AP® Calculus BC (Operational)
2004 Sample Student Responses

NO CALCULATOR ALLOWED

Work for problem 5(a)

If $P(0) = 3$, $\lim_{t\to\infty} P(t) =$ 12

If $P(0) = 20$, $\lim_{t\to\infty} P(t) =$ 12

$\frac{dP}{dt} = \frac{P}{60}(12 - P)$

$P(t) = \frac{12}{1 + Ce^{-t/5}}$

$P(0) = \frac{12}{1+C} = 3$

$C =$

$P(t) = \frac{12}{1+3e^{-t/5}}$

$P(0) = \frac{12}{1+C} = 20$

$C = -.4$

$P(t) = \frac{12}{1-.4e^{-t/5}}$

Work for problem 5(b)

$\frac{1}{2}$ max population is when pop. grows fastest.

and $\lim_{t\to\infty} P(t) = 12$ o.o.

$\frac{12}{2} =$ (6)

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Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\int \frac{dY}{Y} = \int \left(\frac{1}{5} - \frac{t}{60}\right) dt$$

$$\ln Y = \frac{t}{5} - \frac{t^2}{120} + C$$

$$Y = Ce^{\left(\frac{t}{5} - \frac{t^2}{120}\right)}$$

$$Y(0) = Ce^0 = 3$$

$$C = 3$$

$$Y(t) = 3e^{\left(\frac{t}{5} - \frac{t^2}{120}\right)}$$

Work for problem 5(d)

$$\lim_{t \to \infty} 3e^{\left(\frac{t}{5} - \frac{t^2}{120}\right)} = 0$$

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Do not write beyond this border.

NO CALCULATOR ALLOWED

C.1

Work for problem 5(a)

If $P(0) = 3$, $\lim_{t\to\infty} P(t) =$ 12

If $P(0) = 20$, $\lim_{t\to\infty} P(t) =$ 12

The population will reach an equilibrium when $1 - \frac{P}{12} = 0$, i.e., when $P = 12$

Work for problem 5(b)

$\frac{P}{5}\left(1-\frac{P}{12}\right) = \frac{P}{5} - \frac{P^2}{60}$ reaches its maximum when

$\frac{1}{5} - \frac{P}{30} = 0 \quad P = 6$

$\therefore P = 6$ //

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

C_2

Work for problem 5(c)

$\frac{5}{Y}\,dY = \left(1+\frac{t}{12}\right)dt$

$\int \frac{5}{Y}\,dY = \int \left(1+\frac{t}{12}\right)dt$

$(Y>0)\quad 5\ln Y = t + \frac{t^2}{6} + C$

$\ln Y = \frac{t}{5} + \frac{t^2}{30} + C$

$Y = Ce^{\frac{t}{5}}e^{\frac{t^2}{30}}$

$Y(0) = C = 3$

$\therefore\ Y(t) = 3e^{\frac{t}{5}}\cdot e^{\frac{t^2}{30}}$

Work for problem 5(d)

$\lim_{t\to\infty} Y(t) = \infty$

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