

$\Rightarrow y = \frac{7}{3}x - \frac{25}{3} \Rightarrow \boxed{3y = 7x - 25}$

Continue problem 1 on page 5.

Work for problem 1(c)

$$d = \int_0^3 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^3 \sqrt{t^4+9 + (2e^t+5e^{-t})^2}\, dt$$

$$= \int_0^3 \sqrt{t^4+9+4e^{2t}+20+25e^{-2t}}\, dt$$

$$= \int_0^3 \sqrt{t^4+4e^{2t}+25e^{-2t}+29}\, dt$$

$$= 45.227$$

Work for problem 1(d)

$$\frac{dx}{dt} = \sqrt{t^4+9} \Rightarrow dx = \sqrt{t^4+9}\, dt. \Rightarrow x = \int_0^3 \sqrt{t^4+9}\, dt.$$

since $\frac{dx}{dt} \neq 0$ & $\frac{dy}{dt} \neq 0$. $\Rightarrow$ particle does not stop $\rightarrow$ moves in a straight line.

$$x\text{-coordinate at } t=3 = x(0) + \int_0^3 \sqrt{t^4+9}\, dt$$

$$= 4 + 13.931 = 17.931.$$

D,

CALCULUS BC
SECTION II, Part A
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Work for problem 1(a)

Speed = magnitude = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

acceleration vector $\left(\frac{1}{2(t^4+9)^{\frac{1}{2}}}(4t^3),\ 2e^t - 5e^{-t}\right)$

$= \sqrt{\left(\sqrt{t^4+9}\right)^2 + \left(2e^t+5e^{-t}\right)^2}$

$t=0 \rightarrow$ speed $= \sqrt{\left(\sqrt{9}\right)^2 + \left(2e^t+5e^{-t}\right)^2}$

$= \sqrt{58}$

Work for problem 1(b)

$t=0$

point of tangency: $(4,1)$

slope $= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2+5}{3} = \frac{7}{3}$

$y-1 = \frac{7}{3}(x-4)$

$y = \frac{7}{3}x - \frac{28}{3} + 1$

$y = \frac{7}{3}x - \frac{31}{3}$

Continue problem 1 on page 5.

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Work for problem 1(c)

Total dist = length of curve

$$L = \int_0^3 \sqrt{\left(\sqrt{t^4+9}\right)^2 + \left(2e^t + 5e^{-t}\right)^2}$$

$$= 45.227$$

Work for problem 1(d)

$$x(t) = \int_0^t (t^4+9)^{\frac{1}{2}} + C$$

$t = 0$

$$x(0) = 0 + C = 4, \quad C = 4$$

$$x(3) = \int_0^3 (t^4+9)^{\frac{1}{2}} + 4 =$$

$$= 13.9307 + 4 = 17.931$$ (x coordinate)

GO ON TO THE NEXT PAGE.