CollegeBoard
Advanced Placement Program

Student Performance Q&A:

2004 AP® Calculus AB and AP Calculus BC Free-Response Questions

The following comments on the 2004 free-response questions for AP® Calculus AB and AP Calculus BC were written by the Chief Reader, Caren Diefenderfer of Hollins University in Roanoke, Virginia. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop, to learn strategies for improving student performance in specific areas.

AB Question 1/BC Question 1

What was the intent of this question?

This question gave students a function that defined the rate of traffic flow in terms of cars per minute at a given time t. In Part (a), students had to use the definite integral to find the total number of cars that passed through the intersection in a given time period. In Part (b), students had to use the derivative to determine whether the traffic flow was increasing or decreasing at time $t = 7$. Parts (c) and (d) tested whether students understood the difference between the average value of a function and the average rate of change of a function. It was important to use the correct units: cars per minute in Part (c) and cars per minute per minute (or cars/min^2) in Part (d).

How well did students perform on this question?

This was the first free-response problem, and almost all students did work in each part of it. Most students handled the nearest whole number issue in Part (a) successfully. Many used their calculators correctly and appropriately. They recognized that using the calculator was a judicious approach to this problem. There were few "bald" answers (i.e., answers without supporting work). Most students were able to make the distinction between average value and average rate of change in Parts (c) and (d). The fact that Part (b) asked specifically for increasing or decreasing helped students answer this question correctly.

The mean score was 4.89 for the AB students and 6.62 for the BC students (out of a possible nine points). Almost 11 percent of the AB students and over 25 percent of the BC students received a 9, but 13 percent of the AB students earned no points at all—a discouraging result for this standard type of calculus problem.

What were common student errors or omissions?

Students confused average value and average rate of change or did the same work in Parts (c) and (d). Some attempted to solve Parts (a), (b), and (c) using discrete methods. Many students had problems with the units in Parts (c) and (d). Some did not use a calculator and made differentiation and antidifferentiation errors. The same students often made algebra and arithmetic mistakes.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Remind students how to use the graphing calculator effectively and/or appropriately on calculator-active problems. Students need to learn to distinguish between average value and average rate of change. Explain to students that reasoning and justification will continue to be important on the AP Calculus Exams. Students need to write clearly and concisely, and this problem shows that they need to have more experience working with units in problems.

AB Question 2/BC Question 2

What was the intent of this question?

This question gave two functions whose graphs intersected at $x = 0$ and $x = 1$. In Part (a), students were asked to find the area bounded by these graphs. In Part (b), students had to calculate the volume of the solid formed by revolving this region about the horizontal line with equation $y = 2$, a line that lies above the given region. Part (c) tested the students' ability to set up an integral of a solid with square cross sections that lies over a specified region. The upper bounding curve of the region for this part was given as the function h with $h(x) = kx(1 - x)$, where k was an unspecified positive parameter. Students were asked to set up an equation that could be used to find the value of k for which the resulting solid would have volume equal to 15. Students were not asked to find the value of k.

How well did students perform on this question?

Although this was the traditional area/volume problem, the AB students did not seem to score as well as expected. Reasons might include the request to revolve about a line that was not an axis or to find a volume of a different type of solid (square cross sections) in Part (c)—this description required careful reading.

The mean score was 4.09 for the AB students and 5.90 for the BC students (out of a possible nine points). Eleven percent of the AB students and 26 percent of the BC students received a 9, but almost 12 percent of the AB students earned no points at all—a discouraging result for this standard type of calculus problem.

What were common student errors or omissions?

Most students were able to do Part (a). The most common error was integrating the sum of the two functions rather than their difference. This misunderstanding may have occurred because one of the functions was below the x-axis. Parts (b) and (c) were more problematic. Students found rotating a region about an axis that was not a coordinate axis challenging. Unlike in previous years, few students rotated the function about a vertical axis. We believe this was due to the choice to include the words "horizontal line" when describing the axis of rotation in the question. Students had difficulty dealing with the fact that one of the bounding curves was below the x-axis (similar to the Part (a) issue already described). Many had considerable difficulty writing syntactically correct expressions in Part (b). The correct integral included multiple nested parentheses. Students frequently wrote expressions that were not intended, as a result of miscounting or mismatching parentheses; Readers were forced to infer their intentions from other evidence. Part (c) seemed to be the most difficult part of the problem.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

The problem was on the calculator-active part of the exam, but this problem could be done successfully (although tediously) without a calculator—there were a few noncalculator solutions. Writing and entering the correct integrals was problematic for many students. Students well acquainted with AP problems entered the formulas for $f(x)$ and $g(x)$ into their calculators and then referred to $f(x)$ and $g(x)$ in later computations. Different calculators treat missing or mismatched parentheses differently. Teachers could help students by showing them how to enter information into their calculators both efficiently and correctly.

Consciously direct students to express themselves using proper mathematical notation. Students must be reminded that what they write, not what they may *intend* to write, is what will be graded. Continue to have students practice finding the volume of solids formed by rotating a region around a line other than a coordinate axis.

AB Question 3

What was the intent of this question?

This question considered particle motion along a straight line. Students were given a velocity function and an initial position. Part (a) asked for the acceleration at time $t = 2.$ Part (b) tested the students' understanding of the distinction between speed and velocity and their ability to use the values of velocity and acceleration to determine whether the speed was increasing or decreasing at a given time. Part (c) asked students to find and justify a maximum value of the position. For Part (d), students needed to know how to use a definite integral to find the position of the particle at a given time and how to use position and velocity to determine whether the particle was moving toward or away from the origin at a given time.

How well did students perform on this question?

Overall performance on this problem was poor. Students found it difficult to work with a velocity function for which they could not find an explicit position function. The function $v(t) = 1 - \tan^{-1}\left(e^t\right)$ seemed to intimidate many students. The mean score was 2.12 out of a possible nine points. Only 1.3 percent of all students earned a score of 9, and a disappointing 34 percent received no points on this question.

What were common student errors or omissions?

In Part (a), many students were unable to use their calculators to do numerical differentiation. About half tried to do symbolic differentiation, and many were unable to apply the chain rule to calculate $v'(t)$. In Part (b), most students thought that speed was equivalent to velocity. This was especially disappointing since several recent free-response questions were very similar to this one. Students who earned points on this problem usually earned at least two points in Part (c). The justification point for a global argument in Part (c) was rarely earned; most students gave local arguments. Only the strongest students earned points in Part (d), and many students did not know how to get started there. Of the students who *were* able to start the problem, the greatest difficulty was in handling the initial condition.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Have students practice problems that require them to use their calculators to do numerical calculus operations. Students need more exposure to particle motion problems and need to learn the specific connection between speed and velocity. This is clearly a priority on the AP Calculus Exams, and it should be emphasized in the classroom.

Since AP problems demand verbal justifications, students need practice in writing valid justifications. Students are not yet successful at distinguishing between absolute and relative extrema and need more practice working with these ideas.

AB Question 4/BC Question 4

What was the intent of this question?

Students were given an equation relating x and y. In Part (a), they needed to know how to use implicit differentiation and the product rule to find an expression for $\frac{dy}{dx}$. In Part (b), students needed to know that the tangent line is horizontal when the derivative is zero, and they needed to use this fact to solve for y when $x = 3$. They also needed to check that the point they found lies on the curve defined by the equation. Part (c) required students either to apply implicit differentiation a second time to the original equation or to use the quotient rule with implicit differentiation to find the second derivative of y with respect to x. Students then had to use knowledge of the values of the first and second derivatives to determine whether there was a maximum, minimum, or neither at a specified point on the curve and to justify their answers.

How well did students perform on this question?

In general, students performed well. Most could handle calculating the first derivative by implicit differentiation in Part (a) and were able to make progress on Part (b). Part (c) was difficult for many students; some did not realize that one correct approach was to compute the second derivative by using the quotient rule on the result from Part (a).

The mean score was 4.09 for the AB students and 5.70 for the BC students (out of a possible nine points); 2.3 percent of AB students, and 7.4 percent of BC students, earned a score of 9. Fifteen percent of AB students received no points, while only 4 percent of BC students received no points.

What were common student errors or omissions?

The most common error occurred in Part (b) where many students simply forgot to verify that the point they found was on the curve. Those that began by finding all the points on the curve with $x = 3$ often had errors in algebra that lead to incorrect values for y. Many students chose to solve the quadratic equation by factoring and setting each factor equal to 2. Students whose algebra was flawed lost the third point in Part (b).

Students also had difficulty with calculating the symbolic y'' in Part (c). It was clear from their work that many simply had no idea how to go about this calculation. Also in Part (c), a number of students attempted to document the behavior at point P by discussing the signs of velocity and acceleration. They seemed to believe that velocity and acceleration are acceptable synonyms for the first and second derivatives of functions.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Traditional calculus and algebraic manipulations occur on the noncalculator part of the exam. Students who were comfortable with quotient rule derivative problems of explicit functions, as well as implicit differentiation problems, were able to use both of these skills and do well on this problem. Some teachers have found that splitting classroom tests into calculator and noncalculator parts helps students acquire and maintain adequate mechanical and technical skills. It is important to discourage students from attempting justifications based on the use of quick tricks.

AB Question 5

What was the intent of this question?

This was a Fundamental Theorem of Calculus question in which students were given the graph of the function f and asked about the function g defined as the definite integral of f from -3 to x. It was necessary to interpret the graph to answer questions about g, g', and g''. Part (a) asked for the values of $g(0)$ and $g'(0)$. Parts (b) and (c) asked for the locations of the relative maxima and for the absolute minimum value of g and also asked for justifications, testing knowledge of how to use the derivative to find these values and the ability to get this information about the derivative from the graph of f. When justifying the absolute minimum value, it was important for students to explain why each of the other critical values was rejected. Part (d) asked students to find the points of inflection. Students needed to recognize that the second derivative might not exist at a point of inflection.

How well did students perform on this question?

Despite the emphasis on graphical representation of functions within the AP Calculus curriculum for the past several years, and the appearance of questions similar to this one in the last several exams, students performed relatively poorly. Many scores were in the 0–2 range. A number of students confused f and g in one or more parts of the question; many failed to earn one or both justification points due to inadequate or faulty reasoning. Students scoring in the mid-range were most apt to earn both points in Part (d). Among those scoring 6–8 points, the most common errors occurred in Part (c).

The mean score was 2.63 out of a possible nine points. Only 1 percent of students earned a score of 9 on this problem, and almost 28 percent received no points.

What were common student errors or omissions?

Often students confused the graph of f for that of g in several parts of the question. These students scored very few, if any, points.

In Part (a), although most students presented a geometric argument for the computation of $g(0)$, a significant number argued analytically by obtaining and integrating the appropriate linear function. In most cases, they were successful in doing so.

Many students failed to earn the justification point in Part (b) due to inadequate or faulty reasoning. Some students presented true statements about g or g', but without reference to the evidence presented, namely the graph of *f*, these arguments failed to earn the justification point. Others attempted to justify the relative maximum in Part (b) by an accumulation argument, but this approach was rarely successful.

Many students did not earn the justification point in Part (c) due to failure to consider the endpoints of the interval, implying that they were presenting a local argument instead of a global one. A common error in Part (c) was failure to compute the value of g at the x-coordinate of the absolute minimum point. Of those reporting an incorrect value for $g(-4)$, most were at least reporting a negative value, demonstrating that they understood that the area accumulation from -3 to -4 would result in a negative value. Several students who were calculating values in Part (c) mistakenly considered g to be accumulating signed area beginning at $x = -5$ rather than $x = -3$.

A common error in Part (d) was the incorrect inclusion of 0 as a point of inflection, either instead of a correct value or in addition to the correct values. A popular solution, presenting -3, 0, and 2 as the inflection points, received no points. Most students, however, either earned both points or no points in Part (d).

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Continue to provide students with increased opportunities to reason from information presented in graphical form. In particular, students should have increased experience with accumulation functions of the form $\int_a^x f(t)\,dt$ where they must reason about values of x to the left of, as well as to the right of, a. Students should be encouraged to calculate definite integrals by geometric rather than by analytic means when possible.

As was the case with past exams, students often had difficulty distinguishing between a local argument and a global one when dealing with extrema. They need to understand the difference between a relative and an absolute argument and the various means by which a global argument can be made.

For a good number of students the weakest strand of the “Rule of Four” is the verbal strand. This is reflected by their difficulty in deciphering the information given in a question and in recognizing what is being asked. Weakness in verbal communication is especially reflected in many of their written arguments. Often students provided multiple verbal justifications, perhaps hoping that if several arguments were offered, one of them would be good enough to earn some points (whereas, in fact, the opposite is true; one false argument will nullify the sound ones). Many students were successful in constructing a number line argument (although many others failed to label such sign charts, causing the argument to be ineligible for consideration) but then faltered when they attempted to restate the argument verbally. Since sign charts alone will no longer be acceptable as justification beginning with the 2005 AP Calculus Exams, it is vital that students gain significant experience in communicating their reasoning accurately, thoroughly, and succinctly.

AB Question 6

What was the intent of this question?

This question presented students with a differential equation. Part (a) asked them to sketch a slope field, identifying the slopes at 12 specified points. It was not necessary to draw the slopes precisely, but it *was* necessary to distinguish between horizontal, increasing, and decreasing tangent line segments. Part (b) probed understanding of the fact that a slope field only exhibits a small sample of the possible tangent line segments. Students had to use knowledge of the first derivative to determine all points at which the tangent line segments had positive slope. For Part (c), students needed to use separation of variables to solve a differential equation with initial value.

How well did students perform on this question?

Given that this was the first slope field problem on an AB exam, student performance was relatively good. Most students earned points in each of the three parts of this problem. The mean score was 3.50 out of a possible nine points. Only 3.4 percent of students earned a score of 9, and about 25 percent received no points. The large number of students with no points may have been due to the fact that this was the last question on the exam, so some students may have run out of time. It is also possible that some AB students had not been exposed to the topic of slope fields in their classes.

What were common student errors or omissions?

In Part (a), some students forgot to include segments at the three points on the x-axis. In general, the slope fields on some papers were not as precise as they needed to be. Some students lost points because they drew non-linear curves in their slope field, displayed a non-zero slope at $(-1, 1)$, displayed positive slopes at $(-1, 0)$ and/or $(1, 0)$, or had positive slopes decreasing as y increased for $y > 1$. Some forgot to include the condition that x is not zero as the second condition in Part (b), while others wrote a description that considered only the 12 discrete points given in Part (a).

The standard for Part (c) represented what has become the usual approach to grading separable differential equation problems. The typical mistakes for this type of problem occurred. Some students never separated, some had a bad separation, some forgot to include an absolute value in their antiderivative, some forgot the constant of integration, some included a late constant of integration, some had a constant of integration that disappeared, and some were unable to solve for y due to the presence of an exponential function.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Most students seemed to be well prepared for this question, although some indicated that they had not studied slope fields. It is important for teachers to be familiar with the topic outlines in the current *Course Description for AP Calculus* in order to keep up-to-date on current revisions. Changes in the *Course Description* are announced at least one year in advance of new topics being included on an exam; the introduction of slope fields for Calculus AB in 2004 was announced in 2001.

Give students more practice with separable differential equations. Some students could not do the algebra in separating variables. Many were unable to solve for the correct value of the constant of integration due to the presence of the exponential function in this problem. The most frequent errors were due to the lack of algebraic skills.

BC Question 3

What was the intent of this question?

This question presented students with the position at time $t = 2$ and a formula for the rate of change of the x-coordinate of a particle moving in the xy-plane. Part (a) tested students' ability to set up and use a definite integral to find the x-coordinate of the position at a specified time. Part (b) gave the numerical value of $\frac{dy}{dt}$ at time $t = 2$ and asked students to find the equation of the tangent line. Part (c) asked them to find the speed of the object at time $t = 2.$ Part (d) provided a function of t that described the slope of the line tangent to the path of the particle at $(x(t), y(t)).$ By asking for the acceleration vector, this question tested student ability to use the formulas for the slope and the derivative of x with respect to t to find a formula in terms of t for the derivative of y with respect to $t.$ Students then had to use the derivatives of x and y with respect to t to find the acceleration vector.

How well did students perform on this question?

In general, this was a good problem for the students. It asked basic questions about parametric motion, and some students scored well. The mean score was 3.09 out of a possible nine points; 5.2 percent earned a score of 9, and 19.6 percent received no points.

What were common student errors or omissions?

Issues in Part (a) arose when students attempted to solve the problem by using Euler's method to approximate the differential equation $dx = \left(3 + \cos\left(t^2\right)\right) dt.$ This received no points since the student would have to show all the steps, including verification that the solution was accurate to within three digits after the decimal point. In Part (b), students had difficulty putting the information together to calculate $\frac{dy}{dx}$. Many used $-7,$ the value of $\frac{dy}{dt},$ rather than the parametric definition of slope. Surprisingly, many students used the point $(2, 2)$ rather than $(1, 8),$ which was probably due to the reference to time $t = 2.$

In Part (c), many students did not know that speed is the magnitude of the velocity vector. In Part (d), they had difficulty putting together the chain rule to correctly determine $\frac{dy}{dt}$. Poor use of parentheses was an issue. The last point was given for putting the information regarding the second derivatives together, and most students did this well.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Emphasize the importance of the Fundamental Theorem of Integral Calculus. Also, students need to realize that not all continuous functions have closed form antiderivatives. Most students attempted to integrate our function by parts, believing that there had to be a solution. A good starting point for Part (a) would be to have students write a generic accumulation situation: $x(t) = x(a) + \int_a^t \left(3 + \cos\left(w^2\right)\right) dw.$ For Part (b), students need a firm understanding of the parametric representation of slope. Many considered speed as the absolute value of a velocity function, rather than interpreting speed as the magnitude of a velocity vector. Finally, for Part (d), students need to demonstrate algebraic skills when computing the symbolic form of $y'(t)$. The last point indicated that some were not clear on the definition of an acceleration vector.

Student work on this problem showed that many were uncomfortable with the material and need more practice with parametric motion in the *xy*-plane.

BC Question 5

What was the intent of this question?

This question tested students' knowledge of the behavior of a solution to a logistic differential equation. Part (a) tested their knowledge of the limiting behavior of a logistic function: if the initial population is positive, then the population will approach the carrying capacity, which is the positive root of the quadratic polynomial in P. Even without specific knowledge of the logistic function, students should have been able to read this differential equation: it implied that P was increasing for $0 < P < 12$ and that P was decreasing for $P > 12$. This meant that if the initial population was positive, then P would have to approach or equal 12 as t increased. Part (b) tested student recognition that the derivative of P is greatest at the maximum value of the quadratic polynomial in P, which occurs exactly halfway between the two roots. Parts (c) and (d) required solving a separable differential equation with initial condition and determining the long-term behavior of the solution to this differential equation that was superficially similar to, but in fact quite different from, the logistic equation.

How well did students perform on this question?

This question was split graded, with Parts (a) and (b) the BC-only material (a logistic differential equation) and Parts (c) and (d) contributing to the Calculus AB Subscore grade. The mean score on Parts (a) and (b) was a disappointing 0.41 (out of a possible three points), with about 6 percent earning a score of 3 and about 77 percent earning no points. Most Readers felt that students misinterpreted the intent of Parts (a) and (b). The mean score on Parts (c) and (d) was 2.82 (out of a possible six points), with about 12.5 percent earning a score of 6 and about 37 percent earning no points. The total mean score was 3.23 out of a possible nine points.

What were common student errors or omissions?

After the papers were graded, it was clear in Part (a) that students either were not knowledgeable about the behavior of a solution to a logistic growth problem or were not aware that recognition of the carrying capacity was sufficient to answer the problem. Most students tried to solve the given differential equation. Some were able to use partial fractions to obtain a correct solution and then find the limit based on their work; unfortunately, most students using this approach were not able to do so. Some used the concept of carrying capacity but then added on the value of $P(0)$. Others tried to use the form $P = \dfrac{M}{1 + Ae^{-kt}}$ but had trouble. Common incorrect solutions were: $\infty, -\infty$, and 0.

Many students had no work in Part (b). Some tried taking the derivative of the given equation; others used a sign chart with values 0 and 12. The most common incorrect solution was 12. Again, it was obvious that many students were unable to extract information from a logistic growth differential equation.

Part (c) was graded according to the typical differential equations standard. In general, students were comfortable solving this type of problem. In Part (d), students were to find the limit of their solution in Part (c), and most were able to do this.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Remind students to be careful and check their work—some made careless arithmetic errors in Parts (a) and (b) of this problem. Continue to give students practice in solving separable differential equations (see question AB6); in particular, students need to keep practicing algebraic techniques so their solutions will be correct.

BC Question 6

What was the intent of this question?

This question presented a function f that was a composition of a linear and a sine function. Part (a) asked for the third-degree Taylor polynomial of this function about $x = 0.$ Part (b) required that students knew how to find the coefficient of an arbitrary term in the Taylor series and how to find the twenty-second derivative of $f.$ Part (c) required students to be able to use the Lagrange Error Bound to bound the error when the third-degree Taylor polynomial was used to approximate the value of f at $x = \frac{1}{10}.$ Students could answer Part (d) by recognizing that the third-degree Taylor polynomial of the definite integral from 0 to x of f is the definite integral from 0 to x of the second-degree Taylor polynomial of $f.$

How well did students perform on this question?

Most student work indicated experience working with Taylor polynomials, and students did their best work in Part (a). Few students earned both points in Part (b), but most students earned at least one of the two points. Part (c) was the most difficult part of this problem, and few students earned this point. Some used their work from Part (a) to answer Part (d) and others started over. Either approach was acceptable.

The mean score on this problem was 2.77 out of a possible nine points—slightly better than 2.67, the mean on the 2003 BC series problem. Only 0.5 percent earned a score of 9, and almost 43 percent earned no points.

What were common student errors or omissions?

In Part (a), some students forgot to use the chain rule, evaluated the sine and/or cosine functions incorrectly, forgot to include the factorials, differentiated the sine and/or cosine functions incorrectly, forgot to evaluate the derivatives at *a*, or equated $f(x)$ with $P(x).$ Students who missed points in Part (b) either forgot one of the pieces of the coefficient or miscalculated one of the pieces. Many gave no indication of knowing Lagrange's Theorem and were unable to earn the point in Part (c). The most common error among those who attempted this part of the problem was that $\left|f^4(x)\right|$ assumes its maximum value at $x = 0.$ Students lost points in Part (d) because of integration errors and/or including extra terms in their answer; some equated $G(x)$ with a polynomial approximation and also lost points here.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Emphasize that the Taylor polynomial of f about $x = a$ is an approximation to f and usually agrees with $f(x)$ only when $x = a$. Remind students to evaluate the higher derivatives of f at $x = a$ when they compute the coefficients of a Taylor polynomial of f about $x = a$.

Help students cultivate skills in recognizing and describing cyclic patterns. Spending a class period comparing the graphs of well-known functions with the graphs of some of their Taylor polynomial approximations would help students see concretely that Lagrange's theorem gives a method to measure the difference between the approximation and the original function. This graphical comparison could serve as the beginning of an analysis and computation of Lagrange error bounds.

Performance on series problems will improve when students develop a habit of asking of any function, "How does what I now see respond to differentiation and integration? What is preserved? What changes? How does it change?" If questions like these become second nature, approaches to series problems will become natural to students.