CollegeBoard
Advanced Placement Program

AP® Calculus BC
2004 Scoring Commentary
Form B

Question 1

This question presented students with the position at time $t = 0$ and formulas for the rate of change of the x- and y-coordinates of a particle moving in the xy-plane. Part (a) asked for the speed and acceleration vector of the particle at time $t = 0$. Part (b) asked for the equation of the line tangent to the path of the particle at time $t = 0$, requiring students to know how to use the rates of change of the coordinates to find the slope of the tangent line. Part (c) asked for the total distance traveled from $t = 0$ to $t = 3$, requiring a definite integral of the speed. Part (d) asked for the x-coordinate at time $t = 3$, requiring the use of the initial position and a definite integral of the rate of change of the x-coordinate.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: the speed point in part (a), the slope point in part (b), and all points in parts (c) and (d). In part (a), although the student computed the acceleration vector, it was not evaluated at $t = 0$; thus, the student could not earn that point. In part (b), the student made a computational error in writing the equation of the tangent line.

Question 2

Students were given the third-degree Taylor polynomial for f about $x = 2$. Part (a) tested whether they could read the values of $f(2)$ and $f''(2)$ from the Taylor polynomial. Part (b) tested whether students could read the values of $f'(2)$ and $f''(2)$ from the Taylor polynomial and use these values to determine the nature of the critical point at $x = 2$. Part (c) tested whether students realized that although the Taylor polynomial could be used to approximate the value of f at $x = 0$, it could not be used to determine whether f had a critical point at $x = 0$. Part (d) gave students an upper bound on the absolute value of the fourth derivative of f and tested whether students could use the Lagrange Error Bound to put an upper limit on the value of $f(0)$.

<u>Sample A</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: both points in part (a), 1 point in part (b), all points in part (c), and 1 point in part (d). In part (b), the student's argument using the First Derivative Test cannot be supported by the given information. In part (d), the student has an incorrect error bound. Since $T(0)$ plus or minus "the error bound" does yield $f(0) < -1,$ the student earned the second point.

Question 3

This question presented a table of values of the velocity of a test plane (in miles per minute) measured at five-minute intervals from $t = 0$ to $t = 40$ minutes. Part (a) asked for a midpoint Riemann sum approximation to the definite integral of the velocity function over this 40-minute time interval. Students had to know that the definite integral represented the total distance flown by the airplane during this time. Part (b) asked for the least number of times at which the acceleration could equal zero during the 40 minutes. It is implicitly assumed that acceleration must be a continuous function. Students had to justify their answers. The easiest way to do this was to use the Mean Value Theorem. Since velocities were the same at times $t = 0$ and $t = 15$, acceleration must be zero somewhere strictly between these times. Similarly, acceleration must be zero between times $t = 25$ and $t = 30$. Part (c) gave a model for the velocity at any time t in the 40-minute interval and asked for acceleration (with units) at time $t = 23$. Part (d) asked for the average velocity, based on the model given in part (c), over the 40-minute interval.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 1 point in part (b), the point in part (c), and all points in part (d). The student earned the first point in part (a) for the values of $v(5) + v(15) + v(25) + v(35)$ and the second point for the correct answer. The third point was not earned because the time interval of 40 minutes was not mentioned. In part (b), the student did not earn the justification point.

Question 4

Students were given the graph of f' over the interval from $x = -1$ to $x = 5$, were told that it has horizontal tangent lines at $x = 1$ and at $x = 3$, that f is twice differentiable, and that $f(2) = 6$. They had to use this information to answer questions about the function f. Part (a) asked for the x-coordinates of the points of inflection of the graph of f with a reason. Students needed to know that the graph of f has a point of inflection where the graph of its derivative changes from increasing to decreasing or decreasing to increasing. Part (b) asked for the x-coordinates of the absolute minimum and maximum values of f on the closed interval $[-1, 5]$. They had to explain how they used the graph of f' to answer this question. Part (c) defined $g(x) = xf(x)$ and asked for the equation of the line tangent to the graph of g at $x = 2$. The value of $f'(2)$ could be read from the graph.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: all points in parts (a) and (c) and 2 points in part (b). In part (b), the student incorrectly identified $x = 5$ as the x-value of the absolute maximum, and thus was ineligible for the justification point.

AP® CALCULUS BC
2004 SCORING COMMENTARY (Form B)

Question 5

This question presented students with a function g that is positive for all $x > 0$ and whose graph is asymptotic to the x-axis. Parts (a), (b), and (c) considered the function over the interval $[1, 4]$ and asked for the average value of g over this interval, the volume of the solid obtained when the region between the graph of the function and the x-axis over this interval is rotated about the x-axis, and the average value of the cross sectional areas of this solid. Part (c) could have been done by simply realizing that the average value of the cross sectional area was the volume divided by length. Part (d) defined the average value of a function over an unbounded interval—the limit as the upper bound approaches infinity of the average values over bounded intervals. It then asked students to show that the integral of g from 4 to infinity is divergent, but that the average value of g over this infinite interval is finite.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 6)

The student earned 6 points: both points in part (a), both points in part (b), and two points in part (d). The student did not include p in part (c). The student gave a "recipe" for the average value in part (d) but did not address the specific problem.

Question 6

This question presented a region bounded above by the graph of $y = x^n$, below by the x-axis, and to the right by the line tangent to the graph of $y = x^n$ at $(1, 1)$. The problem was to find the value of $n > 1$ that maximizes this area. Part (a) asked students to find the total area under the curve, as a function of n, from $x = 0$ to $x = 1$. Part (b) asked students to show that the area of the triangle formed by the tangent line, the x-axis, and the line $x = 1$ was $\frac{1}{2n}$. Students could use the fact that the slope of the tangent line is n to conclude that the base has length $\frac{1}{n}$. Because the height was 1, the area was $\frac{1}{2n}$. Part (c) asked for the area of the region in question, expressed as a function of n. This was $\frac{1}{n+1} - \frac{1}{2n}$. Students were then asked for the value of $n > 1$ that maximizes this function. The problem was easiest if the terms were not combined over a common denominator, before differentiating.

<u>Sample B</u> (Score 9)

The student earned all 9 points. In part (c), the student chose the correct n based on the fact that it produced a positive area, rather than showing that only one solution satisfies the criteria $n > 1$.

<u>Sample D</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), and 3 points in part (c). The student did not earn the third point in part (b) because of incorrect algebra in the last line at the bottom of the first page. In part (c), the second and third points are awarded for correctly setting the numerator of the derivative equal to 0. The student did not earn the fourth point in part (c).