AP® Calculus BC
2004 Sample Student Responses
Form B

Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2} + \frac{f'''(2)(x-2)^3}{3!}$$

$$= 7 + 0 \quad -9(x-2)^2 - 3(x-2)^3$$

$$\boxed{f(2) = 7}$$

$$\frac{f''(2)\cancel{(x-2)^2}}{2} = -9\cancel{(x-2)^2}$$

$$\boxed{f''(2) = -18}$$

Work for problem 2(b)

Yes there is. to have this Tailor polynomial, $f'(2)=0$ must be true

$f''(2)=-18$, which is negative, meaning curve is concave down

if curve is concave down, and 1st derivative is 0, it is a relative max

$\boxed{f(2) \text{ is a relative maximum}}$

Continue problem 2 on page 7.

Work for problem 2(c)

$T(0) = 7 - 9(0-2)^2 - 3(0-2)^3 = -5$

$f(0) \approx T(0) = -5$

No there isn't, because we do not know any of the derivatives at 0, so nothing can be determined.

Work for problem 2(d)

$\dfrac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1} = \text{max error}$

$\dfrac{f^{(4)}(c)}{(4)!}(x-2)^4 \leq \dfrac{6}{4!}(x-2)^4 = \dfrac{(x-2)^4}{4} = \text{max error}$

$\dfrac{(0-2)^4}{4} = 4 = \text{max error}$

$-5 \pm 4 < 0$, all values within error range is still negative, so $f(0)$ is negative

GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$$T(x) = f(2) + f'(2)(x-2) + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{2!}$$

$$T(x) = 7 + 0 - 9(x-2)^2 - 3(x-2)^3$$

$$\boxed{f(2) = 7}$$

$$\frac{f''(2)\cancel{(x-2)^2}}{2!} = -9\cancel{(x-2)^2}$$

$$f''(2) = -9(2)$$

$$\boxed{f''(2) = -18}$$

Work for problem 2(b)

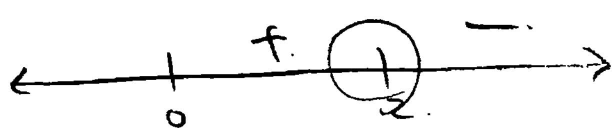
$$T(x) = 7 - 9(x-2)^2 - 3(x-2)^3$$

$$T'(x) = -18(x-2) - 9(x-2)^2 = 0$$

critical pts: $\rightarrow$ $x = 2; 0$

$T'(1) = 9$

$T'(3) = -27$



relative max

yes there is enough information to determine whether f has a critical pt. at $x = 2$.

$f(2)$ is a relative maximum by the 1st derivative test.

Continue problem 2 on page 7.

Work for problem 2(c)

$f(0) \approx T(0) = 7 - 9(0-2)^2 - 3(0-2)^3$

$\boxed{f(0) \approx T(0) = -5}$

No you can't determine whether f has a critical point at $x=0$ because we do not know f. $T(x)$ is a taylor polynomial about $x=2$, thus it is only an approximation. $x=0$ is too far from where $T(x)$ is centered, thus the approximation is very inaccurate.

Work for problem 2(d)

Lagrange error bound $= \dfrac{f^{(n+1)}(z)\,(x-c)^{(n+1)}}{(n+1)!}$

$= \dfrac{f^4(z)\,(x-2)^4}{4!}$

when $x=0$.

$= \dfrac{\overbrace{f^4(z)}^{\text{max is } 6}(-2)^4}{4!} = \dfrac{6\,(-2)^4}{4!} = -4.$

The max error bound is -4, thus $f(0)$ lies somewhere is $T(0) \pm$ error, somewhere in -9 to -1. Thus $f(0)$ is negative.

GO ON TO THE NEXT PAGE.