CollegeBoard
Advanced Placement Program

AP® Calculus BC
2004 Scoring Commentary

Question 1

This question gave students a function that defined the rate of traffic flow in terms of cars per minute at a given time t. In part (a), students had to use the definite integral to find the total number of cars that pass through the intersection in a given time period. In part (b), students had to use the derivative to determine whether the traffic flow was increasing or decreasing at time $t = 7$. Parts (c) and (d) tested whether students understood the difference between the average value of a function and the average rate of change of a function. It was important to use the correct units: cars per minute in part (c) and cars per minute per minute (or cars/min^2) in part (d).

Sample A (Score 9)

The student earned all 9 points. In parts (c) and (d), rounding to the nearest whole number was acceptable since some students applied the instructions from part (a) throughout the question.

Sample D (Score 7)

The student earned 7 points: 3 points in part (a), 3 points in part (c), and 1 point in part (d). In part (b), although the student considered $t = 7$, the student computed $F(7)$ instead of $F'(7)$. The units are incorrect in both parts (c) and (d).

Question 2

This question gave two functions whose graphs intersected at $x = 0$ and $x = 1$. In part (a), students were asked to find the area bounded by these graphs. In part (b), students had to calculate the volume of the solid formed by revolving this region about the horizontal line with equation $y = 2$, a line that lies above the given region. Part (c) tested the students' ability to set up an integral of a solid with square cross sections that lies over a specified region. The upper bounding curve of the region for this part was given as the function h with $h(x) = kx(1-x)$, where k was an unspecified positive parameter. Students were asked to set up an equation that could be used to find the value of k for which the resulting solid would have volume equal to 15. Students were not asked to find the value of k.

Sample A (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: both points in part (a), all 4 points in part (b), and 1 point in part (c). In part (c), the student earned one point for using the side of the cross sectional square in the integrand. The student did not square the side, losing the second integrand point. The student did not earn the answer point as the integral is multiplied by p and is set equal to an incorrect value.

Question 3

This question presented students with the position at time $t = 2$ and a formula for the rate of change of the x-coordinate of a particle moving in the xy-plane. Part (a) tested the students' ability to set up and use a definite integral to find the x-coordinate of the position at a specified time. Part (b) gave the numerical value of $\frac{dy}{dt}$ at time $t = 2$ and asked students to find the equation of the tangent line. Part (c) asked students to find the speed of the object at time $t = 2.$ Part (d) provided a function of t that described the slope of the line tangent to the path of the particle at $(x(t), y(t)).$ By asking for the acceleration vector, this question tested student ability to use the formulas for the slope and the derivative of x with respect to t to find a formula in terms of t for the derivative of y with respect to $t.$ Students then had to use the derivatives of x and y with respect to t to find the acceleration vector.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: all 3 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). The student did not earn the point for $\frac{dy}{dx}$ in part (b). The student earned the first two points in part (d), but did not earn the last point because of the incorrect statement about acceleration.

Question 4

Students were given an equation relating x and y. In part (a), they needed to know how to use implicit differentiation and the product rule to find an expression for $\frac{dy}{dx}$. In part (b), students needed to know that the tangent line is horizontal when the derivative is zero, and they needed to use this fact to solve for y when $x = 3$. They also needed to check that the point they found lies on the curve defined by the equation. Part (c) required students either to apply implicit differentiation a second time to the original equation or to use the quotient rule with implicit differentiation to find the second derivative of y with respect to x. Students then had to use knowledge of the values of the first and second derivatives to determine whether there was a maximum, minimum, or neither at a specified point on the curve and to justify their answers.

<u>Sample A</u> (Score 9)

The student earned all 9 points. In part (b), the student determined that at $x = 3$, $y = 2$ or $\frac{1}{4}$. The student then used the derivative to determine that at $P = (3, 2)$, the slope was 0 and that the line tangent to the curve at P was horizontal. In part (c), the student used implicit differentiation to find the expression for the second derivative and then evaluated it at point P. The student used the Second Derivative Test to determine that the curve had a local maximum at P.

<u>Sample C</u> (Score 7)

The student earned 7 points: both points in part (a), 2 points in part (b), and 3 points in part (c). In part (b), the student determined that the slope at point P was 0, but did not verify that $P = (3, 2)$ was on the given curve. In part (c), the student earned 2 points by finding the expression for the second derivative of y with respect to x using the quotient rule, but made an error when simplifying the numerical value, losing the third point. The student earned the fourth point by determining that point P was a local maximum on the curve, using the facts that the first derivative was 0 at point P, and that the second derivative was negative at point P.

Question 5

This question tested students' knowledge of the behavior of a solution to a logistic differential equation. Part (a) tested the students' knowledge of the limiting behavior of a logistic function, that if the initial population is positive, then the population will approach the carrying capacity which is the positive root of the quadratic polynomial in P. Even without specific knowledge of the logistic function, students should have been able to read this differential equation. It implied that P was increasing for $0 < P < 12$ and that P was decreasing for $P > 12$. This meant that if the initial population was positive, then P would have to approach or equal 12 as t increased. Part (b) tested student recognition that the derivative of P is greatest at the maximum value of the quadratic polynomial in P which occurs exactly halfway between the two roots. Parts (c) and (d) required solving a separable differential equation with initial condition and determining the long-term behavior of the solution to this differential equation that was superficially similar to, but in fact quite different from the logistic equation.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: both points in part (a), the point in part (b), 3 points in part (c), and the point in part (d). The student earned both points in part (a) for correctly identifying the limits of $P(t)$ for both initial conditions. The student earned one point in part (b) for correctly finding the value of P at which P is increasing most rapidly. The student began part (c) by incorrectly copying the given function (replacing a minus sign with a plus sign). For that reason, the student did not earn the point for separating the variables. The student integrated incorrectly (the 6 should be 24) and thus did not earn the point for correct integration of both sides. The student earned points for including a constant of integration, using the initial conditions, and correctly solving for $Y(t)$. The student earned the point in part (d). The student correctly computed the limit as t approaches infinity for $Y(t)$ using a $Y(t)$ that is consistent with the function the student found in part (c).

Question 6

This question presented a function f that was a composition of a linear and a sine function. Part (a) asked for the third-degree Taylor polynomial of this function about $x = 0$. Part (b) required that students knew how to find the coefficient of an arbitrary term in the Taylor series and how to find the twenty-second derivative of f. Part (c) required students to be able to use the Lagrange Error Bound to bound the error when the third-degree Taylor polynomial was used to approximate the value of f at $x = \frac{1}{10}$. Students could answer part (d) by recognizing that the third-degree Taylor polynomial of the definite integral from 0 to x of f is the definite integral from 0 to x of the second-degree Taylor polynomial of f.

Sample B (Score 9)

The student earned all 9 points.

Sample MM (Score 7)

The student earned 7 points: all 4 points in part (a), only the magnitude point in part (b), the point in part (c), and 1 point in part (d). In part (d), the student lost a point for using "$G(x) =$." Since the polynomial is finite in part (d), the student could have used "$G(x) \approx$" or named the result, for example, $H(x)$.