CollegeBoard
Advanced Placement Program

AP® Calculus BC
2005 Sample Student Responses

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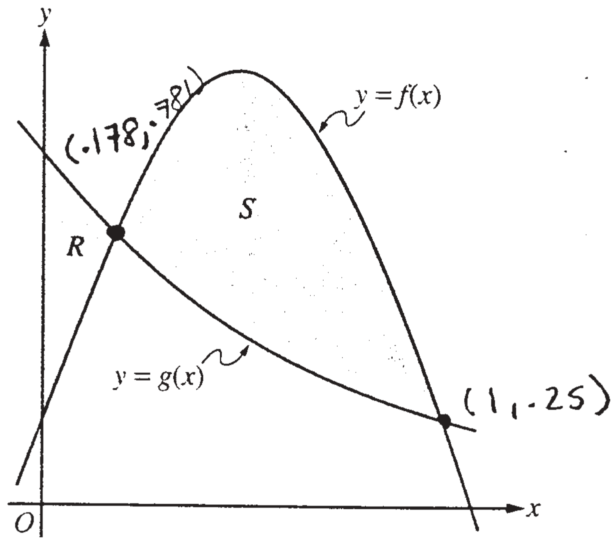
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$f(x) = \frac{1}{4} + \sin(\pi x)$

$g(x) = 4^{-x}$

$$A_R = \int_0^{.178} g(x) - f(x)\, dx$$

$$A_R = \int_0^{.178} 4^{-x} - \frac{1}{4} - \sin(\pi x)\, dx$$

$$\boxed{A_R = .0648\ u^2}$$

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Continue problem 1 on page 5.

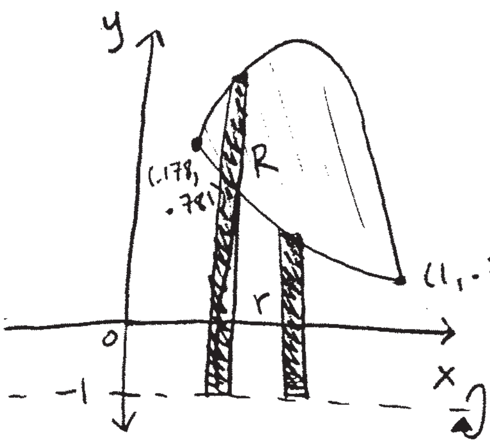
Work for problem 1(b)

$$A_S = \int_{.178}^{1} f(x) - g(x)\,dx$$

$$A_S = \int_{.176}^{1} \tfrac{1}{4} + \sin(\pi x) - 4^{-x}\,dx$$

$$\boxed{A_S = .410\,u^2}$$

Work for problem 1(c)



$$R = f(x) - -1$$

$$r = g(x) - -1$$

$$V_S = \int_{.178}^{1} \pi R^2 - \pi r^2\,dx$$

$$V_S = \int_{.178}^{1} \pi\left(\tfrac{1}{4} + \sin(\pi x) + 1\right)^2 - \pi\left(4^{-x} + 1\right)^2\,dx$$

$$\boxed{V_S = 4.559\,u^3}$$

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B

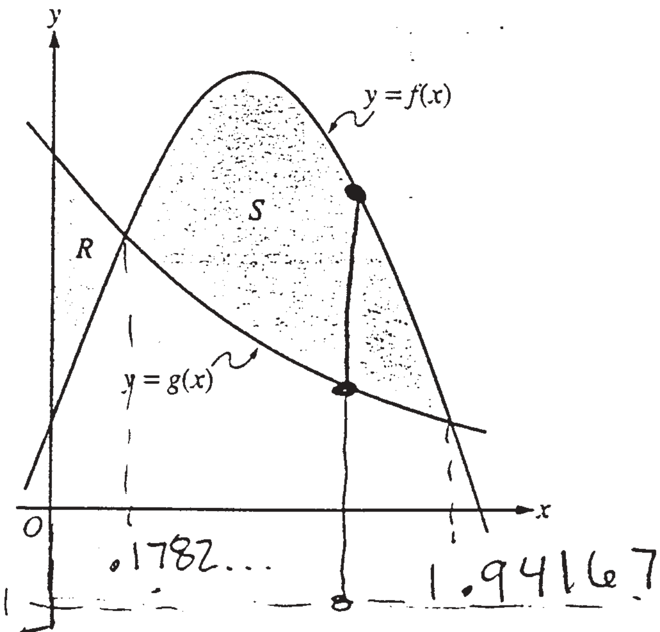
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



rk for problem 1(a)

$$\int_0^{.17821865} g(x) - f(x)\,dx$$

$$= .065$$

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Continue problem 1 on page 5.

Work for problem 1(b)

$$\int_{.17821805}^{1.94167} f(x) - g(x)\, dx$$

$$= .117$$

Work for problem 1(c)

$$\int_{.17821805}^{1.94167} (-1-f(x))^2 - (-1-g(x))^2\, dx$$

$$= .618$$

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C

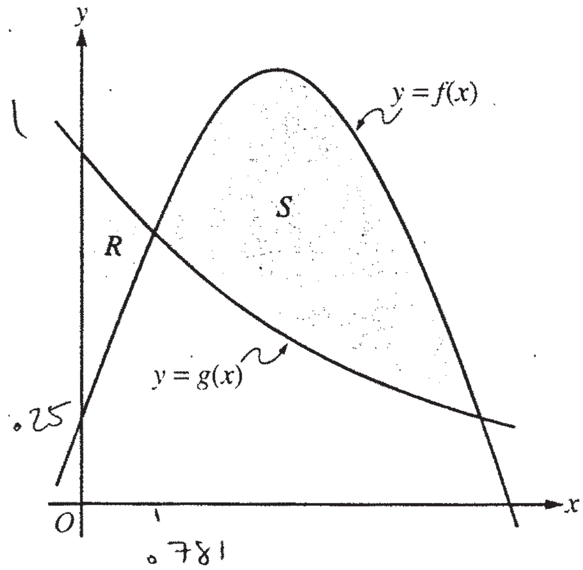
CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\int_0^{.781} 4^{-x} - \left[\frac{1}{4} + \sin(\pi x)\right] dx$$

$\boxed{\text{Area of } R = -.2824 \text{ units}^2}$

via TI-83

intersect
.781

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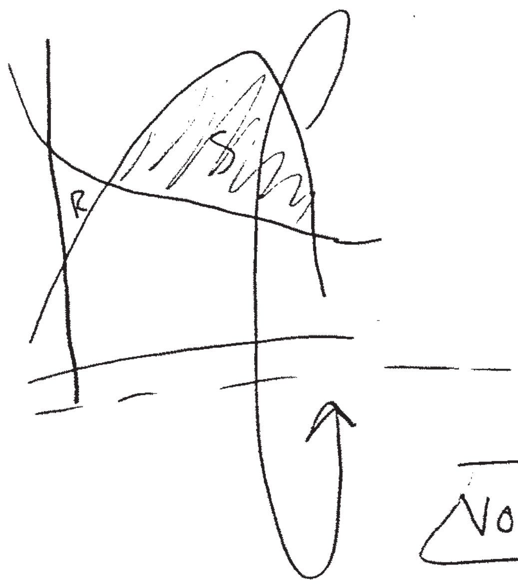
Continue problem 1 on page 5.

Work for problem 1(b)

$$\int_{.781}^{1} f(x) - g(x) = \int_{.781}^{1} \left[\frac{1}{4} + \sin(\pi x)\right] - 4^{-x}$$

Area of S = 0.0632 units² ← via TI-83

Work for problem 1(c)



$$\pi \int_{-1}^{1} [f(x)]^2 - [g(x)]^2 \, dx$$

$$\pi \int_{-1}^{1} \left(\frac{1}{4} \sin(\pi x)\right)^2 - 4^{-2x} \, dx$$

Volume = -14.5243 units³

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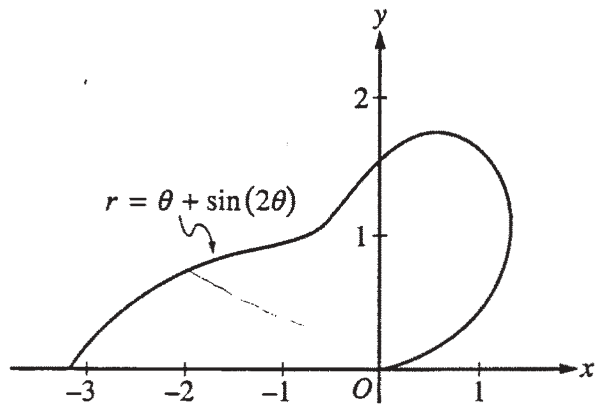
A1

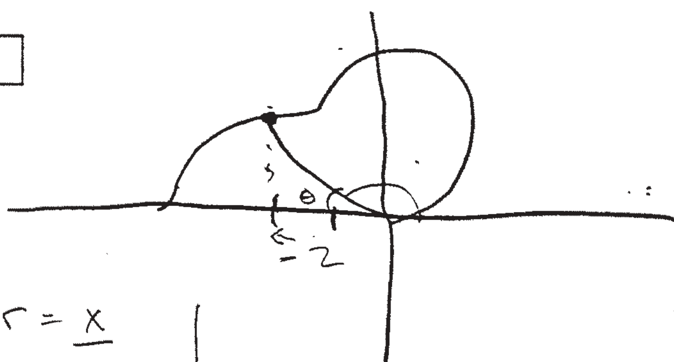


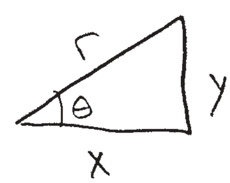
Work for problem 2(a)

Polar Area $= \frac{1}{2}\int r^2\, d\theta$ $\qquad r^2 = (\theta + \sin(2\theta))^2$

$$\frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2\, d\theta = \underline{4.382 \text{ units}^2}$$

Work for problem 2(b)





$\theta + \sin 2\theta = r = \dfrac{x}{\cos\theta}$ $\qquad x = -2$

$r\cos\theta = x$

$r = \dfrac{x}{\cos\theta}$

$\theta + \sin 2\theta = \dfrac{-2}{\cos\theta}$

$\theta = \underline{2.786}$ radians

2.786

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Continue problem 2 on page 7.

A2

Work for problem 2(c)

This means that the length of r is decreasing w/ respect to θ. Therefore the distance of the graph from the origin is getting smaller from $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$.

Work for problem 2(d)

trying to maximize r

$r(\theta) = \theta + \sin(2\theta)$

$r'(\theta) = 1 + 2\cos(2\theta)$

$r'(\theta) = 0$ when $\theta = \pi/3$

$\cos(2\theta) = -1/2$

$2\theta = 2\pi/3$

$\theta = \frac{2\pi}{6} = \frac{\pi}{3}$

$\boxed{\frac{\pi}{3}}$ rad

r: inc. | dec.

r': pos | $\pi/3$ | neg

since r is increasing from $0 < \theta < \pi/3$ since $r'(\theta)$ @ that interval is positive

&

since r is decreasing @ after $\frac{\pi}{2} > \theta > \pi/3$ since $r'(\theta)$ @ that interval is negative

$\boxed{\text{when } \theta = \pi/3,\ r \text{ is max}}$ ∴ there is a max at $\boxed{\theta = \pi/3}$

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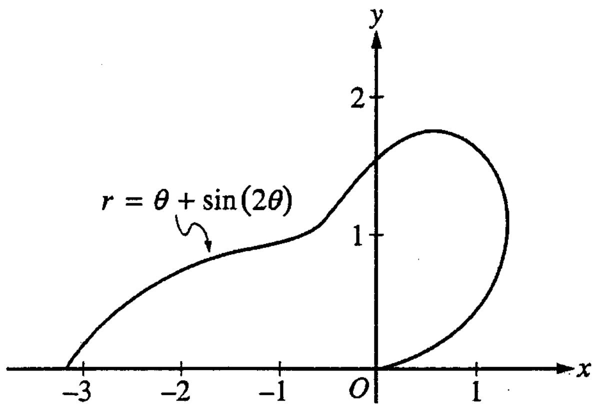

Work for problem 2(a)

The shown area is from $0 \le \theta \le \pi$, so the area is given by:

$$\text{area} = \int_0^{\pi} \frac{1}{2}\left(\theta + \sin(2\theta)\right)^2 d\theta = \boxed{4.382}$$

Work for problem 2(b)

$x = r\cos\theta$

$= (\theta + \sin(2\theta))(\cos\theta)$

Therefore, if $x = -2$,

$-2 = (\theta + \sin(2\theta))(\cos\theta)$

$\therefore \theta = \boxed{2.786}$

Continue problem 2 on page 7.

Work for problem 2(c)

Since from $\frac{\pi}{3} < \theta < \frac{2\pi}{3}$ $\frac{dr}{d\theta} < 0$, r is steadily decreasing in that range. Since this range is between 0 and π, a range in which a constant-r function will be concave down, and since this decreases even more than that does because r is itself diminishing, the curve is concave down in that range.

Work for problem 2(d)

If a point has the greatest distance from the origin, r must be at a maximum. For $r = \theta + \sin(2\theta)$ from $0 \le \theta \le \frac{\pi}{2}$, r reaches a maximum when θ is $\boxed{1.047}$. At that angle, the curve is thus its farthest from the origin in the first quadrant.

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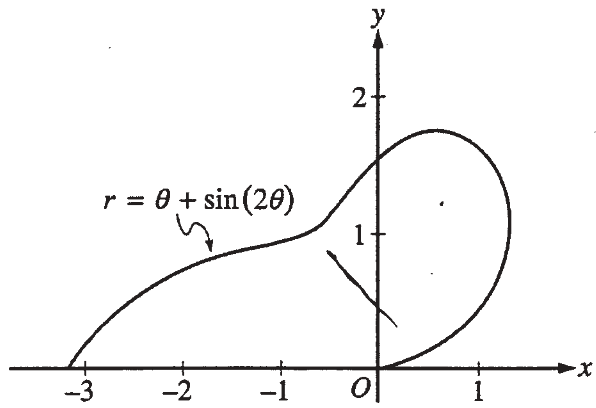

Work for problem 2(a)

$$A = \frac{1}{2}\int_0^{\pi} (\theta + \sin(2\theta))^2 \, d\theta$$

$$= 8.765$$

Work for problem 2(b)

$$x = r\cos\theta$$

$$-2 = r\cos\theta$$

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Continue problem 2 on page 7.

Work for problem 2(c)

This says that as θ increases during the interval from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$ the length of r is decreasing.

It says the curve is bending towards itself more. Because r is decreasing, the curve is forming a tighter loop which also decreases area.

Work for problem 2(d)

$r = \theta + \sin(2\theta)$

$\frac{dr}{d\theta} = 1 + 2\cos(2\theta)$

$0 = 1 + 2\cos(2\theta)$

$\theta = 1.047$

1.047

$\frac{dr}{d\theta}$ + —

A maximum occurs at $\theta = 1.047$ because $\frac{dr}{d\theta}$ changes from positive to negative.

A max. value of R occurs at $\theta = 1.047$

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3 3 3 3 3 3 3 3 3 3

A

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$T'(7) = \frac{55-62}{8-6} = -3.5$ °C/cm

Work for problem 3(b)

Average Temp $= \frac{1}{8}\int_0^8 T(x)\,dx$.

Average $= \frac{1}{8}\cdot\Big[(100+93)(1)(\tfrac{1}{2}) + (93+70)(4)(\tfrac{1}{2})$
$+ (62+70)(1)(\tfrac{1}{2}) + (55+62)(2)(\tfrac{1}{2})\Big]$

$= 75.688$ °C

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Continue problem 3 on page 9.

A

Work for problem 3(c)

$$\int_0^8 T'(x)\,dx = T(8) - T(0)$$
$$= 55 - 100$$
$$= -45\,^{\circ}C$$

$\int_0^8 T'(x)\,dx$ mean the total change (drop) in temperature of the wire from 0 cm to 8 cm.

Work for problem 3(d)

$T''(x) > 0 \Rightarrow T'(x)$ is increasing over the period.

from $x = 0$ to 1
slope $\Rightarrow -7$

$x = 1$ to 5
slope $\Rightarrow \frac{70-93}{5-1} = -5.75$

$x = 5$ to 6
slope $\Rightarrow \frac{62-70}{6-5} = -8.$

$x = 6$ to 8
slope $\Rightarrow \frac{55-62}{8-6} = -3.5$

By MVT
$\therefore$ between 5 to 6 there is a point with slope -8 which means a decrease of $T'(x)$
$\Downarrow$
$T'(x)$ is not always increasing

$\therefore T''(x) > 0$ is not consistent in the table data.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

B

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$$T'(7) \cong \frac{T(8)-T(6)}{8-6}$$

$$\cong \frac{55-62}{2}$$

$$\cong -\frac{7}{2}\ °C/cm$$

Work for problem 3(b)

$$\text{Avg.} = \frac{1}{8}\int_0^8 T(x)\,dx$$

$$\cong \frac{1}{8}\left[\frac{1}{2}(T(0)+T(1)) + \frac{1}{2}(T(1)+T(5)) + \frac{1}{2}(T(5)+T(6)) + \frac{1}{2}(T(6)+T(8))\right]$$

$$\cong \frac{1}{8}\left[\frac{1}{2}(100+93) + \frac{1}{2}(93+70) + \frac{1}{2}(70+62) + \frac{1}{2}(62+55)\right]$$

$$\cong 37.813\ °C$$

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Continue problem 3 on page 9.

B

Work for problem 3(c)

$\int_0^8 T'(x)\,dx = T(8) - T(0)$

$= 55 - 100$

$= -45\,^\circ C$

This is the total change in temperature of the wire, from one end to the other.

Work for problem 3(d)

$T'(0.5) \cong \dfrac{T(1) - T(0)}{1 - 0}$

$\cong -7.00$

$T'(3) \cong \dfrac{T(5) - T(1)}{5 - 1}$

$\cong -5.750$

$T'(7) \cong -3.500$

The table is consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$, since $T'(x)$ is increasing.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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C

Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ (°C)	100	93	70	62	55

Work for problem 3(a)

$$T'(7) \approx \frac{T(8) - T(6)}{8-6}$$

$$\approx \frac{55-62}{2}$$

$$\approx -7/2 \text{ °C/cm}$$

Work for problem 3(b)

$$\text{Avg } T = \frac{\int_0^8 T(x)\,dx}{8}$$

$$\int_0^8 T(x)\,dx \approx \frac{b-a}{2n}\left[f(0) + 2f(1) + 2f(5) + 2f(6) + f(8)\right]$$

$$\approx \frac{8}{2(4)}\left[100 + 2(93) + 2(70) + 2(62) + 55\right]$$

$$\approx 605$$

$$\frac{\int_0^8 T(x)\,dx}{8} \approx \frac{605}{8} = \boxed{75.625\text{ °C}}$$

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Continue problem 3 on page 9.

C

Work for problem 3(c)

$\int_0^8 T'(x)dx = T(x)\Big]_0^8$

$= T(8) - T(0)$

$= 55 - 100$

$= -45°C/cm$

$\int_0^8 T'(x)dx$ represents the average rate of change of the temperature of the wire as x increases from 0 to 8.

Work for problem 3(d)

$T''(x) > 0$ implies that $T(x)$ is concave up, or that the rate of change is increasing.

The data in the table do not show that the rate of change is increasing from $x=0$ to $x=8$. For example: from $x=0$ to $x=1$, $T(x)$ decreases 7°C. In order for $T(x)$ to be concave up, it must decrease by less than 7°C/cm from $x=1$ to $x=5$: $\frac{T(5)-T(1)}{4} = -5.75°C$.

$T(x)$ decreases by 5.75°C, which less than 7°C, so it is changing at an increasing rate.

Therefore $T(x)$ is concave up and $T''(x) > 0$ is true for $x = 0$ to $x = 5$

$\frac{T(6)-T(5)}{1} = -10 \leftarrow$ This is not consistent however, so $T(x)$ is not concave up for all x in $0 < x < 8$.

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

A

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

CALCULUS BC

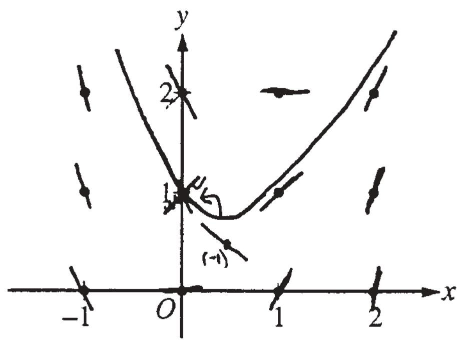
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)



Work for problem 4(b)

Local extrema: $\frac{dy}{dx}=0$

$0=2\ln\left(\frac{3}{2}\right)-y$

$\boxed{y=2\ln\left(\frac{3}{2}\right)}$

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Continue problem 4 on page 11.

Work for problem 4(c)

(x, y)	Δx	$\Delta y = \Delta x \cdot (2x - y)$	next (x, y)
$(0, 1)$	-0.2	$-0.2(-1) = 0.2$	$(-0.2, 1.2)$
$(-0.2, 1.2)$	-0.2	$-0.2(-0.4-1.2) = 0.32$	$(-0.4, 1.52)$

$\downarrow$

$f(-0.4) = \boxed{1.52}$

Work for problem 4(d)

$$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - 2x + y$$

x negative & y positive for values in question;

$\frac{d^2y}{dx^2}$ positive, graph concave up

The estimation is less than the actual value. The slopes used were greater than the actual slopes; when working from right to left, this means a lesser value.

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B

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

CALCULUS BC

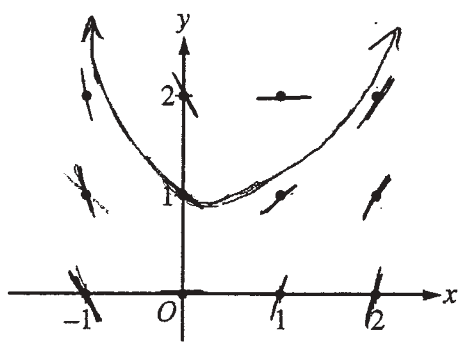
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)



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Work for problem 4(b)

$\frac{dy}{dx} = 2(\ln(\frac{3}{2})) - y$

$2(\ln(\frac{3}{2})) - y = 0$

$-y = -2(\ln(\frac{3}{2}))$

$y = 2(\ln(\frac{3}{2}))$

Continue problem 4 on page 11.

Work for problem 4(c)

$\Delta x = -.2$

$(0,1)$ $\frac{dy}{dx} = 2x-y$ $\Delta y = -1(-.2) = .2$

$= 2(0)-1$

$= -1$

$(-.2, 1.2)$ $2(-.2)-1.2$ $\Delta y = .8(-.2) = \begin{array}{r} -.2 \\ \underline{\times .8} \\ -.16 \end{array}$

$\begin{array}{r} 2 \\ \underline{\times .2} \\ -.4 \end{array}$

$-.4-1.2$

$\begin{array}{r} .1.2 \\ \underline{-.4} \\ .8 \end{array}$

$\begin{array}{r} 1.2 \\ \underline{+ .16} \\ 1.04 \end{array}$

$(-.4, 1.04)$

$f(-0.4) = 1.04$

Work for problem 4(d)

$$\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}$$

$$\frac{d^2y}{dx^2} = 2-(2x-y)$$

$$\frac{d^2y}{dx^2} = 2-2x-y$$

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NO CALCULATOR ALLOWED

CALCULUS BC

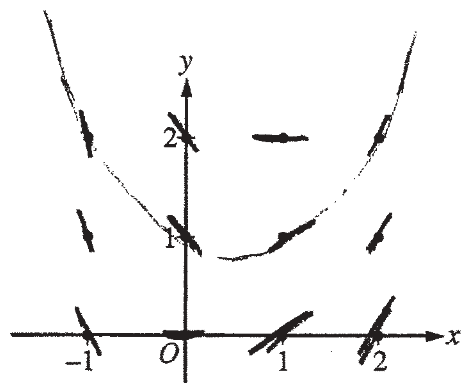
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)



Work for problem 4(b)

$\frac{dy}{dx} = 2x - y$

$\int \frac{1}{-y}\, dy = \int 2x\, dx$

$-\ln y = x^2 + C$

$-\ln 1 = 0 + C$

$C = -\ln 1$

$-\ln y = x^2 - \ln 1$

$-\ln y = (\ln \frac{3}{2})^2 - \ln 1$

$-\ln y = \ln \frac{\frac{9}{4}}{1}$

$-\frac{9}{4} = y$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(c)

x	y	dx	dy
0	1	-.2	.2
-.2	1.2	-.2	.32
-.4	1.52		

$2x - y = \frac{dy}{dx}$

$2(0) - 1 = \frac{dy}{-.2}$

$2(-.2) - 1.2 = \frac{dy}{-.2}$

$dx \cdot 2x - y = dy$

1.52

Work for problem 4(d)

$\frac{d^2y}{dx^2} = 2 - y'$

$\frac{d^2y}{dx^2} = 2 - 2x - y$

$y' = 2x - y$

$\frac{d^2y}{dx^2}$ + | +

-.4

less because the graph is concave up for (-.4, 0) ~~that means when~~ it is estimated from the tangent line at 0, it'll be less than actual

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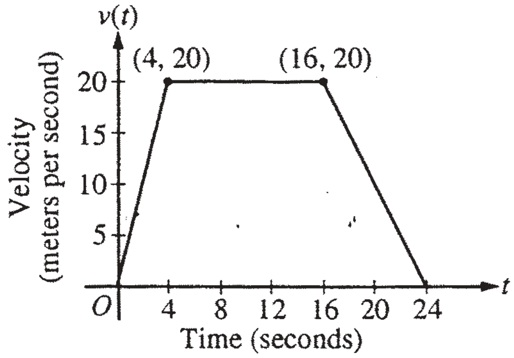
A

NO CALCULATOR ALLOWED



Work for problem 5(a)

$\int_0^{24} v(t)\,dt = \frac{1}{2}(12+24)(20) = \boxed{360 \text{ meters}}$

$\int_0^{24} v(t)\,dt$ is the displacement ~~of the~~ car in meters from time $t=0$ seconds to $t=24$ seconds - since the integral is positive, the car is 360 meters in the positive direction at time 24 seconds as compared with its position at time 0 seconds

Work for problem 5(b)

$v'(4)$ does not exist because $\lim_{x\to 4^-} v'(x) = 5 \neq 0 = \lim_{x \to 4^+} v'(x)$

$v'(20) = \frac{-20}{8} = \boxed{\frac{-5}{2} \frac{\text{meters}}{\text{second}^2}}$

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Continue problem 5 on page 13.

Work for problem 5(c)

$$a(t) = \begin{cases} 5 \text{ m/s}^2 \text{ for } 0 \le t < 4 \text{ seconds} \\ 0 \text{ m/s}^2 \text{ for } 4 < t < 16 \text{ seconds} \\ -5/2 \text{ m/s}^2 \text{ for } 16 < t \le 24 \text{ seconds} \end{cases}$$

* a(t) undefined for t = 4, 16 seconds

Work for problem 5(d)

$$\frac{10-20}{20-8} = \frac{-10}{12} = \boxed{\frac{-5}{6}\ \frac{m}{s^2}}$$

The Mean Value Theorem does NOT guarantee a value for c, $8<c<20$, such that $v'(c)$ equals this average rate of change because $v(t)$ does not fulfill the requirements for the Mean Value Theorem since $v(t)$ is not differentiable on the interval $(8, 20)$ since $v'(t)$ is undefined at $t = 16$ seconds

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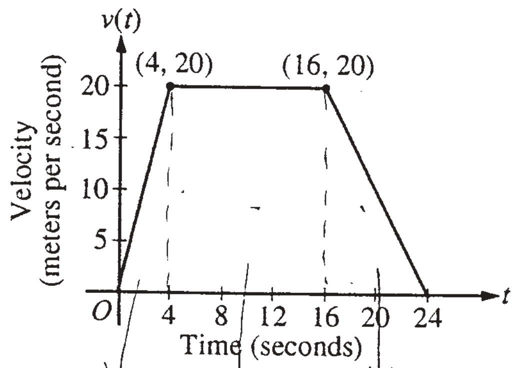

Work for problem 5(a)

$\frac{1}{2}(4)(20) + 12(20) + \frac{1}{2}(8)(20)$

$40 + 240 + 80 = 360 = \int_0^{24} v(t)\,dt$

$\int_0^{24} v(t)\,dt = 360$ meters which is the total distance the car traveled from $t=0$ to $t=24$.

Work for problem 5(b)

The derivative of v at $t=4$ does not exist because it is located at a corner.

$v'(20) = \frac{20-0}{16-24} = \frac{20}{-8} = -\frac{5}{2}\ \frac{\text{meters}}{\text{sec}}$

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Continue problem 5 on page 13.

Work for problem 5(c)

slope of $v(t)$ from $t=0$ to $t=4$: 5
slope " $t=4$ to $t=16$: 0
slope " $t=16$ to $t=24$: $-\frac{5}{2}$

$$a(t) = \begin{cases} 5 , & 0 \le x \le 4 \\ 0 , & 4 < x \le 16 \\ -5/2 , & 16 < x \le 24 \end{cases}$$

Work for problem 5(d)

The Mean Value Theorem does not guarantee this because $v(t)$ is not differentiable over $8 \le t \le 20$.

$$\cancel{\frac{\int [v(20) - v(8)] dv}{20-8} = \frac{\int (10-20) dv}{12} = \frac{\int -10 \, dv}{12}}$$

$4(20) + 4(20) + \frac{1}{2}(4)(20+10) = 80 + 80 + 60 = 220$

$\frac{220}{12} \frac{\text{meters}}{\text{sec}^2}$

160
60
220

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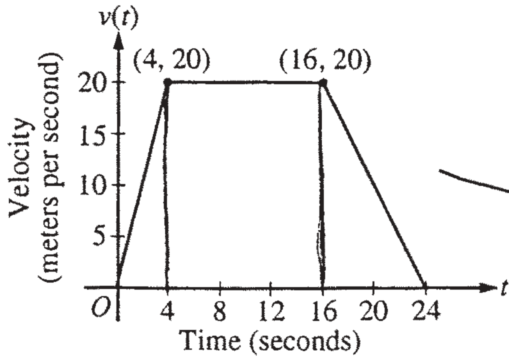
 C

NO CALCULATOR ALLOWED



Work for problem 5(a)

$\int_0^{24} (v(t))\,dt$ is asking for the total distance traveled in these 24 seconds. This is the total Area underneath the graph.

$A = \frac{1}{2}(4)(20) + 12(20) + \frac{1}{2}(8)(20)$

$= 40 + 240 + 80$

$= 360$

Work for problem 5(b)

$v'(t)$ is asking for the slope at a certain point in time. A $t=4$ and $t=20$ the slope is undefined because there is a corner at both of these times.

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Continue problem 5 on page 13.

 C

Work for problem 5(c)

$A(t) = v'(t)$

$$v(t) = \begin{cases} 5x & 0 \le x \le 4 \\ 20 & 4 < x \le 16 \\ -\frac{5}{2}x + 60 & 16 < x \le 24 \end{cases}$$

$$\boxed{A(t) = \begin{cases} 5 & 0 \le x \le 4 \\ 0 & 4 < x \le 16 \\ -\frac{5}{2} & 16 < x \le 24 \end{cases}}$$

$\leftarrow 5$ is the deriv of $5x$

$\leftarrow 0$ is the deriv of 20

$\leftarrow -5/2$ is the deriv of $-5/2\,x + 60$

Work for problem 5(d)

avg rate of chg $= \dfrac{f(b) - f(a)}{b - a} = \dfrac{f(20) - f(8)}{20 - 8} = \dfrac{10 - 20}{20 - 8} = \boxed{-\dfrac{5}{6}}$

• yes C has a garunteed value because $v(t)$ is continuous from $8 \le x \le 20$.

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GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 6(a)

$$\begin{array}{r} 81 \\ \times\ 3 \\ \hline 243 \end{array}$$

$f(2)=7$

$f'(2)=f'''(2)=f^{V}(2)=0$

$^{''}(2)=\frac{(2-1)!}{3^2}=\frac{1}{9}$

$^{IV}(2)=\frac{(4-1)!}{3^4}=\frac{6}{81}=\frac{2}{27}$

$^{VI}(2)=\frac{(6-1)!}{3^6}=\frac{5\cdot4\cdot\cancel{3}\cdot2}{3^{\cancel{6}5}}=\frac{40}{243}$

$$P_6(x)=7+\frac{1/9\,(x-2)^2}{2!}+\frac{2/27\,(x-2)^4}{4!}+\frac{40/243\,(x-2)^6}{6!}$$

Work for problem 6(b)

$$\text{coefficient}=\frac{(2n-1)!}{3^{2n}}\Big/(2n)!=\frac{(2n-1)!}{(3^{2n})(2n)!}=\frac{1}{2n(3^{2n})}$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$1+\sum_{n=1} \frac{(x-2)^{2n}}{2n(9^n)}$$

$$\lim_{n\to\infty} \left|\frac{(x-2)^{\cancel{2n+2}\ 2}}{(2n+2)(9^{n+1})} \cdot \frac{2n(\cancel{9^n})}{\cancel{(x-2)^{2n}}}\right|$$

$$\lim_{n\to\infty} \left|\underbrace{\frac{2n}{2n+2}}_{\downarrow\ 1} \cdot \frac{(x-2)^2}{9}\right|$$

$$\left|\frac{(x-2)^2}{9}\right| < 1$$

$$(x-2)^2 < 9$$

$$-3 < x-2 < 3$$

$$-1 < x < 5$$

If $x=-1$: $\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{2n(\cancel{3^{2n}})} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2n} = \sum \frac{1}{2n}$

$\frac{1}{n}$ = harmonic series ∴ DN

If $x=5$: $\sum_{n=1}^{\infty} \frac{\cancel{3^{2n}}}{2n(\cancel{3^{2n}})} = \sum_{n=1}^{\infty} \frac{1}{2}\cdot\frac{1}{n}$

harmonic series ∴ DIV

END OF EXAM

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No odd terms

Work for problem 6(a)

$$P(x) = 7 + \cancel{(x-2)(f'(2))}^{\,0} + \frac{\frac{1}{9}\cdot(x-2)^2}{2!}$$

$f^{(n)}$

$\frac{6}{81}$

$$P_{6}(x) = 7 + \frac{1!(x-2)^2}{9\cdot 2!} + \frac{3!(x-2)^4}{3^4\cdot 4!} + \frac{5!(x-2)^6}{3^6\, 6!}$$

$\frac{5!}{3^5}$

$$\boxed{P_{6}(x) = 7 + \frac{(x-2)^2}{9\cdot 2} + \frac{(x-2)^4}{3^4\cdot 4} + \frac{(x-2)^6}{3^6\cdot 6}}$$

Work for problem 6(b)

$$\frac{(x-2)^{2n}}{3^{2n}\cdot(2n)} \qquad n \geq 1$$

$$\boxed{\text{Coefficient is } \frac{1}{3^{2n}\cdot(2n)}}$$

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$7 + \sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{3^{2n}\cdot(2n)}$$

$$\lim_{a_n \to \infty} \left| \frac{(x-2)^{2n+2}}{3^{2n+2}\cdot(2n+2)} \cdot \frac{3^{2n}(2n)}{(x-2)^{2n}} \right| =$$

$$\lim_{a_n \to \infty} = \left| \frac{(x-2)^2}{9} \frac{(2n)}{(2n+2} \right| < 1$$

$$-1 < \frac{(x-2)^2}{9} < 1$$

But same has to be 0

$$-9 < (x-2)^2 < 9$$

$$0 < x-2 < 3$$

$$2 < x < 5$$

Left hand $x = 2$

$$\sum_{n=1}^{\infty} \frac{0}{3^{2n}\cdot(2n)} = 0$$

converges

2 inclusive

Right hand $x = 5$

$$\sum_{n=1}^{\infty} \frac{(3)^{\cancel{2n}}}{\cancel{3^{2n}}(2n} \quad \frac{1}{2n}$$

harmonic series diverges

Therefore $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{3^{2n}\cdot 2n}$ has interval of convergence of $[2,5)$

END OF EXAM

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Work for problem 6(a)

$f(2)=7.$

1st, 3rd, 5th derivatives = 0.

$f'(2)=0$ $\quad f''(2)=\frac{1}{9}$

$f'''(2)=0$ $\quad f^4(2)=\frac{3!}{3^4}$

$f^5(2)=0$ $\quad f^6(2)=\frac{5!}{3^6}$

Taylor Series

$$f(x)+f'(x)(x-a)+\frac{f''(x)(x-a)^2}{2!}+\frac{f'''(x)(x-a)^3}{3!}+\ldots$$

$$7+\cancel{0}+\frac{\frac{1}{9}(x-2)^2}{2!}+\cancel{0}+\frac{\frac{3!}{3^4}(x-2)^4}{4!}+\cancel{0}+\frac{\frac{5!}{3^6}(x-2)^6}{6!\rightarrow 6(5!)}$$

$$\boxed{=7+\frac{1}{18}(x-2)^2+\frac{(x-2)^4}{3^4\cdot 4}+\frac{(x-2)^6}{3^6\cdot 6}}$$

Work for problem 6(b)

general term for Taylor series about $x=2$

$$\frac{(x-2)^{2n}}{3^{2n}\cdot n}$$

The coefficient of $(x-2)^{2n}$ for $n\geq 1$ is 0 when n is odd and $\frac{1}{3^{2n}\cdot n}$ when n is even and $n\geq 2$.

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Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

Interval of convergence using Ratio test

$$\lim_{n\to\infty} \left| \frac{(x-2)^{(2n+2)}}{3^{(2n+2)}\cdot(n-1)} \cdot \frac{3^{2n}\cdot n}{(x-2)^{2n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(x-2)^2 n}{3^{2n}(n+1)} \right| = \lim_{n\to\infty} \Big) (x-2)^2$$

*as $n \to \infty$, the

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END OF EXAM

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