AP® Calculus BC
2005 Scoring Commentary

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Question 1

Overview

This problem gives two functions whose graphs intersect at $x \approx 0.178218$ and at $x = 1$. Students have to find these first two points of intersection to the right of the y-axis. In part (a) students have to find the area of the region bounded by the y-axis and the two graphs. In part (b) they have to find the area bounded by the two graphs between the first two points of intersection. Part (c) tests whether students can set up and then evaluate the definite integral that represents the volume of the solid of revolution obtained when the region, whose area is found in part (b), is revolved about the horizontal line $y = -1$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 3 points in part (a), 1 point in part (b), and 2 points in part (c). The work in part (a) is correct. In part (b) the upper limit in the integral is incorrect, so the limits point was not earned. The integrand in part (b) is correct, but the student was not eligible for the answer point because the limits point was not earned. In part (c) the student has a correct integrand and earned two points. Even though the limits of integration are consistent with those in part (b), the student omits the constant π and did not earn the answer point.

Sample: 1C
Score: 4

The student earned 1 point in part (a) and 3 points in part (b). In part (a) the student earned only the integrand point for a correct integrand. The student has incorrect limits of integration; the upper limit is the y-coordinate of the point of intersection rather than the x-coordinate. The student was already penalized for using 0.781 as the x-coordinate of the first point of intersection and so is allowed to use this value in part (b). The answer to part (b) is consistent with the student's limits, so the student earned all 3 points. In part (c) the student's integrand does not reflect rotation about the horizontal line $y = -1$; it is set up as if the region were being rotated about the x-axis. This simplifies the problem so the student could not earn the two integrand points. Since the integrand points were not earned, it was not possible for the student to earn the answer point. Note also the incorrect limits of integration and the negative answer for the volume.

Question 2

Overview

Students are given a curve described in polar coordinates with r as a function of θ. To avoid an unfair advantage for students with calculators that have a computer algebra system, the derivative of r with respect to θ is also given. Part (a) tests the ability to find the area of a region bounded by a curve described in polar coordinates. Part (b) tests the ability to convert between rectangular and polar coordinates, giving the x-coordinate of a point on the curve and asking for its angle θ. In part (c) students are asked to explain what the sign of $\frac{dr}{d\theta}$ tells them about the behavior of the curve. Students are expected to recognize that when this derivative is negative, the curve is getting closer to the origin. Part (d) asks students to find the $\theta-$coordinate of the point on the curve farthest from the origin, testing whether they can recognize that this must happen at a point where $\frac{dr}{d\theta}$ changes from positive to negative.

Sample: 2A
Score: 9

The student earned all 9 points. Note that reference to open intervals in part (d) is acceptable.

Sample: 2B
Score: 6

The student earned 3 points in part (a), 2 points in part (b), and 1 point in part (c). In part (c) the student earned the first point but did not earn the second point because of the statement that the curve is concave down. In part (d) the student did not earn the first point because there is no connection between the first derivative and a critical value. The second point was not earned because there is no justification.

Sample: 2C
Score: 4

The student earned 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student earned the limits and constant point. The integrand and answer points were not earned because a closing parenthesis is missing and the answer is incorrect. In part (b) the student earned the first point by writing a correct equation that uses -2 as the value for x. (It is not necessary to substitute $\theta + \sin\theta$ for r since r is defined in the problem.) In part (c) the student earned the first point but did not earn the second point because of the statement about the curve. In part (d) the student earned the first point but did not earn the second point because the justification does not reference the required interval.

Question 3

Overview

This problem presents students with a table of the temperatures, $T(x)$, of a wire taken at intervals of unequal lengths, where x is the distance from the heated end of the wire. Part (a) asks for an estimation of the derivative of T at a point midway between two of the table values. It is expected that students will use the average rate of change of T over this interval to approximate the derivative. Part (b) requires setting up an integral for the average temperature and using all of the information in the table to find a trapezoidal approximation of the integral over the length of the wire. Stating correct units for average temperature is an important part of this problem. Part (c) tests student knowledge of the Fundamental Theorem of Calculus. Students have to recognize $\int_0^8 T'(x)\,dx$ as the total change in the value of T between the endpoints at $x = 0$ and at $x = 8$. Part (d) asks whether the tabular data are consistent with a strictly positive second derivative at each point between $x = 0$ and $x = 8$. One way to do this is to find average rates of change over successive intervals of unequal lengths and to recognize that since these average rates of change are not increasing, the second derivative cannot be positive on the entire interval. Ideally, students should invoke the Mean Value Theorem to make the connection between the average rate of change and the value of the first derivative at some point in the interval.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 1 point in part (a), 1 point in part (b), 2 points in part (c), 1 point in part (d), and the units point. In part (b) the student earned the first point for correctly setting up the integral. No points were awarded for the trapezoidal sum or answer, since the student assumes intervals of equal length instead of using the data from the table. In part (d) the first point was earned for two correct slopes of secant lines. The second point was not earned. The student skips over the interval from 5 to 6, which shows that T'' decreased.

Sample: 3C
Score: 4

The student earned 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). The student did not earn the units point since part (c) includes $°\text{C/cm}$. In part (b) the student earned the first point but does not use the data to set up intervals of the correct lengths for the trapezoidal approximation and thus did not earn the setup or answer points. In part (c) the student earned the first point but did not receive credit for the incorrect explanation involving average rate of change. In part (d) the student earned the first point with "decreases 7" and "-5.75," but since there is an incorrect slope of -10, the answer point was not earned.

Question 4

Overview

Students are given a differential equation and in part (a) are asked to sketch its slope field. In part (b) they need to show how to use the differential equation to find the minimum value of the solution that has its minimum at $x = \ln\left(\frac{3}{2}\right)$. Part (c) asks students to demonstrate their knowledge of Euler's method by approximating the value of f at $x = -0.4$, where f is the solution that satisfies $f(0) = 1$. Part (d) asks students to find the second derivative of y with respect to x and to use it to determine, with explanation, whether the estimate obtained from Euler's method is necessarily too low or too high.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 3 points in part (a), 2 points in part (b), and 1 point in part (c). In part (a) the student includes a correct slope field and solution curve and earned all 3 points. Part (b) is correct. In part (c) the student uses a table for Euler's method with an increment of x as -0.2 but makes an algebraic error in the second step so did not earn the answer point. In part (d) the student did not earn any points. The student makes an algebraic error in computing the second derivative and does not include a condition on y in the reasoning.

Sample: 4C
Score: 4

The student earned 2 points in part (a) and 2 points in part (c). In part (a) the student earned points for the slope field but did not earn the point for the solution curve because it does not pass through $(0, 1)$. In part (b) the student does not set the derivative equal to 0 and did not earn any points. In part (c) the student correctly constructs a table for Euler's method and earned both points. In part (d) the student did not earn any points. The student makes an algebraic error in computing the second derivative and does not include a condition on y in the reasoning.

Question 5

Overview

Students are given a piecewise-linear graph that models a car's velocity at time t. Testing the ability to connect integrals with area under a graph, part (a) asks students to find the integral of v from $t = 0$ to $t = 24$ and to explain the meaning of this integral, that is to say that it represents the change in position over that time interval. Part (b) asks students to determine values of the derivative of v from the graph and to indicate units of measure. Part (c) asks for a piecewise-defined function that describes the car's acceleration. Part (d) requires students to calculate the average rate of change of v and whether the Mean Value Theorem would guarantee the existence of a point on the interval where the derivative took on that value. In parts (b) and (c) students are led to discover points at which the derivative of v is not defined. This question tests whether students recognize that the Mean Value Theorem does not apply when the derivative of v is not defined over the entire interval.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In part (b) the student does not exhibit correct units of $v'(t)$ and did not earn the third point. In part (c) the student earned the first point for the values of $a(t)$ but not the second point, due to the incorrect treatment at $t = 4$ and at $t = 16.$ In part (d) the student earned the second point by stating which hypothesis of the Mean Value Theorem is not satisfied. Endpoint behavior is not considered relevant unless the student specifically invokes it in the justification.

Sample: 5C
Score: 4

The student earned 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). In part (a) there is no reference to meters, and the student did not earn the second point. In part (b) the student did not earn the point for $v'(20).$ In part (c) the student earned the first point for the values of $a(t)$ but not the second point, due to the incorrect treatment at $t = 4$ and at $t = 16.$ In part (d) units are not necessary since they are not requested in the problem. The student did not earn the second point since the statement regarding the Mean Value Theorem is incorrect.

Question 6

Overview

This problem gives the value of f and of all of its derivatives at $x = 2.$ Part (a) asks for the sixth-degree Taylor polynomial about $x = 2.$ Part (b) asks for the coefficient of the general term in the Taylor series for f. Part (c) asks students to find the interval of convergence of this series with explanation. This requires finding the radius of convergence, testing for convergence at each endpoint, and centering the resulting interval at $x = 2.$

Sample: 6A
Score: 9

The student earned all 9 points.

Sample: 6B
Score: 6

The student earned 3 points in part (a), 1 point in part (b), and 2 points in part (c). Parts (a) and (b) are correct, and the student earned all points. In part (c) the student has the correct series, sets up the ratio correctly, and earned the first point. The student evaluates the limit correctly and earned the second point. The student fails to produce an interval centered at $x = 2,$ did not earn the third point, and was not eligible to earn the other points.

Sample: 6C
Score: 4

The student earned 3 points in part (a) and 1 point in part (c). Part (a) is correct, and the student earned all points. The coefficient produced in part (b) is incorrect, but it is eligible to be used in part (c). In part (c) the student sets up the ratio correctly for the imported series but never computes a limit and never produces an interval centered at $x = 2.$ The student earned only the first point in part (c).