CollegeBoard
Advanced Placement Program

AP® Calculus BC
2005 Sample Student Responses
Form B

The College Board: Connecting Students to College Success

The College Board is a not-for-profit membership association whose mission is to connect students to college success and opportunity. Founded in 1900, the association is composed of more than 4,700 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three and a half million students and their parents, 23,000 high schools, and 3,500 colleges through major programs and services in college admissions, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT®, and the Advanced Placement Program® (AP®). The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

Visit the College Board on the Web: www.collegeboard.com.
AP Central is the official online home for the AP Program and Pre-AP: apcentral.collegeboard.com.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\left.\frac{dx}{dt}\right|_{t=2} = 12(2)-3(2)^2 = 12$

$\left.\frac{dy}{dt}\right|_{t=2} = \ln(1+(2-4)^4) = \ln(17)$

$\frac{d^2x}{dt^2} = 12-6t$

$\frac{d^2y}{dt^2} = \frac{4(t-4)^3}{1+(t-4)^4}$

$\left.\frac{d^2x}{dt^2}\right|_{t=2} = 0$

$\left.\frac{d^2y}{dt^2}\right|_{t=2} = -1.882$

$\text{Speed} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$= \sqrt{12^2 + \ln(17)^2}$

$\boxed{= 12.330}$

$\boxed{a_{t=2} = (0, -1.882)}$

Work for problem 1(b)

$y(2) = y(0) + \int_0^2 \ln(1+(t-4)^4)\, dt$

$= 5 + 8.671$

$\boxed{= 13.671}$

Continue problem 1 on page 5.

Work for problem 1(c)

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln(17)}{12} = 0.236$ $P(3, 13.671)$

$y = 0.236(x-3) + 13.671$

Work for problem 1(d)

Speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 0$ when $-\left(\frac{dx}{dt}\right)^2 = \left(\frac{dy}{dt}\right)^2$

$-(12t-3t^2)^2 = (\ln(1+(t-4)^4))^2$

$\boxed{t=4}$

$\therefore$ at $t=4$ the object is at rest.

GO ON TO THE NEXT PAGE.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$\frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d}{dt}(12t - 3t^2) = 12 - 6t$

$\frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}\left(\ln\left(1+(t-4)^4\right)\right) = \frac{4(t-4)^3}{1+(t-4)^4}$

$\therefore (x'(2), y'(2)) = \left(0, \frac{32}{17}\right)$ is acceleration vector

---

Work for problem 1(b)

13.671

$\frac{dy}{dt} = \ln(1+(t-4)^4) \rightarrow y = \int dy = \int \ln(1+(t-4)^4)\,dt$

$\therefore y(2) = y(0) + \int_0^2 \ln(1+(t-4)^4)\,dt$

$\fallingdotseq 5 + 8.671 = 13.671$

Continue problem 1 on page 5.

Work for problem 1(c)

$$y - y(2) = \frac{\frac{dy(2)}{dt}}{\frac{dx(2)}{dt}}(x - x(2))$$

$$\rightarrow y = \frac{\ln 9}{12}(x-3) + 13.671$$

Work for problem 1(d)

$\frac{dx}{dt} = 12t - 3t^2 = 3t(4-t) = 0$ when $t = 0$ or 4

$\frac{dy}{dt} = \ln(1 + (t-4)^4) = 0$ when $t = 4$

$\therefore$ both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are zero when $t = 4$, indicating th both x and y component of velocity is zero, and the the object is at rest

GO ON TO THE NEXT PAGE.

CALCULUS BC
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Acceleration vector : $\langle \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2} \rangle$

$a(t) = \langle -6t+12, \frac{1}{1+(t-4)^4} \rangle$

$a(2) = \langle 0, \frac{1}{17} \rangle$

Speed $= \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}$

$= \sqrt{(12t-3t^2)^2 + (\ln(1+(t-4)^4))^2}$

$x=2 \;\; = \sqrt{16^2 + \ln 17^2}$

$\cong \sqrt{264.027}$

Work for problem 1(b)

$t=0 \quad (-13, 5) \quad t=2 \quad x=3$

$\int \frac{dx}{dt} = -t^3+6t^2+C$

$x(t=0) = 0+0+C = -13$

$x = -t^3+6t^2-13$

$x(t=2) = -8+24-13$

$= 3$

$\int \frac{dy}{dt} = \int \ln(1+(t-4)^4)$

$= 5 + \int_0^2 \ln(1+(t-4)^4)$

$= 10.1833.$

Continue problem 1 on page 5.

Work for problem 1(c)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\ln(1+(t-4)^4)}{12t-3t^2}$$

$P = (3, 10.1833)$
$t=2$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\ln 17}{12}$$

$$P: \ y - 10.1833 = \frac{\ln 17}{12}(x-3)$$

Work for problem 1(d)

$$\frac{dy}{dx} = \frac{\ln(1+(t-4)^4)}{12t-3t^2} = 0$$

GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$W(15) = 131.782$ gal/hour (water pumped into)
$R(15) = 252.872$ gal/hour (water removed)

Therefore, the amount of water is decreasing b/c $R(t) > W(t)$ when $t=15$, meaning that water is being removed at a higher rate than is water being pumped into the tank.

Work for problem 2(b)

$1200 + \int_0^{18} (W(t) - R(t))\,dt$ = total gallons of water in the tank at $t=18$

$= 1200 + 109.788$

$= 1309.79$

≈ 1310 gallons of water.

Continue problem 2 on page 7.

A2

Work for problem 2(c)

$W(t) - R(t) = 0 \rightarrow$ indicates a max or min.

$t = 0, 6.49484, 12.9748$

End points : 0, 18.

$1200 + \int_0^{6.49484} (W(t) - R(t))\,dt = \boxed{525 \text{ gallons}}$

$1200 + \int_0^{12.9748} (W(t) - R(t))\,dt = 1697.44$ gallons

$1200 + \int_0^{0} (W(t) - R(t))\,dt = 1200$ gallons

$1200 + \int_0^{18} (W(t) - R(t))\,dt = 1310$ gallons.

Therefore, the amount of water reaches an absolute minimum when $t = 6.49484$.

Work for problem 2(d)

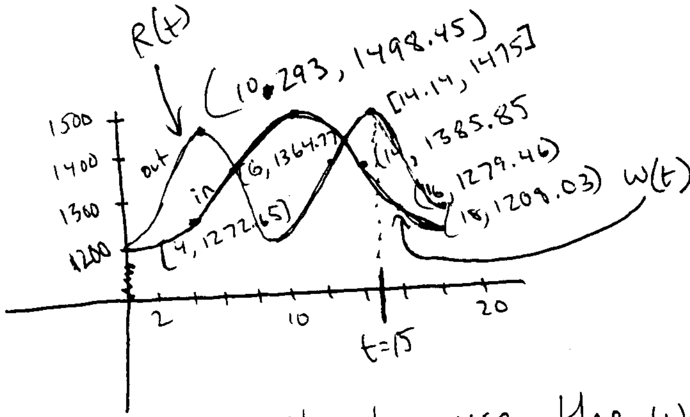
$\rightarrow$ the amount of water, in gallons, when $t = 18$

$1310 - \int_{18}^{k} R(t)\,dt = 0$

GO ON TO THE NEXT PAGE.

B.

Work for problem 2(a)



No, because the water is being removed at a greater rate than it is being pumped in.

Work for problem 2(b)

1200 + gallons in − gallons out

in: $\int_0^{18} 95\sqrt{t}\,\sin^2(t/6)\,dt \approx 2695.46$ gallons

out: $\int_0^{18} 275\sin^2(t/3)\,dt \approx 2585.67$ gallons

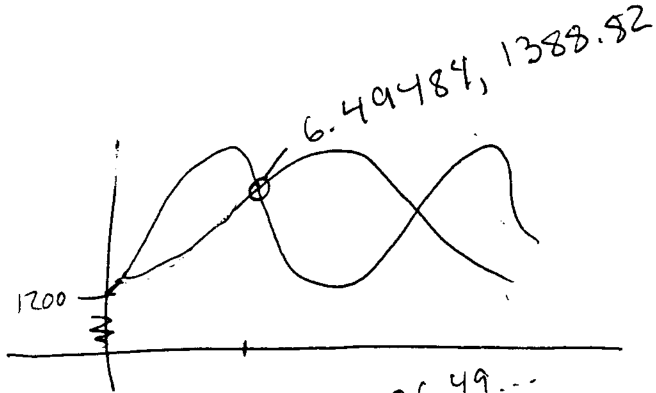
$\approx 1200 + 2695.46 - 2585.67 \approx$

1309.79 gallons

$\approx \boxed{1310 \text{ gallons}}$

Continue problem 2 on page 7.

Work for problem 2(c)



$$1200 + \int_0^{6.49\ldots} W(t) - \int_0^{6.49\ldots} R(t)$$

$= 525.242$ gallons

at time $t = 6.49484$, the greatest amout of water has been removed to the amount pumped in; up to $t = 6.49484$, the water level is decreasing.

Work for problem 2(d)

$$\int_{18}^{k} 275 \sin^2 (t/3)\, dt = 1310 \text{ gal.}$$

(the amount removed from time $t = 18$ to some time k is approximately 1310 gallons)

C 1

2 2 2 2 2 2 2 2 2 2

Work for problem 2(a)

$S(t) = 95\sqrt{t}\sin^2(\frac{1}{6}t) - 275\sin^2(\frac{1}{3}t)$

$S(15) = 95\sqrt{15}\sin^2(\frac{15}{6}) - 275\sin^2(\frac{1}{3}(15)) = -121.09$ gallons

so, no water isn't increasing but decreasing

Work for problem 2(b)

$\int_0^{18} (95\sqrt{t}\sin^2(\frac{1}{6}t) - 275\sin^2(\frac{1}{3}t))dt = \boxed{109.79}$ gallons

Continue problem 2 on page 7.

Work for problem 2(c)

$$95\sqrt{t}\sin^2\left(\tfrac{1}{6}t\right) - 275\sin^2\left(\tfrac{1}{3}t\right) = 0$$

$$95\sqrt{t}\sin^2\left(\tfrac{1}{8}t\right) = 275\sin^2\left(\tfrac{1}{3}t\right)$$

$$\frac{\sin^2\left(\frac{1}{8}t\right)}{\sin^2\left(\frac{1}{3}t\right)} = \frac{275}{95\sqrt{t}}$$

$$\frac{\sqrt{t}\sin^2\left(\frac{1}{6}t\right)}{\sin^2\left(\frac{1}{3}t\right)} = \frac{27}{95}$$

Work for problem 2(d)

$$\int_{18}^{K}\left(275\sin^2\left(\tfrac{t}{3}\right)\right)dt = 0$$

GO ON TO THE NEXT PAGE.

Work for problem 3(a)

since when $x=0$

- $f'(0)=0$

$f''(0) = \frac{(-1)^3(3!)}{5^2} = -\frac{6}{25} < 0$

$f(x)$ has a relative maximum at $x=0$

Work for problem 3(b)

$T(x) \approx f(0) + f'(0)x + f''(0)\frac{x^2}{2} + f'''(0)\frac{x^3}{3!}$

$\approx 6 - \frac{3}{25}x^2 + \frac{1}{125}x^3$

since $f'(0)=0$, $f(0)=6$, $f''(0) = -\frac{6}{25}$ and $f'''(0) = \frac{6}{125}$

Continue problem 3 on page 9.

A

Work for problem 3(c)

f converges absolutely for $\lim\limits_{n\to\infty}\left|\frac{U_{n+1}}{U_n}\right|<1$

$\left|\lim\limits_{n\to\infty}\frac{(n+2)}{5^{n+1}\cdot n^2}\cdot x^{n+1}\cdot\frac{5^n(n-1)^2}{(n+1)}\cdot\frac{1}{x^n}\right|<1$

$\Rightarrow \left|\frac{x}{5}\right|<1$

$-5<x<5$

the radius of convergence is 5

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$$f^{(2)}(0) = \frac{-1 \cdot 3!}{5^2 \cdot 1} < 0.$$

$$f^{(1)}(0) = 0$$

$\therefore$ maybe relative maximum

$(\because f''(0) < 0,\ f'(0) = 0)$

Work for problem 3(b)

$$f(0) + f'(0) \cdot f(0) x + \frac{f''(0) \cdot f(0)}{2!} x^2 + \frac{f'''(0) \cdot f(0)}{3!} \cdot x^3$$

$$6 + 0 + \left(\frac{6 \cdot}{2} \times \frac{-3!}{25} x^2\right) + \left(\frac{6}{3!} \times \frac{4!}{5^3 \cdot 4} \cdot x^3\right)$$

$$= 6 + 0 + \frac{-18}{25} x^2 + \frac{6}{125} x^3$$

Continue problem 3 on page 9.

B.

Work for problem 3(c)

$$\ldots \frac{f^{(n)}(0)}{n!}$$

$$\left(\frac{\dfrac{(-1)^{n+2}\cdot(n+2)!}{5^{n+1}\cdot n^2\cdot(n+1)!}}{\dfrac{(-1)^{n+1}\cdot(n+1)!}{5^{n}\cdot(n-1)^2\cdot n!}}\right)$$

$$= \frac{\cancel{(-1)^{n+1}}^{\,-1}\cdot\cancel{(n+2)!}\cdot\cancel{5^n}\cdot(n-1)^2\cdot\cancel{n!}}{5^{n+1}\cdot n^2\cdot\cancel{(n+1)!}\cdot\cancel{(-1)^{n+1}}\cdot\cancel{(n+1)!}}$$

$$\left|\lim_{n\to\infty} -1\cdot\frac{(n+2)}{(n+1)}\cdot\left(\frac{n-1}{n}\right)^2\cdot\frac{1}{5}\cdot x\right| < 1.$$

$\therefore$ radius : 5.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$f^{(n)}(0)$ is defined at $n \geq 2$

Therefore, $f'(0)$ and $f''(0)$ are not defined

$\therefore$ f has neither a relative maximum nor a relative minimum at $x=0$

Work for problem 3(b)

$$f(x) = \frac{f^{(n)}(0)}{n!} x^n$$

$$= \frac{f^{(2)}(0)}{2!} x^2 + \frac{f^{(3)}(0)}{3!} x^3 \quad (n \geq 2)$$

$$= \frac{\frac{(-1)^3(3!)}{5^2 \cdot 1^2}}{2!} x^2 + \frac{\frac{(-1)^4 \cdot 4!}{5^3(2^2)}}{3!} x^3$$

$$= \frac{(-1)\, 3\cancel{!}}{25 \times \cancel{2!}} x^2 + \frac{1 \times \cancel{4!}}{125 \times \cancel{4 \times 3!}} x^3$$

$$= -\frac{3}{25} x^2 + \frac{1}{125} x^3$$

$$\therefore f(x) = -\frac{3}{25} x^2 + \frac{1}{125} x^3$$

Continue problem 3 on page 9.

 C

Work for problem 3(c)

coefficient :
$$\frac{\frac{(-1)^{n+1}\cdot(n+1)!}{5^n(n-1)^2}}{n!} = \frac{(-1)^{n+1}\cdot(n+1)!}{n!\,5^n(n-1)^2} = \frac{(-1)^{n+1}\cdot(n+1)}{5^n(n-1)^2}$$

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n} = \frac{\frac{(-1)^{n+2}\cdot(n+2)}{5^{n+1}(n^2)}}{\frac{(-1)^{n+1}\cdot(n+1)}{5^n(n-1)^2}} = \frac{(-1)^{n+2}\cdot(n+2)(n-1)^2}{5\,n^2\cdot(-1)^{n+1}\cdot(n+1)}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

A1

NO CALCULATOR ALLOWED

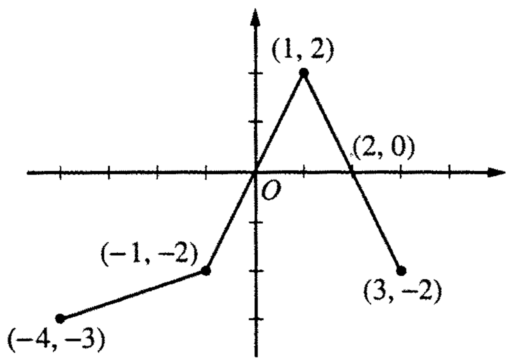
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Vork for problem 4(a)

$g(-1) = \int_{-4}^{-1} f(t)\,dt$

$= \frac{-3+(-2)}{2}(3)$

$= \frac{-15}{2}$

$g'(-1) = f(-1)$

$= -2$

$g''(-1) = f'(-1)$

$g''(-1)$ does not exist

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$g(x)$ has an inflection pt when
$x=1$

$g''(x) = f'(x)$

inflection pt occurs when $f'(x)$ goes from + to − or from − to +
(concavity change). This happens when $x=1$

Work for problem 4(c)

$h(x) = \int_x^3 f(t)\,dt$

$h(x)$ is 0 when $x = 3, 1, -1$

$\int_3^3 f(t)\,dt$, $\int_1^3 f(t)\,dt$, $\int_{-1}^3 f(t)\,dt$ are all zero.

Work for problem 4(d)

$h(x)$ decreases when $0<x<2$.

$h'(x) = -f(t)$ $\left(h(x) = \int_x^3 f(t)\,dt \right)$

$\therefore$ $h'(x)<0$ when $f(t)>0$.

$f(t)>0$ when $0<x<2$.

B1

NO CALCULATOR ALLOWED

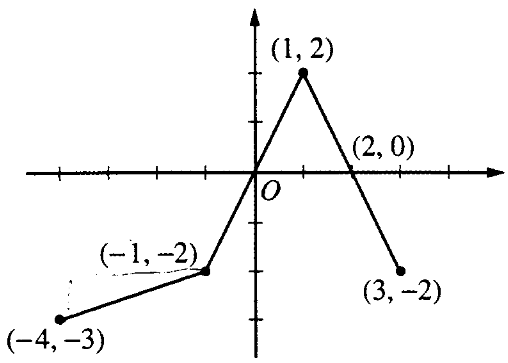
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

Work for problem 4(a)

$g(-1) = \int_{-4}^{-1} f(t)\,dt = -\frac{3}{2} - 6 = -7.5$

$g'(-1) = f(-1) = -2$

$g''(-1)$ does not exist, since $f'(-1)$ does not exis

($f(x)$ is not differentiable at point $x=-1$)

Continue problem 4 on page 11.

B2

NO CALCULATOR ALLOWED

Work for problem 4(b)

$g''(x) = f'(x)$

$g''(x) = 0 \Rightarrow f'(x) = 0$, but there are no points where $f'(x) = 0$ on the interval $(-4;3) \Rightarrow$ there are no points of inflection of function $g(x)$ on the same interval $(-4;3)$

Work for problem 4(c)

There is only ~~one~~ two values of $x \Rightarrow$

$x = 1,\ x = -1$

$h(x) = \int_x^3 f(t)\,dt$

$h(1) = 1 - 1 = 0$, $h(-1) = 1 - 1 + 1 - 1 = 0$

Work for problem 4(d)

$h'(x) = \left(-\int_3^x f(t)\,dt\right)' = -f(x)$

$h'(x) < 0$ for $h(x)$ to decrease $\Rightarrow f(x) > 0 \Rightarrow x \in [0;2]$

GO ON TO THE NEXT PAGE.

C_1

NO CALCULATOR ALLOWED

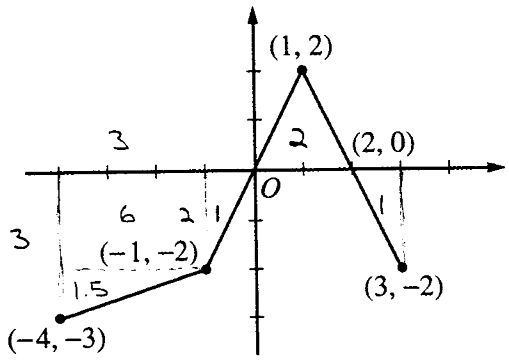
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Graph of f

$g'(x) = f(t)$

$g''(x) = f'(t)$

Work for problem 4(a)

$g(-1) = \int_{-4}^{-1} f(t)\,dt = 7.5$

$g'(-1) = -2$

$g''(-1) =$ does not exist

Continue problem 4 on page 11.

C_2

NO CALCULATOR ALLOWED

Work for problem 4(b)

g(x) experiences a point of inflection where $g''(x) = 0$ $(f'(t) = 0)$, hence it is where f(t) has critical points. $x = 1$

Work for problem 4(c)

$x = -1$, where $h(x) = 0$

Work for problem 4(d)

where $h'(x)$ = negative, hence $(-4, -1)(-1, 0)(2, 3)$.

GO ON TO THE NEXT PAGE.

Work for problem 5(a)

$$2y\frac{dy}{dx} = y + x\frac{dy}{dx}$$

$$(2y - x)\frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

Work for problem 5(b)

$\frac{dy}{dx}$ = slope of the tangent $\qquad \frac{1}{2} = \frac{y}{2y-x}$

$2y = 2y - x \Rightarrow x = 0$

$x=0 \Rightarrow y^2 = 2 \Rightarrow y = +\sqrt{2}$ or $-\sqrt{2}$

$(0, \sqrt{2})$ and $(0, -\sqrt{2})$ are points on the curve where the tangent has slope $\frac{1}{2}$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

~~line~~ line tangent is horizontal $\Rightarrow$ slope of tangent is zero $\Rightarrow \frac{dy}{dx} = 0$

if ~~for~~ $\frac{dy}{dx} = 0$, then $y = 0$

substituting $y = 0$ in $y^2 = 2 + xy$ gives $0 = 2$ which is false.

$\therefore \frac{dy}{dx}$ cannot be zero $\Rightarrow$ there is no point (x, y) where the line tangent to the curve is horizontal

---

Work for problem 5(d)

$$2y \frac{dy}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x$$

$y = 3 \Rightarrow 9 = 2 + 3x \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$

at $t = 5$:

$$2(3)(6) = \frac{dx}{dt}(3) + (6)\left(\frac{7}{3}\right)$$

$$36 = 3\frac{dx}{dt} + 14$$

$$\frac{dx}{dt} = \frac{36 - 14}{3}$$

$$\frac{dx}{dt} = \frac{22}{3}$$

GO ON TO THE NEXT PAGE.

B-1

NO CALCULATOR ALLOWED

Work for problem 5(a)

$y^2 = 2 + xy$

$2y\frac{dy}{dx} = \frac{dx}{dx}y + x\frac{dy}{dx}$

$2y\frac{dy}{dx} = y + x\frac{dy}{dx}$

$2y\frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx}(2y - x) = y$

$\boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$

$\frac{dy}{dx} = \frac{y}{2y-x}$

Work for problem 5(b)

(x, y) $\quad y - y_1 = m(x - x_1)$

$\frac{y}{2y-x} = \frac{1}{2}$

$2y = 2y - x$

$y = y - \frac{1}{2}x$

$y + \frac{1}{2}x = y$

$-y \qquad -y$

$\frac{1}{2}x = 0$

$x = 0$

$y^2 = 2 + xy$

$y^2 = 2 + 0(y)$

$y^2 = 2 + 0$

$y = \pm\sqrt{2}$

$\boxed{(0, \sqrt{2}) \quad (0, -\sqrt{2})}$

Continue problem 5 on page 13.

Work for problem 5(c)

$f'(c) = 0$

$\frac{y}{2y-x} = 0$ $\qquad \frac{y}{2y-x} = 0 \qquad \frac{0}{-x} = 0$

$y = 0(2y-x)$

$y = 0$ can't solve for x at $f'(c) = 0$.

Work for problem 5(d)

$y^2 = 2 + xy$

$2y\frac{dy}{dt} = \frac{dx}{dt}y + \frac{dy}{dt}x$

$y = 3$

$\frac{dy}{dt} = 6$

$t = 5$

$\frac{2y\frac{dy}{dt} - \frac{dy}{dt}x}{y} = \frac{dx}{dt}$

$\frac{2(3)(6) - 6(5)}{3} = \frac{dx}{dt}$

$\frac{36 - 30}{3} = \frac{dx}{dt}$

$\frac{6}{3} = \frac{dx}{dt} = 2$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 5(a)

$y^2 = 2 + xy$

$2y \frac{dy}{dx} = x \frac{\delta y}{dx} + y$

$(2y - x)\frac{\delta y}{dx} = y$

$\frac{dy}{dx} = \frac{y}{2y - x}$

Work for problem 5(b)

$\frac{1}{2} = \frac{y}{2y - x}$

$-x = 2y$

$x = 0$

$+ \quad x = 0, \; y = \sqrt{2})$

$0, \sqrt{2})$

Continue problem 5 on page 13.

 C-2

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

$y^2 = 2 + xy$

$\therefore 2y \neq x$

Work for problem 5(d)

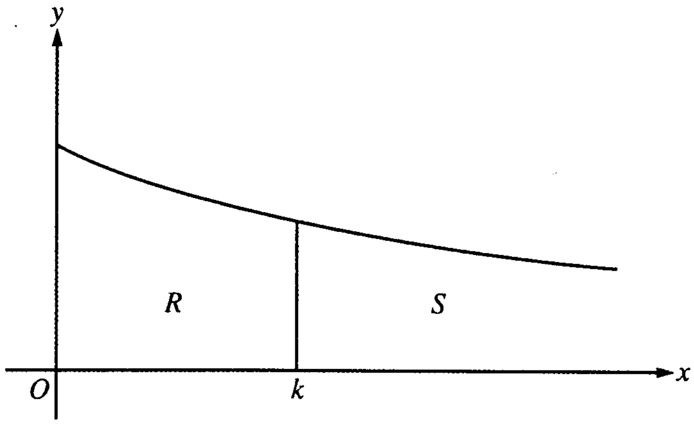
$y^2 = 2 + xy$

$2y\frac{dy}{dt} = x\frac{\delta y}{dt} + y\frac{\delta x}{dt}$

GO ON TO THE NEXT PAGE.

A₁

NO CALCULATOR ALLOWED



Work for problem 6(a)

$$R = \int_0^k \frac{1}{x+2}\,dx$$

$$= \Big[\ln x+2\Big]_0^k$$

$$= \ln(k+2) - \ln 2$$

Work for problem 6(b)

$$V = \int_0^k \pi\left(\frac{1}{x+2}\right)^2 dx$$

$$= \pi\int_0^k \frac{1}{(x+2)^2}\,dx$$

$$= \pi\Big[-(x+2)^{-1}\Big]_0^k$$

$$= \pi\left(\frac{-1}{k+2} + \frac{1}{0+2}\right) = \frac{\pi}{2} - \frac{\pi}{k+2}$$

Continue problem 6 on page 15.

A2

6 6 6 6 6 6 6 6 6 6

NO CALCULATOR ALLOWED

Work for problem 6(c)

First we find an expression for volume S.

$$S = \int_K^{\infty} \pi\left(\frac{1}{x+2}\right)^2 dx$$

$$= \pi\left[\frac{-1}{x+2}\right]_K^{\infty}$$

$$= \pi \lim_{b\to\infty}\left[\frac{-1}{x+2}\right]_K^b$$

$$= \pi\left(\lim_{b\to\infty}\frac{-1}{b+2} + \frac{1}{K+2}\right)$$

$$= \pi\left(0 + \frac{1}{K+2}\right)$$

$$= \frac{\pi}{K+2}$$

we have the equation $\frac{\pi}{2} - \frac{\pi}{K+2} = \frac{\pi}{K+2}$

$$\frac{1}{2} - \frac{1}{K+2} = \frac{1}{K+2}$$

$$\frac{2}{K+2} = \frac{1}{2}$$

$$K+2 = 4$$

$$\boxed{K=2}$$

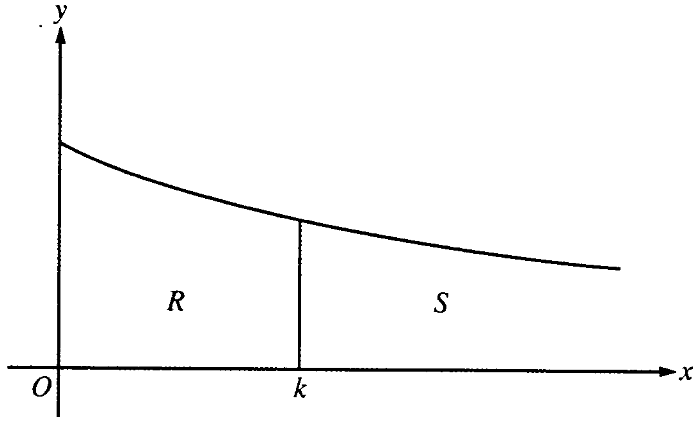
END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

B

NO CALCULATOR ALLOWED



Work for problem 6(a)

$$\text{Area}_R = \int_0^k \frac{1}{x+2}$$

$$A_R = \ln(x+2)\Big]_0^k$$

$$A_R = \ln(k+2) - \ln(2)$$

Work for problem 6(b)

using answer frompart A

$$\text{Volume}_R = \pi \int_0^k \left(\frac{1}{x+2}\right)^2$$

$$V_R = \frac{\pi}{3}\left(\frac{1}{x+2}\right)^3 \frac{1}{\ln(x+2)}\Big]_0^k$$

$$V_R = \frac{\pi}{3}\left(\frac{1}{k+2}\right)^3 \frac{1}{\ln(k+2)} - \frac{\pi}{3}\left(\frac{1}{2}\right)^3 \frac{1}{\ln(2)}$$

Continue problem 6 on page 15.

B 2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\cancel{\pi}\int_0^K \left(\frac{1}{x+2}\right)^2 = \cancel{\pi}\int_K^\infty \left(\frac{1}{x+2}\right)^2$$

use t for ∞

$$\int_0^K \left(\frac{1}{x+2}\right)^2 = \int_K^t \left(\frac{1}{x+2}\right)^2$$

$$\frac{1}{3}\left(\frac{1}{x+2}\right)^3 \frac{1}{\ln(x+2)}\Bigg]_0^K = \frac{1}{3}\left(\frac{1}{x+2}\right)^3 \frac{1}{\ln(x+2)}\Bigg]_K^t$$

$$\frac{1}{3}\left(\frac{1}{K+2}\right)^3 \frac{1}{\ln(x+2)} - \frac{1}{24\ln(2)} = \frac{1}{3}\left(\frac{1}{t+2}\right)^3 \frac{1}{\ln(t+2)} - \frac{1}{3}\left(\frac{1}{K+2}\right)^3 \frac{1}{\ln(K+2)}$$

$$\frac{2}{3}\left(\frac{1}{K+2}\right)\frac{1}{\ln(K+2)} = \underbrace{\cancel{\frac{1}{3}\left(\frac{1}{t+2}\right)^3 \frac{1}{\ln(t+2)}}}_{\text{reduces to 0 as } t \to \infty} + \frac{1}{24\ln(2)}$$

$$\frac{1}{\ln(K+2)}\left(\frac{1}{K+2}\right) = \frac{3}{48\ln(2)}$$

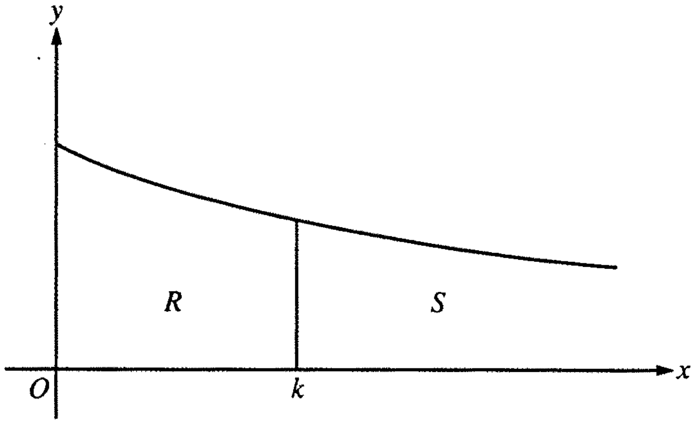
END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMS YOU HAVE TAKEN THIS YEAR.**

C1

NO CALCULATOR ALLOWED



Work for problem 6(a)

$= \int_0^K f(x)\,dx \rightarrow \int_0^K \left(\frac{1}{x+2}\right) dx$

Work for problem 6(b)

$V_R = \int_0^K dA\,dx \quad A = \pi r^2 \quad r = \frac{1}{x+2}$

$dA = \pi \left(\frac{1}{x+2}\right)^2 dx$

$V_R = \pi \int_0^K \left(\frac{1}{x+2}\right)^2 dx$

Continue problem 6 on page 15.

C2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$V_S = V_R$

$V_R = \pi \int_0^K \left(\frac{1}{x+2}\right)^2 dx$

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.**
- **MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**